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INTRODUCTORY TALK[†*]

H. Pierre Noyes

Stanford Linear Accelerator Center, Stanford University, Stanford, California, USA

ABSTRACT

The Faddeev formulation of the quantum mechanical three-body problem immediately confronts us with the question of whether we know the two-particle wave function inside the range of forces. Since there is good reason to believe, on general grounds, that the exchange of a pair of pseudo-scalar pions between two nucleons will yield a non-local interaction of range $\hbar/2m_\pi c$, this question must be answered before further progress can be made on the three-nucleon problem. Existing calculations reveal that the gross features of the three-nucleon system can be explained by the non-local single nucleon exchange mechanism, and that so far only the binding energy of the triton and the doublet n-d scattering length can be shown to be sensitive to the detailed physical assumptions about the two-nucleon interaction used in the calculation. It is shown that the three-particle continuum state always contains a long-range non-local effect even when the interactions between pairs are strictly local and of finite range. It is conjectured that this long-range effect might offer an experimental tool for measuring the two-particle wave function inside the range of forces.

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It is a great pleasure to be back in Birmingham again, and to recall my first year here in 1950 as a Fulbright student. At that time I was hoping to extend the phenomenological analysis of "high energy" (32 and 350 MeV) proton-proton scattering that Christian and I had made into the relativistic domain. Like many another physicist before and since, I found no clean way in which to anchor relativistic quantum mechanics in the non-relativistic limit, and had to turn to other problems. Recently, interest in the three-body problem has forced me to re-examine this problem, and has led me to what I believe is a clue to the requirements for the non-relativistic limit of any strong interaction quantum dynamics. Quite simply, I now believe that any theory of a finite number of strongly interacting particles will require a non-local description of the interaction, even though the external energies of the separated particles are small compared to their rest energies. If this is true, the attempt to derive a "local potential" description of nuclear forces has been a mistake of serious dimensions, and may have been a major source of the confusion characterized by Goldberger[1] with the phrase "Scarcely ever has the world of physics owed so little to so many".

That the strong interactions are basically non-local is by no means a novel idea, and there are many ways to arrive at it. However, study of the physics needed to solve any problem involving three strongly interacting particles brings out the underlying non-locality of the interactions in an interesting way. I assume that at this conference and this date, there is no point in deriving the Faddeev equations, and that I can go immediately to the driving terms and kernels which, for a single angular momentum state of a single interacting pair, depend on the two-particle t matrices $t_\ell(p,q;z)$ in the three-particle

Hilbert space. It is also presumably well known[2] that it is unnecessary to start with the full dependence on all three variables, since a knowledge of the half off-shell $t_\ell(p,q;q^2/2\mu) = t_\ell(q,p;q^2/2\mu)$ allows both the full t and the interaction term $V_\ell(p,q) = V_\ell(q,p)$ to be computed from the Low equation $t(z) = V + VG(z)V$. The physical significance of this half off-shell t matrix is that it contains both the scattering phase shift $\delta_\ell\,[t_\ell(k,k;k^2/2\mu) = \exp(i\delta_\ell)\sin\delta_\ell/k]$ and the behavior of the two-particle wave function inside the range of forces. Explicitly, if $w_\ell(kr)$ is the wave function which approaches $n_\ell(kr) - \mathrm{ctg}\,\delta_\ell\, j_\ell(kr)$ asymptotically, then[3]

$$\frac{t_\ell(p,k;k^2/2\mu)}{t_\ell(k,k;k^2/2\mu)} = \left(\frac{p}{k}\right)^\ell + (p^2-k^2)\int_0^\infty r^2 dr\, j_\ell(pr)\, k\left[n_\ell(kr) - \mathrm{ctg}\,\delta_\ell\, j_\ell(kr) - w_\ell(kr)\right]$$

Since two-particle scattering experiments only provide information on $\delta_\ell(k)$ (and only over a finite range of energies at that), the structure of the three-body problem forces us to ask the question of what we know about the wave function inside the range of forces. Attempts have been made to avoid the question by arbitrarily assuming that if some model for the interaction is fitted to the phase shift, and the wave function then computed from that model, the off-shell effects will be unimportant. But this has been proved false. Local models fitted identically either to the same phase shift or to the same bound-state wave function as a separable model give several MeV differences in the binding calculated for the symmetric S state of the triton[4]. The locality of non-locality of the nuclear force is thus a matter of practical significance and not just of theoretical interest.

Direct experimental information on the non-locality of strong interactions is hard to come by. Insofar as the electromagnetic and weak charges and currents follow the strongly interacting particle distribution, electromagnetic and weak form factors of bound two-particle systems do provide a measure of the type we are seeking. However, the way to connect the charge-current to the matter distribution is uncertain for reasons that involve the same physical question of non-locality in the strong interactions, so precise interpretation of these results is frustrated. In particular, the indication[5] that the non-local Feshbach-Lomon wave function for the deuteron gives a better fit to the low energy e-d scattering data than the local Partovi wave function is controversial because of these (and other) uncertainties. In other instances as well, uncertainties in the theoretical interpretation set in at just about the same level as the effects expected from local-nonlocal differences.

Although information from elastic scattering in a single two-particle state gives no way to discriminate local from non-local interactions, if we assume (arbitrarily) that the same interaction is responsible for scattering in several different angular momentum states some progress is possible. The only case I know of with sufficient data to make the test is that of the singlet-even two-nucleon states. Since this system has no bound states, the Gelfand-Levitan theorem asserts that knowledge of the $^1S_0$ phase shift at all energies would uniquely determine the local potential responsible also for scattering in $^1D_2$, $^1G_4$.... states. Of course, phase shifts above 400 MeV are not reliable (or even real), but we do know that the longest range interaction in the system is due to one pion exchange (OPE). Fixing this, and fitting the intermediate range attraction and short range repulsion to the scattering length, effective range, and zero in the phase shift near 250 MeV, we find[6] that the $^1D_2$ and $^1G_4$ phase shifts are in fact uniquely predicted over the same energy range, as illustrated in fig. 1. Thus the lack of knowledge of whether to use an infinitely repulsive hard core, or a soft (Yukawa) core with the ω mass, does not prevent a prediction of these higher phases from knowledge of the $^1S_0$ phase, <u>if</u> the interaction is local. The figure also shows that this prediction fails at several energies by several standard deviations, conclusively disproving the assumption of locality for the singlet-even two-nucleon interaction. One might distrust this simple calculation, but after two years of strenuous efforts to find a local potential with an OPE tail which would fit both $^1S_0$ and $^1D_2$ phases, Reid[7] was forced to conclude that it is impossible. But disproof of locality dies not tell us much about the structure of the non-locality. For instance, the sign and approximate magnitude of the discrepancy shown here is predicted as a velocity-dependent effect of vector meson exchange, but this explanation is obviously not unique.

Turning to theory, we encounter either controversy or lack of clarity in the available results. The successful extraction of the local e^2/r potential from quantum electrodynamics (once the infrared and untraviolet problems were manipulated away) stimulated the search for similar results in meson theory. The corresponding approximation in meson theory does give the longest-range part of the potential (OPEP), and if the pion were scalar, the fact that $f^2/\hbar c = 0.08$ might have allowed a useful local approximation for nuclear forces; after all, non-local effects in QED (vacuum polarization, anomalous magnetic moment, Lamb shift, ...) come in only in order $(e^2/\hbar c)^2$ and higher, and can be ignored for many problems. However, the emission and absorption of two pseudoscalar pions goes primarily through nucleon-antinucleon states, and brings in $G^2 = (2M_n/m_\pi)^2 f^2 = 14.64$ as the basic coupling constant. This means that in any region less than $\hbar/2m_\pi c$ from a nucleon, we are likely to find the nucleon in this nucleon-antinucleon pair, and since this is indistinguishable from the original nucleon, it is impossible to localize a nucleon within $\hbar/2m_\pi c = 0.7$ fm. The conclusion seems inescapable on general grounds, and tells us immediately that the two-nucleon interaction must be intrinsically non-local over regions of this size. A number of theoretical calculations have tried to show this over the years, the latest I know of being that of Hussein Partovi[8]. Starting with Breit and Bouricius, the non-local boundary condition model has, with various refinements, been the basis of successful phenomenologies. Yet somehow, the idea that any theory of the strong interactions should start (rather than end) with a non-local framework has not won wide acceptance. To speculate on why this idea has been resisted would take us too far away from our task at hand.

Regardless of whether or not the elementary particle interactions are non-local, any three- (and a fotiriori any multi-) particle quantum mechanical strongly interacting system will exhibit significant non-local effects. This is easy to show for the specific case of the three-nucleon system, since it has by now been amply demonstrated that the basic driving mechanism for that system is the non-local single nucleon exchange process. The superficially apparent cause of this non-locality for this specific mechanism is the large size of the deuteron coupled with the identity of two of the particles. However, a simple example developed below shows that for any system with local two-particle interactions of strictly finite range, and whether or not there is a two-particle bound state or identity of particles, there must be a long-range non-local effect of this type. That analysis will also illustrate how three-body continuum final states might be used as a tool for measuring non-locality in the two-particle subsystems, but unfortunately the last part of the analysis is still incomplete.

Rather than give a historical account of the growing awareness of the dominant importance of single nucleon exchange in the three-nucleon system, I will start with my current understanding of the situation — which has been derived primarily from useful discussions with A. C. Phillips — and bring in references to earlier work at the points hwere they seem most relevant. The most striking qualitative feature of low energy n-d scattering is the strong energy variation of the doublet effective range function k ctg δ_2 just above elastic scattering threshold. The experimental situation a couple of years ago[9] is illustrated in fig. 2, and the latest data bearing on this point will be discussed by Seagrave later in this conference. Phenomenologically, this behavior can be accurately described by a pole in k ctg δ_2 (a zero in T) lying just below elastic scattering threshold; the possible existence of this pole was pointed out long ago by Gammel, Baker and Delves[10]. If such a phenomenological model is properly fitted to the triton pole, a_2, and the effective range parameter which gives approximately the right variation to k ctg δ_2 above threshold, then a ghost pole appears in T between the triton pole and threshold. No simple potential model will reproduce this behavior. If a potential is used which gives the correct binding energy to the triton (treated as an n-d bound state) and a virtual state above elastic threshold adjusted so as to give a zero in T just below threshold, then a_2 had the wrong sign. It has sometimes been assumed, by myself[2] as well as others, that the zero in T is in fact just this necessary zero between a bound and a virtual state, and that this interpretation was supported by the existence of two bound states of three bosons, the upper one of which would become virtual and the inclusion of spin. That this "explanation" is too naive is clear from the wrong sign simple models of it predict for a_2.

All these facts, and much more, fall into place when we examine the singularity structure of the n-d elastic amplitude which is shown in fig. 3. As noted by Phillips and Barton[11], by Reiner[12], and by Blankenbecler, Goldberger and Halpern[13], by far the nearest singularity to elastic threshold is single nucleon exchange. For the quartet state, this corresponds to a long-range repulsive interaction (due to the exclusion principle acting between the two neutrons), and can be approximated by the simple effective range formula using a repulsive pole near threshold and a short range (large negative k^2) pole to account for the effective range. Following Phillips and Barton[11], we use the deuteron binding energy as the unit, by taking $z = E/\epsilon_d = 3k^2/4M\epsilon_d$; the single nucleon exchange cut then lies between $z = -3$ and $-1/3$, and breakup threshold at $z = +1$. Fitting the quartet scattering length and effective range, the repulsive pole falls at $z = -0.5$, near to the beginning of the single nucleon exchange cut, which begins to demonstrate the reasonableness of the model. Since the single nucleon cut in the doublet state is attractive and of half the magnitude of that in the quartet state (as a consequence of the exchange nature of the force and the symmetry of the wave function), the next step is to approximate the cut by a pole at the same position as in the quartet state but with a positive residue of half the magnitude (this is the "ghost" pole mentioned earlier), put in the triton pole, and adjust the position of the short-range pole to fit a_2; the rapid energy variation of k ctg δ_2 is then reproduced.

Rather than approximating the single nucleon exchange cut between $z = -3$ and $z = -1/3$ by a single pole, it is also possible to include it exactly if some assumption is made about the d-(n, p) vertex. The residue at the deuteron pole in the two-nucleon t matrix, which determines the strength of this cut, is just the square of the asymptotic normalization of the deuteron wave function. For forces of zero range, $N^2 = -2\gamma = -2M\epsilon_d/\hbar^2$, and in the shape-independent approximation $N^2 = -2\gamma(1-\gamma r_t)$, where $r_t = 2(1-1/\gamma a_t)/\gamma$ is the triplet effective range. The latter approximation is still in a sense zero range, since it gives no momentum dependence to the vertex; for instance, the Hulthén wave function and the Yamaguchi separable potential wave function would give an additional $(\beta^2 + k^2)^{-2}$ structure to the vertex. Ignoring this structure, Barton and Phillips[14] find that the solution of the N/D equations for the quartet state, using only single nucleon exchange and elastic unitarity, gives $a_4 = 6.3$ fm as compared with the experimental value of 6.13 ± 0.04 fm which will be reported by Seagrave here. This works for the quartet state primarily because the single nucleon exchange interaction is repulsive. For the doublet state, the long range attractive interaction due to single nucleon exchange concentrates the wave function at shorter range and makes the calculation sensitive to shorter range forces, a sensitivity which is excaberated by the pole in k ctn δ_2. Therefore, for the doublet state, the experimental value of a_2 is used as a subtraction constant. This in effect introduces a zero range interaction to mock up the rest of the complicated cuts on the left and the inelasticity corrections on the right and leads to reasonable agreement with the rather poorly known ^2S phase shifts below breakup threshold. Having a simple analytic approximation for k ctg δ_2, Barton and Phillips[14] can easily show that for values of a_2 near that given by experiment, the prediction is independent of a_2 except in the immediate neighborhood of threshold ($z \lesssim 0.05$); in other words, single nucleon exchange predicts the doublet "effective range" as accurately as it does the quartet parameters. This model also predicts a triton pole at -6.42 MeV rather than at the experimental position of - 8.48 MeV. Exact agreement is hardly to be expected since the model here treats the triton as a bound state of the n-d system, while the actual triton is primarily in a symmetric-S state of rather different structure than the wave function implied by this N/D calculation; further, the triton pole lies below the energy range for which the model is valid. It may be of interest, however, that in the simple pole calculation reported above, the residue of the triton pole — which is precisely the asymptotic normalization of the neutron in the n-d decomposition of the triton wave function, corresponds closely to the value for this normalization computed from the separable models discussed below.

A much more elaborate, numerical N/D calculation has been carried through by Avishai, Ebenhöh, and Reiner[15], who include two-nucleon exchange, through it the virtual singlet state, the Yamaguchi vertex correction, and the effect of the inelastic breakup cut as given by a model for the quartet state, and by "experiment" for the doublet state. Since a number of simplifying assumptions went into the "experimental" phase shift analysis which produced the inelasticity and with which the calculation is compared, the significance of the global agreement of the results with experiment up to 25 MeV is a little hard to interpret. It does seem that the gross features of low energy n-d scattering can be reproduced by this type of on-shell model independent of more detailed dynamical considerations, once the sensitive parameter a_2 is given the experimental value. However, the complexity of the calculation is comparable to that of calculations made with separable interactions; the latter require no arbitrary inelasticity corrections, but have less flexibility in the choice of driving terms.

For a system of three identical bosons, the "single nucleon" exchange cut has twice the strength of that in the doublet state discussed above. Barton and Phillips[14] show that this gives an immediate and simple explanation of the first excited of this system found by Osborn using local potentials. Their N/D calculation shows that this state necessarily appears at that interaction strength which produces a bound pair. Since it is a direct consequence of particle exchange, it must occur in the local model of Osborn[16], or in the separable model of Aaron, Amado, and Yam[17], as indeed it does. Why Bander[18] found instead a ghost state is still not clear; it may be that he mistook the branch point at the start of the single particle exchange cut for a pole, since his conclusion was based on numerical calculations rather than analytic formulae.

If, instead of using the single nucleon exchange diagram as the driving term in an N/D calculation, it is used to formulate an integral equation for the T matrix, one obtains the Amado[19] model. If the freedom to treat the deuteron as partly an elementary particle in that model is not exploited, this becomes identical to the separable potential model pioneered by Sitenko and Kharchenko[20] and by Mitra[21]. Since Amado will discuss the successes of this model in more detail later in this conference, I will only note here that it gives a reasonably accurate representation of the higher partial waves as well as the

S waves. In the zero range approximation, this model was already correctly formulated by Skornyakov and Ter-Martirosyan[22] in 1956. They show that it gives 5.9 fm for a_4; it has not been sufficiently appreciated that in fact a_4 is determined to 10% knowing only the binding energy of the deuteron. In the doublet state, the zero range approximation gives infinite binding to the triton, as pointed out in the thirties by Thomas[23], which helped to obscure the fundamental significance of their result. So far as the higher partial waves go, in 1953 Christian and Gammel[24] showed that the loose structure of the deuteron allows them to be calculated to reasonable accuracy in the Born approximation, which again amounts to computing them from single nucleon exchange; hence if the S phases are treated phenomenologically (e.g. by an effective range expansion), the n-d differential cross section can be accurately reproduced in this way. More recently, Purrington and Gammel[25] have shown that if the S and D phases are fitted phenomenologically, and all others taken from single nucleon exchange (Born approximation), the n-d polarization and differential cross section can both be explained at 9.3 MeV.

The conclusion to which this work converges is that all the gross features of n-d elastic scattering and polarization at low energy are readily understood in terms of single nucleon exchange; when supplemented by the impulse approximation, this understanding can be extended into the higher energy region. This is a great theoretical triumph, but has the melancholy corrolary that theorists will have to work very hard to get much more information about three-body dynamics from the three-nucleon system. The only aspects of the system which have so far been shown to be at all sensitive to the details of the physical assumptions behind the calculations are the two numbers ϵ_t and a_2, which does not give us much theoretical leverage. This sensitivity is clearly brought out by the type of plot instituted by Phillips[26], as illustrated in fig. 4. We see that it is necessary for accurate calculations of ϵ_t and a_2 to know both the n-p singlet effective range and the percentage D state in the deuteron to high precision if these uncertainties are not to vitiate any calculation. Fortunately, it appears likely that the charge-independent prediction $r_s^{np} = 2.73 \pm 0.03$ F is now in agreement with experiment[2], although continued experimental scrutiny of that problem is still called for. But the percentage D state is only known indirectly. Models such as the Hamada-Johnston or Yale potentials which have the one pion exchange potential plus shorter range phenomenological attraction outside an infinite repulsive core give about 7% D state. Models such as that of Lomon and Feshbach, which are quite similar to the hard core models outside about 1 fm but replace the strong attraction from 0.48 - 0.7 fm plus hard core at 0.48 fm by an energy independent boundary condition at 0.7 fm, require only about $4\frac{1}{2}$% D state. The latter models seem to be in better agreement with low energy e-d scattering[5], but this is controversial.

The Hamada-Johnston potential raises almost insuperable obstacles to accurate variational calculations of ϵ_t, and only after herculean efforts extending over a decade have Delves, Blatt, Pask and Davies[27] succeeded in arriving at a convincing result of 6.7 ± 1.0 MeV as compared to the experimental value of 8.48 MeV. Unfortunately, no other model which gives a comparable fit to the n-p and p-p elastic scattering data has been given this much attention. We therefore cannot know whether to ascribe the discrepancy to the neglect of three-body forces, to the Hamada-Johnston potential having too much tensor force (too high percentage D state), or to the local potential plus hard core wave function being unrealistic because of an intrinsic non-locality in the nuclear force. That the latter is a serious possibility is illustrated by comparison of the extremely non-local (separable) Yamaguchi model with equivalent local potentials by van Wageningen's group[4]. If one requires equivalence by making the phase shift the same at all energies, ϵ_t shifts by a couple of MeV one way, while requiring equivalence by fitting the same deuteron wave function shifts in a comparable amount the other way. Work on these comparisons is becoming easier. For instance, Fiedeldey[28], using the method of Chadan, has shown how to construct second rank separable potentials which fit both the wave function and the phase shift, and that this ties down the off-shell behavior pretty closely, at least in his example. Malfleit and Tjon[29] have found a rapidly convergent series for computing both ϵ_t and the wave function for local potentials including short-range repulsion. Brayshaw[30] has found a way to remove the singularity from the continuum problem without contour deformation. Kim[31] has a new numerical method that looks simpler than either two-dimensional Faddeev or variational calculations. Many of these advances in technique will be reported later in the conference. But the fact remains that none of these improvements by themselves,except possibly that of Brayshaw, will increase our knowledge of the three-nucleon system one iota. The time is past when simple model calculations of ϵ_t and and a_2 are useful. Only if these are used for the starting point of a calculation of theoretically viable models of the nuclear force, and the effects of physical uncertainties in those models on the calculations are carefully explored, can progress be made on this aspect of the problem; that is hard work, but almost anything less is by now meaningless.

Anticipating the fact that it will be some time before there is a theoretical consesus on the non-local structure of the nuclear force, and sufficiently precise understanding of that structure to incorporate it into three-nucleon calculations, it behooves us to look for an experimental method to explore the non-locality of the interaction. The single nucleon exchange mechanism is already a non-local effect in the n-d system, considered as a two-body system, since the identity of the two neutrons in the system prevents our distinguishing which is the incident particle and which is the bound particle at distances of the order of the dueteron radius. But we have seen above that this effect is both well understood and sufficient to explain most of the features of the n-d system. However, there is a more general long-range non-locality in the three-particle continuum final state, which might give us a new handle on the problem, and which I now demonstrate.

Since the effect occurs even in the simplest three-particle system — that of three identical bosons interacting with finite range local potentials in states of zero relative angular momentum — and the reduction of the general problem in configurations space has already been given[2], I turn immediately to the J = 0 state of that system, which obeys the simple equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+Z-W(x)\right]U(x,y)=W(x)\int_{-1}^{1}d(\cos\gamma)\,\frac{xy}{x'y'}\,U(x',y')$$

$$=\frac{4}{\sqrt{3}}W(x)\int_{\Delta_-}^{\Delta_+}d\theta\;U(r\cos\theta,\,r\sin\theta)$$

,where

$$x'=\sqrt{\tfrac{3}{4}x^2+\tfrac{1}{4}y^2-\tfrac{\sqrt{3}}{2}xy\cos\gamma}=r\cos\theta$$

$$y'=\sqrt{\tfrac{1}{4}x^2+\tfrac{3}{4}y^2+\tfrac{\sqrt{3}}{2}xy\cos\gamma}=r\sin\theta$$

$$r=\sqrt{x^2+y^2}$$

and the domain of integration in θ is given in fig. 5. If the term on the right were known and of short range, we could solve the problem immediately by constructing the Green's function for the left hand side in the usual way from the complete set $u_p(x) \sin qy$ (where $u_p'' + p^2 u_p = W(x) u_p$, $u_p(0) = 0$, and $u_p = \sin(px + \delta_p)$ outside the range of forces) and applying it to the source term on the right. However, we can see immediately that, since U is of order unity asymptotically, the source term falls off only like $1/y$, and the integral will converge, if at all, only due to the sinusoidal oscillations of the integrand. The origin of this singularity is illustrated in fig. 6. Since the source term is an outgoing circular wave from the other two Faddeev channels (here identical to the initial channel), the amplitude falls off only like $1/y$, and so long as the remaining pair with relative coordinate x are within the range of forces, a scattering can occur to the final continuum state. Thus there is a long-range (non-local) effect in <u>any</u> three-particle quantum mechanical system, even though the pairwise forces are local and of finite range. Note also that this effect will persist if the particles are distinguishable. Further, this effect <u>is</u> a probe of the two-particle wave function <u>within</u> the range of forces using directly the strong interaction wave function which we wish to explore. Hence a practical scheme for exploiting this effect in any three-body system will provide the tool we seek.

The analysis of this system can be taken one step further. If the interaction $W(x) = 0$ for $x > R$, then the source term exists in the infinite strip illustrated in fig. 7a. However, if we examine the region in which we need to know $U(x, y)$ in order to compute this source term, we find it is given by the diagonal strip illustrated in fig. 7b. The overlap region (fig. 7c) is clearly the region where all three particles are within the range of each others forces. If we assume the wave function known within this region, then in the remainder of fig. 7b the particles are free and on-shell, and can be expanded in terms of the complete set $\exp(i(px+\delta_p))\sin(z-p^2)^{\frac{1}{2}}y$, and the coefficient of this wave function is just proportional to the three-body T matrix in this state. Thus T(p) can be expressed in terms of a one-variable integral equation with an inhomogeneous term coming from the overlab region in fig. 7c and the initial inhomogeneous term. Note that the wave functions in this region are on-shell, so this formulation does eliminate all multiple scattering singularities. Unfortunately, closer examination reveals that the kernel still contains the three-particle branch cut in z, so more work is required before an explicit method for inverting this equation can be developed.* However, if this can be done, the method gives immediately an integral equation for U(x, y) in the finite region of fig. 7c. Further, even without a solution in that region, any parametrization of this interior U (or its value on the boundary) will yield immediately an expansion of the three-particle T matrix in terms of two-particle off-shell t matrices. Thus it will give a phenomenology, applicable for instance to overlapping resonances in the Dalitz plot, comparable to the phase-shift analysis of two-particle final states. It would also give the correct three-body generalization of the Watson-Migdal final state formalism for short-range production mechanisms.

I hope someone will soon be able to turn the crank on this problem the final notch and produce this phenomenology; it could provide the tool we need for exploring strong interaction wave functions within the range of forces.

* It has now proved possible to demonstrate the compactness of the kernel and hence to guarantee that at least a numerical inversion is possible; cf. SLAC-PUB-668 (submitted to Phys. Rev. Letters).

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31) Y. E. Kim, Purdue preprint

FIGURE CAPTIONS

1. Proof of the failure of the locality assumption for the singlet-even nucleon-nucleon state (from ref. 2).
2. Low energy behavior of n-d ^2S scattering (from ref. 9).
3. Singularity structure of the n-d elastic amplitude and the simple pole approximations to it.
4. Comparison of calculations of ϵ_t and a_2 as given by A. C. Phillips, Nuclear Physics A107 (1968) 209; V. F. Kharchenko, N. M. Petrov and S. A. Storozhenko, Nuclear Physics A106 (1968) 464; and G. L. Schrenk and A. N. Mitra, preprint and Brela Symposium. The much more extensive results obtained by the second group in the reference cited and in the earlier publication by A. G. Sitenko, V. F. Kharchenko and N. M. Petrov, Physics Letters 21 (1966) 54 have mostly been omitted in order not to confuse the plot. They use only 4% D state, and are slightly shifted from Phillips' results (open circle, solid circle, open triangle) because of the slightly different value for a_s, as is illustrated for $r_s = 2.7$ fm by the open square labeled 1_t. Values for other values of r_s also agree with Phillips if shifted by about the same amount. The value labeled 3_t is obtained by these authors by cubing the Yamaguchi form factor in the central but not the tensor parts of the interaction. Results from Schrenk and Mitra are not directly comparable, since they include a second rank singlet potential fitted by Naqvi and Gupta. The $(C + T)_Y$ points use the same (Yamaguchi) triplet interaction as Phillips 4% D state points. The $(C + T)_N$ points use the Naqvi triplet parameters, omitting the L·S term. The designation of the singlet model used (N, G_1, G_1', G_1'', G_2', G_2, G_3) refers to

parameters taken from Naqvi and Gupta by Schrenk and Mitra, and occurs in the same order along both dotted curves; for clarity the points are labeled only along the $(C + T)_N$ curve (from ref. 2).

5. Domain of integration for the source term in the equal-mass three body problem.
6. Origin of the long-range non-local effect in any three-body system (see text).
7. Domains where (a) there is a long-range source term, (b) the wave function need be known to compute this term, and (c) the overlap, which is also the region in which all three particles are within the range of forces.

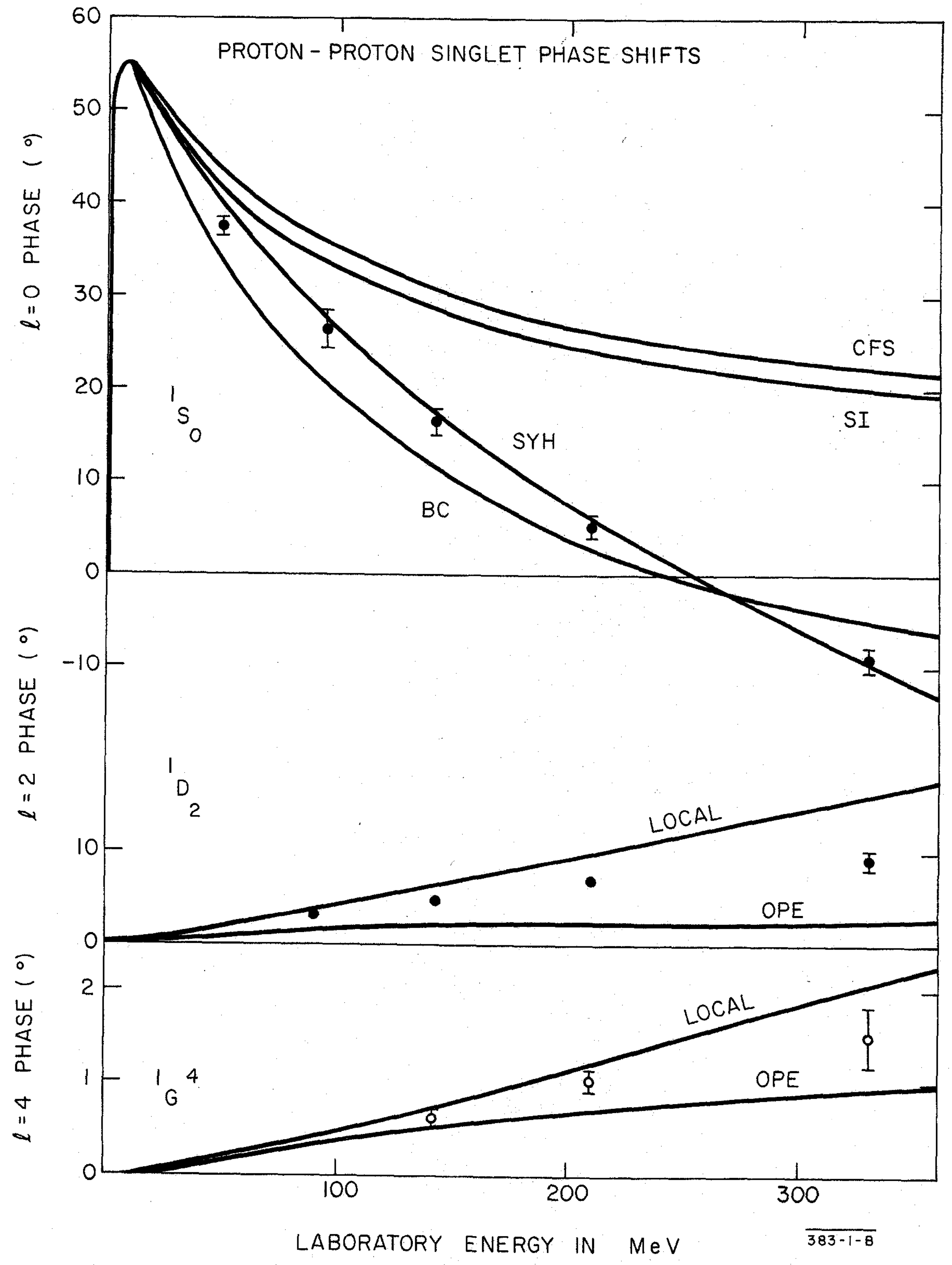
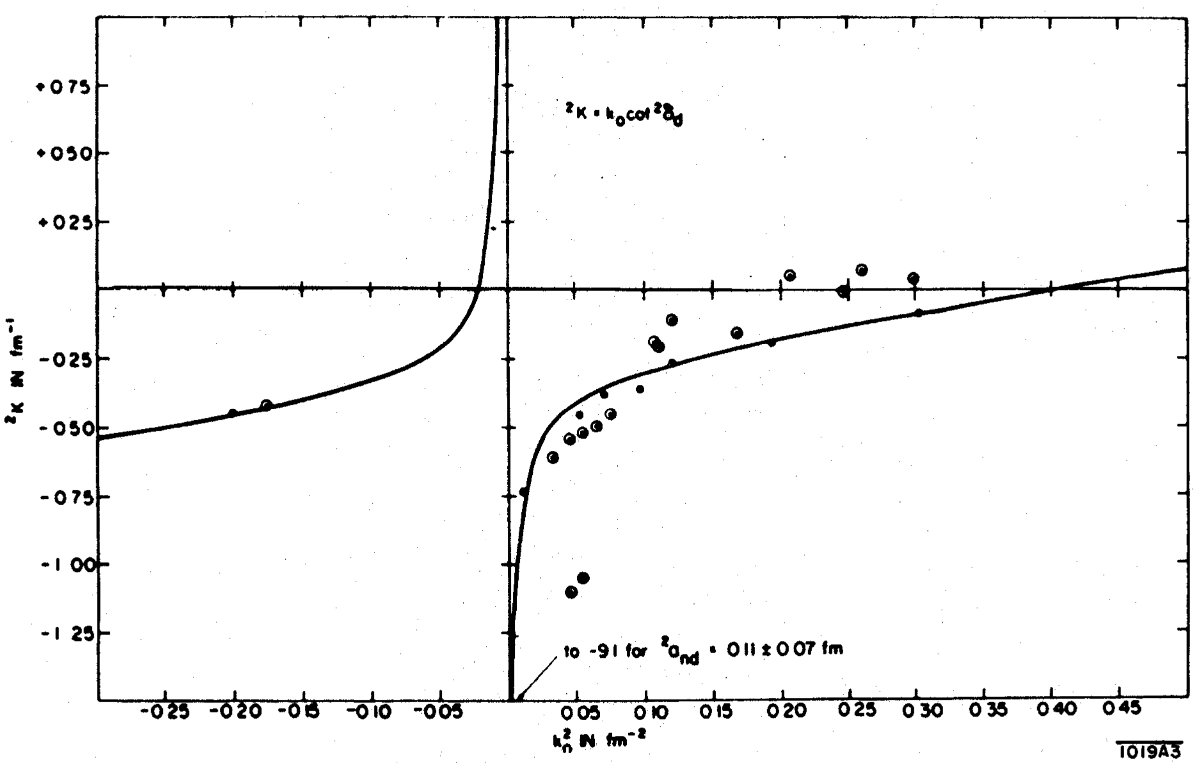

PROTON - PROTON SINGLET PHASE SHIFTS
ℓ = 0 PHASE (°)
ℓ = 2 PHASE (°)
ℓ = 4 PHASE (°)
60
50
40
30
20
10
0
-10
10
0
2
1
0
1S0
1D2
1G4
CFS
SI
SYH
BC
LOCAL
OPE
LOCAL
OPE
100
200
300
LABORATORY ENERGY IN MeV
383-1-B

Fig. 1



+0.75
+0.50
+0.25
-0.25
-0.50
-0.75
-1.00
-1.25
²K = k₀ cot ²δ_d
²K IN fm⁻¹
to -91 for ²a_nd = 0.11 ± 0.07 fm
-0.25
-0.20
-0.15
-0.10
-0.05
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
k²_n IN fm⁻²
1019A3

Fig. 2

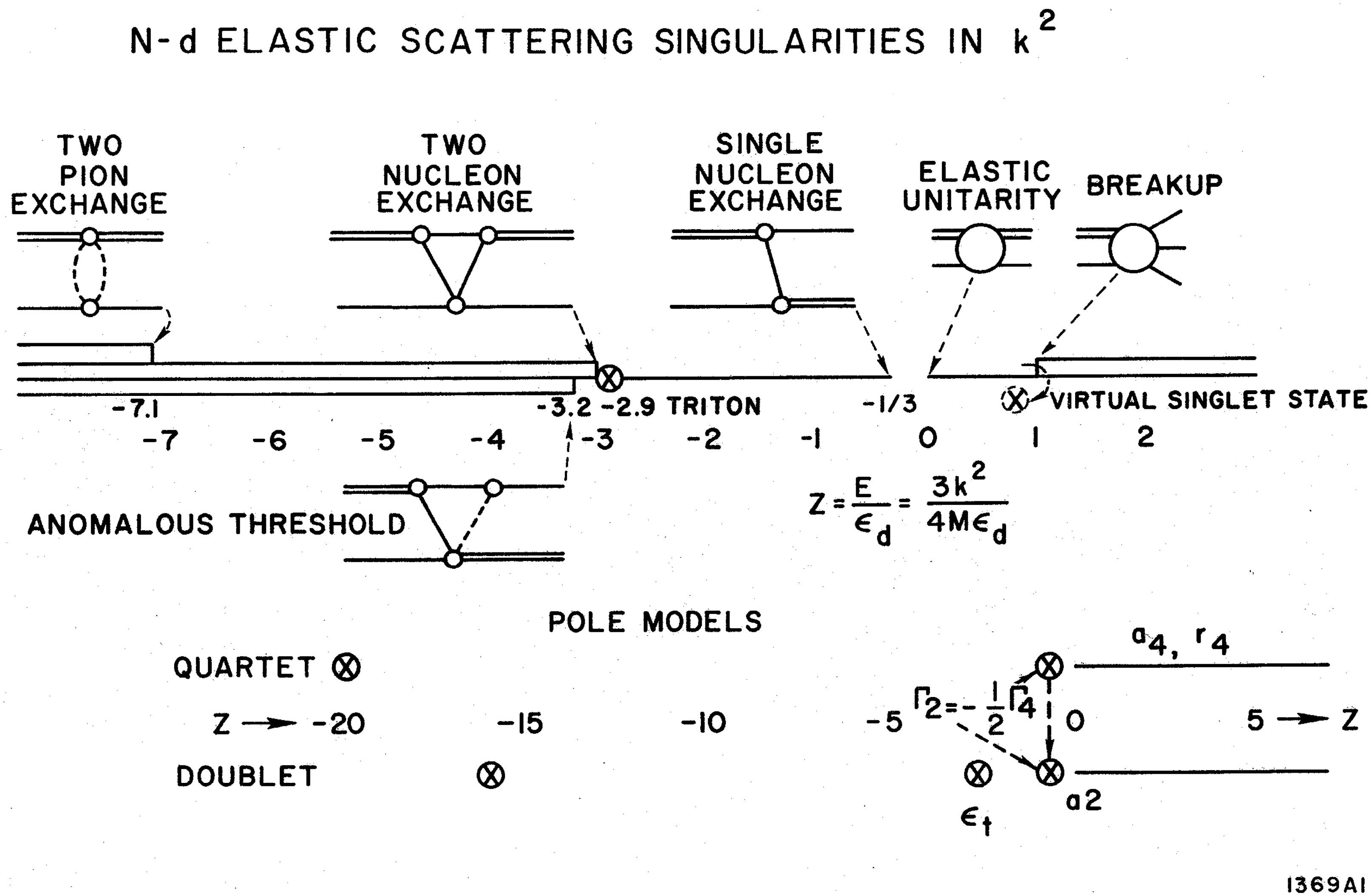
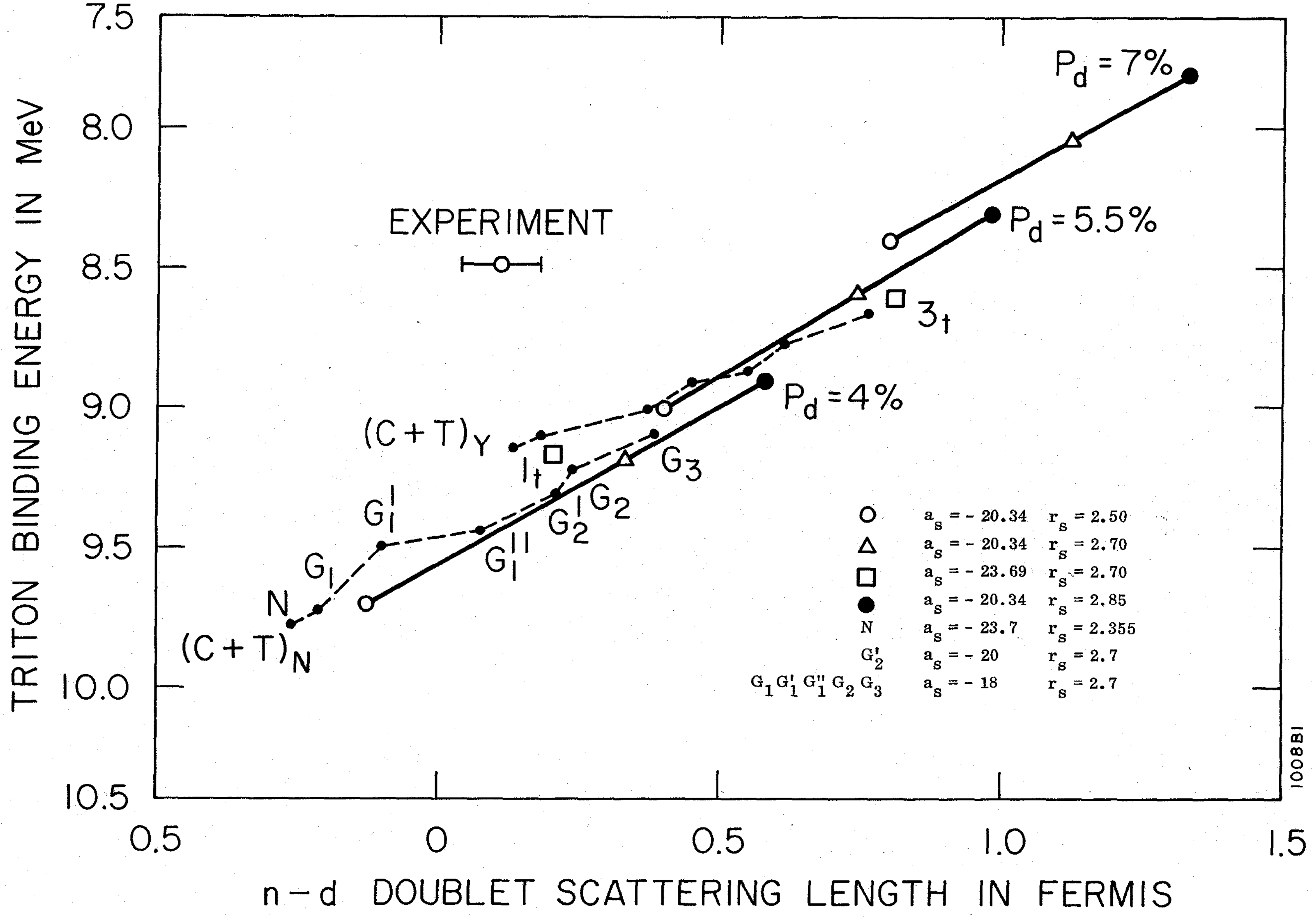
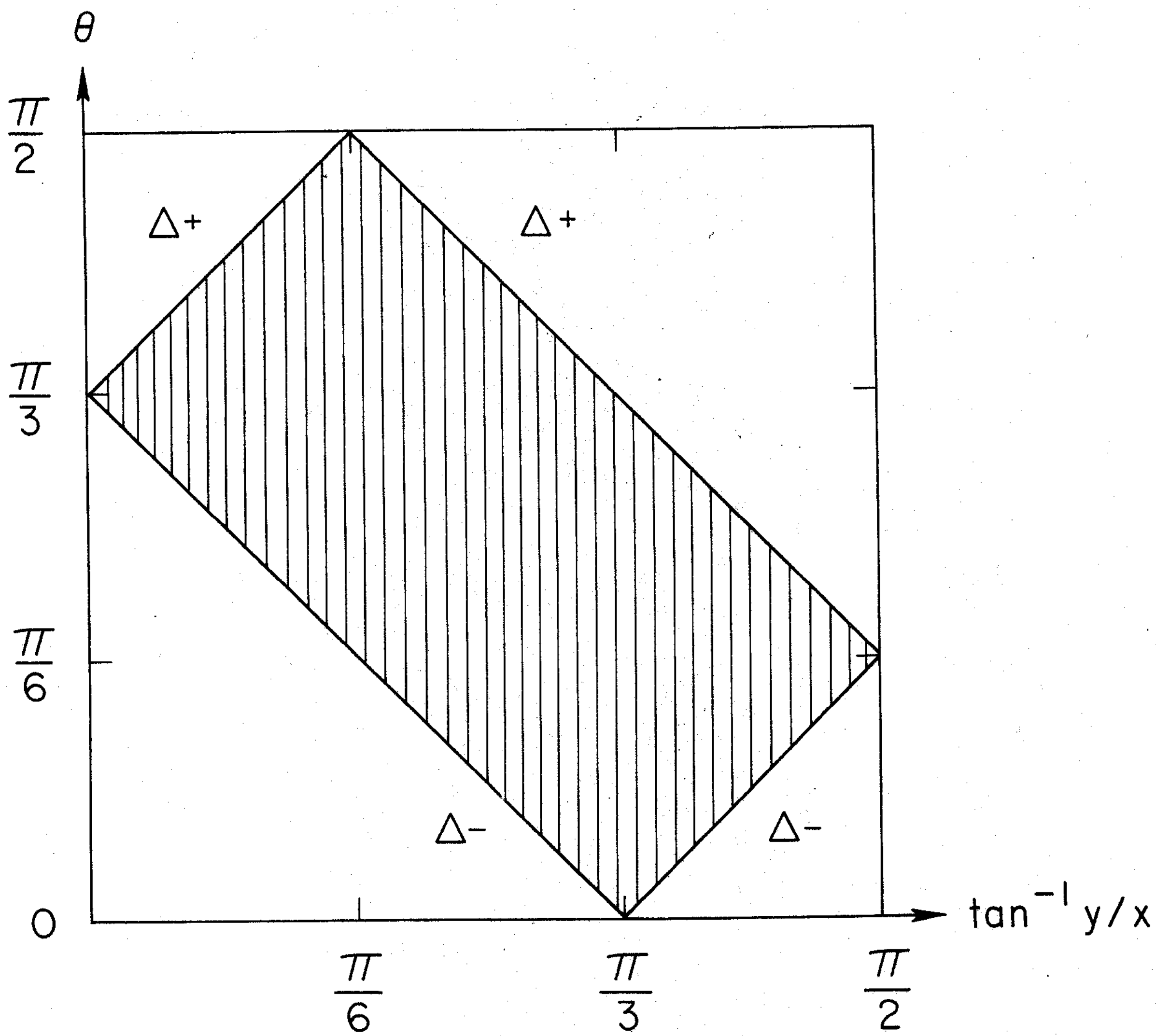

N-d ELASTIC SCATTERING SINGULARITIES IN k²
TWO PION EXCHANGE
TWO NUCLEON EXCHANGE
SINGLE NUCLEON EXCHANGE
ELASTIC UNITARITY
BREAKUP
-7.1
-3.2 -2.9 TRITON
-1/3
VIRTUAL SINGLET STATE
-7 -6 -5 -4 -3 -2 -1 0 1 2
ANOMALOUS THRESHOLD
$Z = \frac{E}{\epsilon_d} = \frac{3k^2}{4M\epsilon_d}$
POLE MODELS
QUARTET
Z → -20 -15 -10 -5 0 5 → Z
DOUBLET
a_4, r_4
$\Gamma_2 = -\frac{1}{2}\Gamma_4$
ϵ_t
a2
1369A1

Fig. 3



TRITON BINDING ENERGY IN MeV
7.5
8.0
8.5
9.0
9.5
10.0
10.5
0.5
0
0.5
1.0
1.5
n – d DOUBLET SCATTERING LENGTH IN FERMIS
EXPERIMENT
$P_d = 7\%$
$P_d = 5.5\%$
$P_d = 4\%$
3_t
1_t
$(C+T)_Y$
$(C+T)_N$
N
G_1
G_1'
G_1''
G_2'
G_2
G_3
$a_s = -20.34$ $r_s = 2.50$
$a_s = -20.34$ $r_s = 2.70$
$a_s = -23.69$ $r_s = 2.70$
$a_s = -20.34$ $r_s = 2.85$
N $a_s = -23.7$ $r_s = 2.355$
G_2' $a_s = -20$ $r_s = 2.7$
$G_1 G_1' G_1'' G_2 G_3$ $a_s = -18$ $r_s = 2.7$
1008B1

Fig. 4



$$\Delta_+ = \frac{\pi}{3} + \tan^{-1} y/x \qquad 0 < \tan^{-1} y/x < \frac{\pi}{6}$$

$$= \frac{2\pi}{3} - \tan^{-1} y/x \qquad \frac{\pi}{6} < \tan^{-1} y/x < \frac{\pi}{2}$$

$$\Delta_- = \left| \frac{\pi}{3} - \tan^{-1} y/x \right| \qquad 0 < \tan^{-1} y/x < \frac{\pi}{2}$$

1097B1

Fig. 5

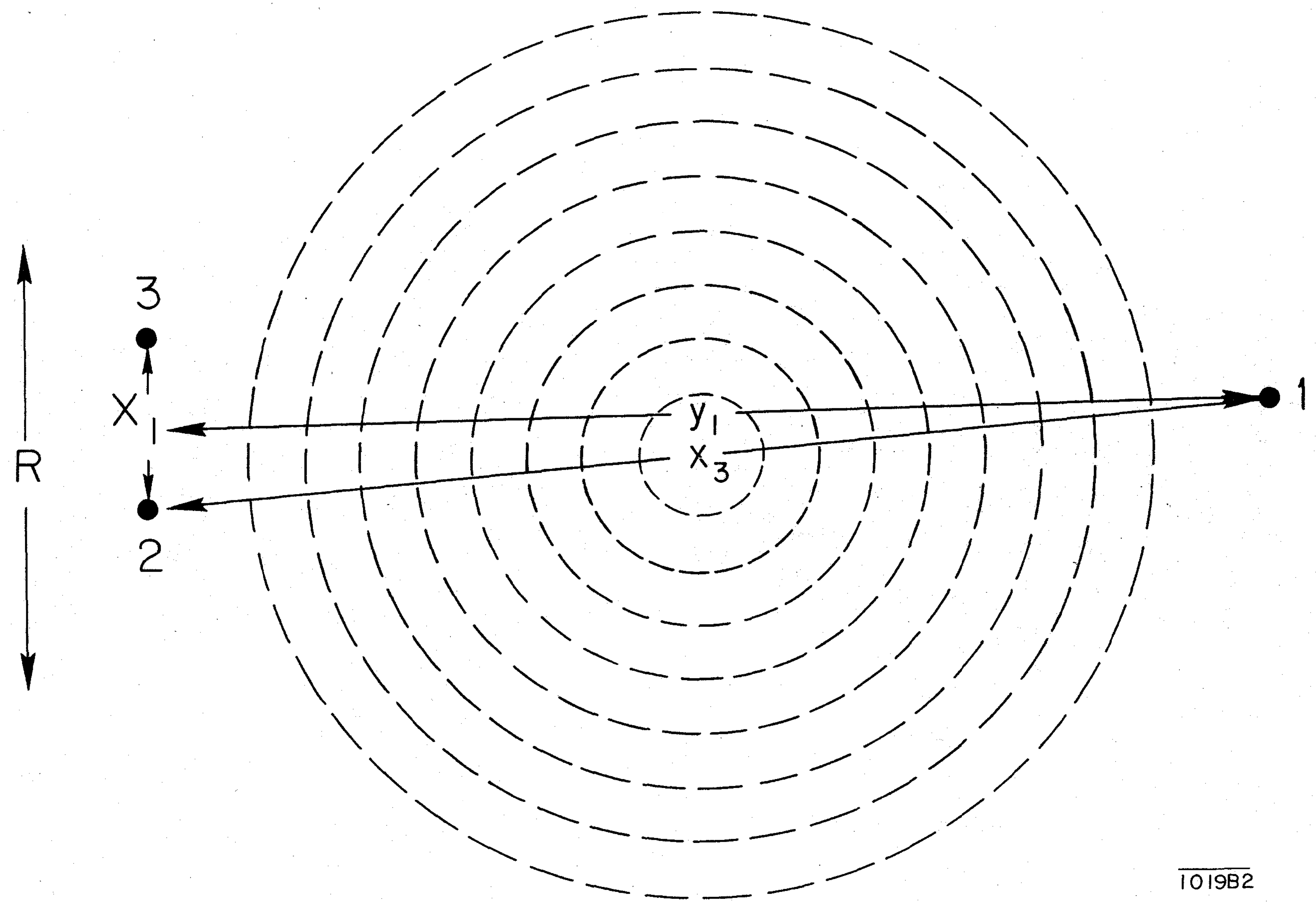
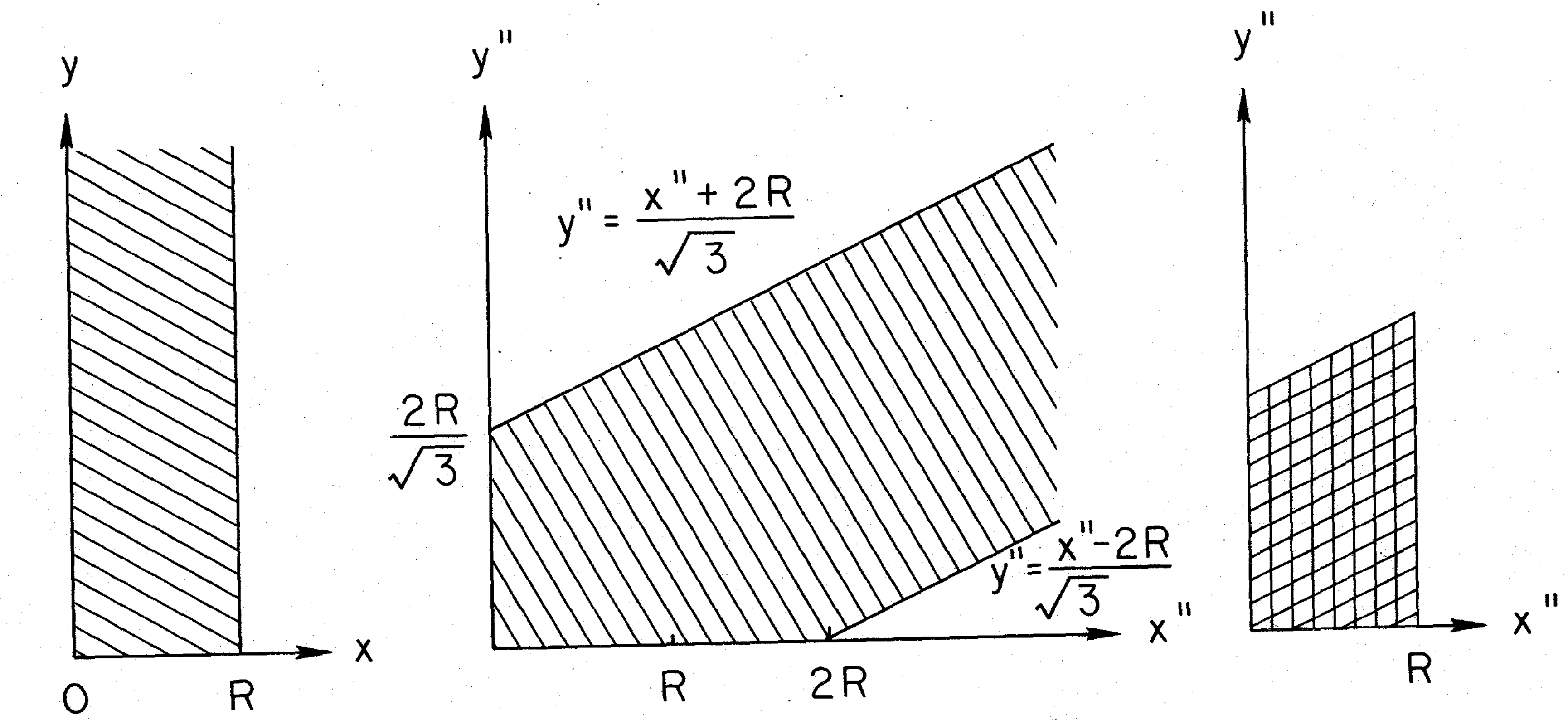


Fig. 6



a) Source: Direct Channel

b) Region of Integration over U (x",y")

c) Overlap

1097A2

Fig. 7