SLAC-PUB-652
August 1969
(EXP) & (TH)

MUON-PROTON INELASTIC SCATTERING AND VECTOR DOMINANCE*

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ABSTRACT

Results from inelastic muon-proton scattering are used to investigate the form of $\sigma_{exp}(q^2, K)$, the quantity commonly called the "virtual photon-proton total cross section." Fits to σ_{exp} have been made for photon energies $0.6 \leq K\ (\mathrm{GeV}) \leq 6.5$ and 4-momentum transfer squared $|q^2|$ up to $1.2\,(\mathrm{GeV}/c)^2$. Measurements of the real photon-proton total cross section $\sigma_{\gamma p}(K)$ have been included in the fits. The forms of rho-dominance theory which predict $\sigma_{exp}(q^2, K) = \sigma_{\gamma p}(K)/(1 + |q^2|/m_\rho^2)$ are in agreement with the data.

*Work supported by the U. S. Atomic Energy Commission.

In this letter, we will discuss the interpretation of the muon-proton inelastic scattering cross sections presented in the preceeding paper,[1] which we will refer to hereafter as I. The one photon exchange mechanism can be assumed to dominate the inelastic scattering cross section, as it does the elastic,[2] since the exchange of a second photon would be expected to be suppressed by a factor of approximately 1/137 (the fine structure constant). At the proton vertex we are studying inelastic processes in which additional hadrons are produced. Thus we have a situation, unique in elementary particle physics, in which a single virtual particle coupled to the strongly interacting particles is exchanged. These processes are then not only a function of the energy of the particle, but also of its "virtuality" or q^2 value. In experiments in which only the scattered lepton is detected, the total cross sections for the absorption of virtual photons can be determined as functions of the q^2 and energy of the exchanged photons. These cross sections, $\sigma_T(q^2, K)$ for transverse photons and $\sigma_S(q^2, K)$ for scalar photons are related to the experimentally determined differential cross section:[3]

$$
\begin{aligned}
d^2\sigma/dq^2dK &= \Gamma_T(q^2, K)\ \sigma_T(q^2, K) + \Gamma_S(q^2, K)\,\sigma_S(q^2, K) \\
&= \Gamma_T(q^2, K)\left[\sigma_T(q^2, K) + \epsilon\sigma_S(q^2, K)\right] \\
&= \Gamma_T(q^2, K)\ \sigma_{exp}\ (q^2, K)
\end{aligned}
$$

$\sigma_{exp} = (\sigma_T + \epsilon\sigma_S)$ is the total cross section combination which will be considered in this paper. q^2 is the square of the four-momentum transfer from the muon, and K is given by $K = \nu - \left|q^2\right|/2M$, where ν is the laboratory energy of the virtual photon and M is the proton mass.

Γ_T is the flux of transversely polarized virtual photons defined by

$$\Gamma_T = \frac{\alpha}{2\pi|q^2|}\left(\frac{K}{p^2}\right)\left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right)$$

$$\epsilon = \left(\frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \Big/ \left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{(E-E')^2 + |q^2|}\right) \tag{1}$$

ϵ is the ratio of the flux of scalar to transverse photons. Here α is the fine structure constant, $E(E')$ and $p(p')$ are the laboratory energy and momentum of the incident (outgoing) muon and m is the muon mass.

The values of σ_{exp} given in Table I of I are shown in Fig. 1. The points at $|q^2| = 0$ are derived from recent bubble chamber measurements[4,5,6] of $\sigma_{\gamma p}(K)$, the photon-proton total cross section. As $|q^2|$ goes toward zero the values of $\sigma_{exp}(q^2, K)$ approach $\sigma_{\gamma p}(K)$ within our errors, as they should.[3] The inelastic cross sections fall off slowly with $|q^2|$, decreasing on the average by a factor of 2 from $|q^2|$ 0 to $|q^2| = 0.5$. If the inelastic cross section had a form factor term such as the $(1 + 1.41\,|q^2|)^{-4}$ term in elastic lepton-proton scattering,[2] we would expect more like a factor of 10 decrease.

The application of the vector dominance model to inelastic lepton-proton scattering invokes the concept that the virtual photon couples to the proton through the vector mesons, and that the strength of the coupling is independent of q^2. We assume throughout this letter that the contribution of the ω and ϕ mesons can be neglected. Naively one would expect that σ_{exp} would have a q^2 dependence given by the square of the rho meson propagator, namely: $(1 + |q^2|/m_\rho^2)^{-2}$. However, preliminary results from this experiment[7] and from inelastic electron-proton scattering[8] showed that the q^2 dependence of σ_{exp} is more like $(1 + |q^2|/m_\rho^2)^{-1}$.

Sakurai[9] then showed that when polarization considerations are taken into account, it is possible to obtain the relationship, $\sigma_S(q^2, K) \approx \left(|q^2|/m_\rho^2\right)\sigma_T(q^2, K)$. The use of these concepts leads to the equation:

$$\sigma_{exp}(q^2, K) \approx \left[1 + |q^2|/m_\rho^2\right]^{-2} \left[1 + \epsilon(|q^2|/m_\rho^2)\right] \sigma_{\gamma p} (K) \qquad (2)$$

Now for our data, ϵ given in Table I of I is close to unity, so that

$$\sigma_{exp}(q^2, K) \approx \left[1 + |q^2|/m_\rho^2\right]^{-1} \sigma_{\gamma p}(K) \qquad (3)$$

Detailed considerations by Sakurai[9] lead to a variation of Eq. (2), namely:

$$\sigma_{exp}(q^2, K) = \left[1 + |q^2|/m_\rho^2\right]^{-2} \left[1 + \epsilon(|q^2|/m_\rho^2) (K/\nu)^2 \xi(K)\right] \sigma_{\gamma p}(K) \qquad (4)$$

$\xi(K)$ is the ratio of the total cross sections for scattering of longitudinally and transversely polarized ρ mesons on protons and is expected to be close to 1.

Tsai[10] has derived a vector dominance prediction which differs from Eq. (4) by factors like (K/ν) and in the interpretation of the energy at which $\sigma_{\gamma p}(K)$ is to be evaluated. All predictions must agree when $|q^2|$ becomes zero, but, as discussed by Tsai,[10] there is really no unique way to make the extrapolation away from zero, so that it must be left up to experiment to determine the correct form. The vector dominance predictions are supposed to be best when $|q^2|$ is small and ν is large; therefore, it is not clear that the refinements of the theory are very meaningful in the region where they can be tested. For this reason and because of statistical uncertainties in the data we shall restrict our attention to the cruder forms of the vector dominance predictions.

We have fit the data in Fig. 1 to the expressions.

Linear: $$\sigma_{exp}(q^2, K) = S(K) \left(1 + R_L |q^2|\right) \qquad (5)$$

Inverse Linear: $$\sigma_{exp}(q^2, K) = S(K)\ (1 + R_I \left|q^2\right|)^{-1} \tag{6}$$

Inverse Quadratic: $$\sigma_{exp}(q^2, K) = S(K)\ (1 + R_Q \left|q^2\right|)^{-2} \tag{7}$$

Equation (5), the linear form, has no theoretical significance. Equation (6) is the vector dominance prediction of Eq. (3). Equation (7) fits the supposition that σ_S is very small in this region so that only the square of the rho meson propagator appears.

Since $\sigma_{exp}(q^2, K)$ must approach $\sigma_{\gamma p}(K)$ when $|q^2| \to 0$, we must expect S to be energy dependent. The fitting procedure allows S to be separately determined for each K bin, but R is considered to be independent of K. Therefore, there are six parameters, the five values of S and the single value of R in each fit. The values were selected to give the minimum χ^2 and the uncertainties in the parameters are the standard deviations. Fits to our data were made both with and without the use of experimental values of $\sigma_{\gamma p}(K)$, but most of the analysis presented in this paper makes use of them. We obtained these values of $\sigma_{\gamma p}(K)$, listed in Table I, by interpolating and averaging the published values[4,5,6] to match our K bins. The errors include our relative normalization uncertainties. The results in Table I obtained using the $\sigma_{\gamma p}(K)$ values show that the linear fit is poor (6% confidence level), but the inverse linear and inverse quadratic are good (65% and 50% confidence levels, respectively). The values of σ_{exp} predicted by these two fits differ by only a few percent in the q^2 range covered by our data. The curves shown in Fig. 1 are for the inverse linear fit with $R = 1.38\ (\mathrm{GeV}/c)^{-2}$. If we constrain R to be $1/m_\rho^2$, the confidence level for the inverse linear form is 50%. Our results are therefore in agreement with the crude vector dominance predictions of Eq. (3), which assume $\sigma_S/\sigma_T = |q^2|/m_\rho^2$. There is as yet no direct experimental verification of this assumption. If σ_S is small in this q^2 range, vector dominance would predict $\sigma_T \approx \sigma_{exp} \approx \sigma_{\gamma p}(1 + |q^2|/m_\rho^2)^{-2}$. The χ^2 for this assumption is 146 for 26 degrees of freedom if we use the $\sigma_{\gamma p}(K)$ values and it is 63 for 21 degrees of freedom for the muon data alone. Vector dominance can, therefore, only fit the data if $\sigma_S/\sigma_T \sim |q^2|/m_\rho^2$. We have also fit the data to the more detailed prediction of Sakurai given in Eq. (4). For $\xi = 1.0$ we find $R = .99\ (\mathrm{GeV/c})^{-2}$ with a confidence level of 30%. The best fit to ξ, constraining R to be $1/m_\rho^2$, is $\xi = 1.6 \pm .2$ with a confidence level for the fit of 11%. If the same fit is made using data with $K \geq 2$ GeV, where the considerations of Sakurai are more appropriate, $\xi = 1.2 \pm .2$ with a 50% level of confidence.

Although we have been able to fit the data with an R value independent of K, it can be seen from Fig. 1 that there is a slight tendency for σ_{exp} to fall more rapidly with q^2 at the higher values of K. Now, in the region where K and ν are significantly different, there is an arbitrariness in the distinction between virtual photon flux and cross section which can affect the interpretation of the data. As an example, we have considered the effect of using the quantities σ_{trans} and $-\sigma_{long}$ defined by Gilman.[11] σ_{trans} and $-\sigma_{long}$ are $|\vec{q}|/K$ times σ_T and σ_S, respectively, where $|\vec{q}|$ is the laboratory momentum of the virtual photon. Then we have $\sigma'_{exp} = |\vec{q}|/K\ \sigma_{exp}$, and fits to σ'_{exp} with the inverse linear and inverse quadratic forms are also shown in Table I. The R values increase, and the χ^2 probability for the fit improves, showing that we have taken out some of the residual energy dependence. However, we have insufficient statistical precision to warrant choosing between Gilman cross sections and the Hand cross sections. If we require $R = 1/(m_\rho^2)$, the inverse quadratic form is still ruled out by our data.

When this experiment was proposed, there were no reliable measurements of $\sigma_{\gamma p}(K)$, and this experiment, when extrapolated to a $|q^2|$ of zero, was considered a method of determining $\sigma_{\gamma p}(K)$. There are now reliable measurements of $\sigma_{\gamma p}(K)$. It is interesting to examine the effect on the fits, of fitting the various equations to just the muon-proton inelastic scattering data. These fits are given in Table I for the linear and inverse linear forms. The extrapolation to $|q^2| = 0$ leads to predictions of $\sigma_{\gamma p}(K)$, which fluctuate somewhat with the type of fit used and because of statistical error. But $\sigma_{\gamma p}(K)$ is obtained with an uncertainty of the order of 10 to 20 %.

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TABLE CAPTION

I. Table I values of the parameters R and S(K) for the best fits to various equations for σ_{exp}. $\sigma_{\gamma p}$ is the experimental value of the photon-proton total cross section, integrated over the indicated K bins. K is the equivalent photon energy defined in the text. σ'_{exp} is the Gilman form of the virtual photon cross section defined in the text. Prob. is χ^2 probability for the fit. The errors in the parameters are the standard deviations.

FIGURE CAPTION

1. The "virtual photon-proton cross section," $\sigma_{exp}(q^2, K)$, as determined by muon-proton inelastic scattering. The data at $|q^2| = 0$ are bubble chamber measurements of the total photon-proton cross sections. The curves are for the fit $\sigma_{exp}(q^2, K) = S(K)/(1 + R\,|q^2|)$ which has a 65% probability of fitting the combined data. The fitted value of R is $1.38 \pm .22$ $(\mathrm{GeV}/c)^2$.

TABLE I

FITS	Prob.	R $(\mathrm{GeV}/c)^{-2}$	S (microbarns) for K (GeV/c) of 0.8	1.5	2.5	4.0	5.75	$\sigma_{\gamma p}$ used
Linear, Eq. 5	.06	$-.63\pm.06$	195 ± 12	146 ± 7	119 ± 6	122 ± 7	107 ± 8	yes
Inverse Linear, Eq. 6	.65	$1.38\pm.22$	211 ± 13	156 ± 7	126 ± 7	129 ± 7	113 ± 9	yes
Inverse Linear, Eq. 6 with $R = 1/(m_\rho)^2$	.50	1.71	222 ± 13	162 ± 7	130 ± 7	133 ± 7	116 ± 9	yes
Inverse Quadratic, Eq. 7	.50	$.58\pm.09$	208 ± 13	154 ± 7	124 ± 6	127 ± 7	112 ± 9	yes
Sakurai, Eq. 4, $\xi(K) = 1$	.30	$.99\pm.17$	212 ± 13	154 ± 7	123 ± 6	127 ± 7	112 ± 9	yes
Inverse Linear to σ'_{exp}	.85	$2.16\pm.26$	184 ± 13	155 ± 7	130 ± 7	135 ± 7	118 ± 9	yes
Inverse Quadratic to σ'_{exp}	.75	$.86\pm.10$	182 ± 13	153 ± 7	128 ± 7	134 ± 7	116 ± 9	yes
Inverse Linear, Eq. 6	.75	$1.10\pm.42$	202 ± 23	150 ± 17	111 ± 13	122 ± 14	92 ± 16	no
Inverse Linear, Eq. 6 with $R = 1/(m_\rho)^2$	.55	1.71	230 ± 23	171 ± 17	126 ± 13	139 ± 14	103 ± 16	no
Inverse Quadratic, Eq. 7	.65	$.42\pm.14$	194 ± 19	144 ± 14	106 ± 11	118 ± 13	106 ± 14	no
$\sigma_{\gamma p}(K)$ (microbarns)			201 ± 20	151 ± 9	134 ± 8	127 ± 8	125 ± 11	

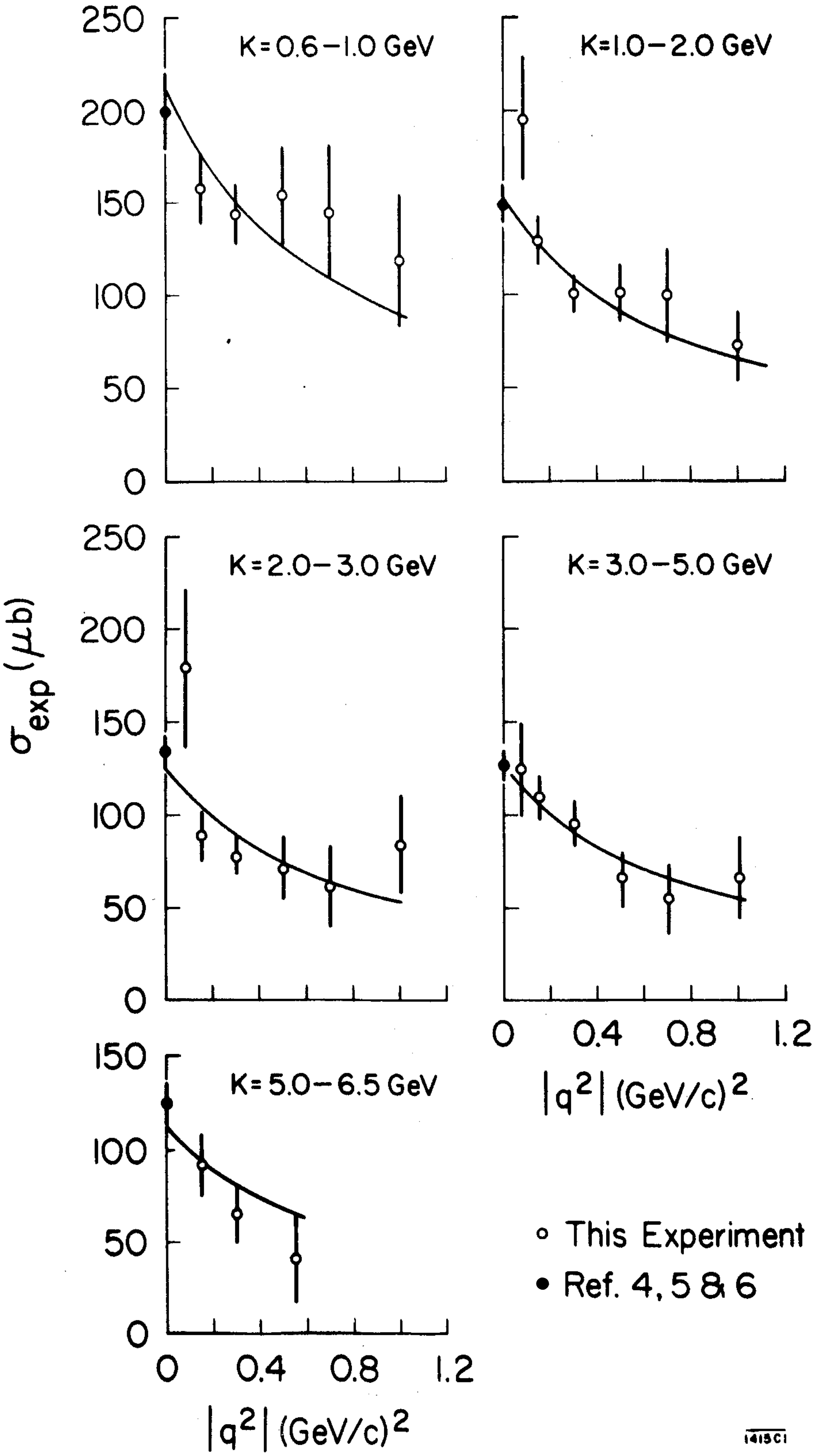
K=0.6−1.0 GeV
K=1.0−2.0 GeV
K=2.0−3.0 GeV
K=3.0−5.0 GeV
K=5.0−6.5 GeV
σ_{exp} (μb)
250
200
150
100
50
0
0
0.4
0.8
1.2
$|q^2|$ (GeV/c)^2
○ This Experiment
● Ref. 4, 5 & 6
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Fig. 1