SLAC-PUB-636
August 1969
(TH) and (EXP)

RELATIONS BETWEEN CROSSING SYMMETRIC PROCESSES*

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ABSTRACT

Relations between processes related by crossing are proposed as a test of (i) exchange degeneracy, (ii) dominance by trajectories of the same signature. Various possible experiments are discussed.

*Work supported by the U.S. Atomic Energy Commission.

The Regge analytic continuation of the u-channel amplitudes gives the high energy behavior for both the s- and t-channel processes in the neighborhood of $u = 0$. Therefore, both regions are mediated by the same Regge trajectories. For two general classes of exchanges the cross sections of the two channels are identical in the high energy limit, except for known phase-space factors.[1] We review here the argument that leads to these relations, and then discuss processes which must obey them, as well as the kinematic regions where they are expected to hold.

Figure 1 shows the paths for the analytic continuation of the u-channel amplitudes into two different regions of the Mandelstam plane. We define two different sets of points (u, s_s) and (u, s_t), such that

$$z_u(u, s_s) = - z_u(u, s_t) \quad . \tag{1}$$

The u-channel Jacob-Wick helicity amplitudes with the half angles removed[2]

$$A_i(u, s) = f^u_{cd, ab}(u, s) \left(\cos \frac{\theta_u}{2}\right)^{-|\lambda + \mu|} \left(\sin \frac{\theta_u}{2}\right)^{-|\lambda - \mu|} \tag{2}$$

can be analytically continued into the s- and t-channels using the equations:

$$A_i(u, s_s) = \sum \beta_{\alpha(u)}(u) \frac{P_{\alpha(u)}\left(z_u(u, s_s)\right) \pm P_{\alpha(u)}\left(-z_u(u, s_s)\right)}{\sin \pi \alpha(u)} \tag{3}$$

$$\begin{aligned} A_i(u, s_t) &= \sum \beta_{\alpha(u)}(u) \frac{P_{\alpha(u)}\left(z_u(u, s_t)\right) \pm P_{\alpha(u)}\left(-z_u(u, s_t)\right)}{\sin \pi \alpha(u)} \\ &= \sum \beta_{\alpha(u)}(u) \frac{P_{\alpha(u)}\left(-z_u(u, s_s)\right) \pm P_{\alpha(u)}\left(z_u(u, s_s)\right)}{\sin \pi \alpha(u)} \quad . \end{aligned} \tag{4}$$

There are two classes of trajectories for which

$$\left|A_i(u, s_s)\right|^2 = \left|A_i(u, s_t)\right|^2 . \tag{5}$$

Class 1: All the contributing trajectories have the same signature. In this case all the $A_i(u, s_s)$ and $A_i(u, s_t)$ either have the same sign or they differ by an overall minus sign and Eq. (5) follows.

Class 2: There are two trajectories contributing which are exchange degenerate.[3] In this case the phases of the contributions from the two trajectories are given by:

$$(1 - e^{-i\pi\alpha}) = 2e^{i\pi/2}\, e^{-i\pi\alpha/2} \sin \frac{\pi\alpha}{2}$$

and

$$(1 + e^{-i\pi\alpha}) = 2e^{-i\pi\alpha/2} \cos \frac{\pi\alpha}{2}$$

respectively. They are out of phase by exactly 90°. There are no interference terms and Eq. (5) again follows.

To translate (5) into relations between cross sections, we note that the s-channel differential cross section is given by:

$$\frac{d\sigma}{du} = \frac{\pi}{(2s_a + 1)(2s_b + 1)} \frac{1}{q_s^2} \sum_i \left|A_i\right|^2 \left|\cos^2 \frac{\theta_u}{2}\right|^{|\lambda+\mu|} \cdot \left|\sin^2 \frac{\theta_u}{2}\right|^{|\lambda-\mu|} . \tag{6}$$

For values of (u, s_s) where $|z(u, s_s)| \gg 1$, Eq. (6) is identical to the corresponding t-channel cross section evaluated at (u, s_t) except for the factor $\frac{1}{(2s_a+1)(2s_b+1)} \frac{1}{q_s^2}$. Therefore with the exception of known phase-space factors the cross sections at the two sets of points are identical.

Next we come to the classification of the processes.

1. Processes to which only $S = 0$, $I = 3/2$, $B = 1$ trajectories contribute belong to Class 1. In such cases it is believed that only the Δ trajectory contributes significantly. The trajectory that passes through the $J^P = 5/2^+(1880)$ resonance could interfere with the above theorem, but its effects have not yet

been revealed in the differential cross sections for backward scattering.[4] Violation of the following relations implies that other I = 3/2 trajectories are also contributing.

(a) Connection between the $\pi^- p$ elastic cross section in the backward direction and $\bar{p}p \to \pi^+\pi^-$ is the best example of this case, as it has already been discussed in the literature.[1]

(b) Recent data[5] on $\pi^- p \to p\rho^-$ and $\pi^- p \to pA_2^-$ indicate that these processes are also dominated by the Δ trajectory. Therefore we expect

$$\frac{1}{2}\frac{d\sigma}{du}(\pi^- p \to p\rho^-) = \left(\frac{q_t}{q_s}\right)^2 \frac{d\sigma}{du}(\bar{p}p \to \pi^+\rho^-) \tag{7}$$

$$\frac{1}{2}\frac{d\sigma}{du}(\pi^- p \to pA_2^-) = \left(\frac{q_t}{q_s}\right)^2 \frac{d\sigma}{du}(\bar{p}p \to \pi^+ A_2^-) \tag{8}$$

to hold at high energies and extended regions of the momentum transfer. In the above reactions the π^+ meson is the fast particle moving in the direction of $\bar{p}$. Both of these relations can and should be checked experimentally.

At low energies the cross sections for the two processes in either (7) or (8) differ by the half angles occurring in (3) and they are in general different. Similar relation can be established at lower energies, provided that one of the helicity amplitudes dominates and provided that there are no direct channel contributions.

(c) The photoproduction of Δ^{++} in the backward direction is also dominated by the Δ trajectory,[6] and must be related to $\bar{p}\,\Delta^{++} \to \pi^+\gamma$ by

$$\frac{d\sigma}{du}(\bar{p}\,\Delta^{++} \to \pi^+\gamma) = \frac{1}{2}\left(\frac{q_s}{q_t}\right)^2 \frac{d\sigma}{du}(\gamma p \to \Delta^{++}\pi^-) \ . \tag{9}$$

Unfortunately there is no hope presently of checking this relation experimentally.

2. In $\pi^+ p \to p\pi^+$ there are several trajectories contributing, but in the $u \approx 0$ region it is believed that the N_α trajectory dominates. Therefore in this restricted

region of small u we expect:

$$\frac{1}{2}\frac{d\sigma}{du}(\pi^+ p \to p\pi^+) = \left(\frac{q_t}{q_s}\right)^2 \frac{d\sigma}{du}(\bar{p}p \to \pi^- \pi^+) \ . \tag{10}$$

3. The classic example for Class 2 is a relation between backward $K^+ p \to pK^+$ and $\bar{p}p \to K^+ K^-$ scattering. In this case there are no direct channel resonances in $K^+ p$ system, nor is there a backward peak in $K^- p$ elastic scattering. As a result the u-channel exchanges must be strongly degenerate, as it is confirmed in part by the Chew-Frautschi plots. An equation similar to (10) must hold. Such a relation can also hold at low energies since the direct channel effects are small.

4. There have been several recent proposals for an approximate exchange degeneracy between the $N_\alpha(1/2^+, 5/2^+, \ldots)$ and the $N_\gamma(3/2^-, 7/2^-, \ldots)$ trajectories:

(a) Several theoretical models[7] predict them to be exchange degenerate.

(b) The absence of the N_α dip in $pp \to D\pi^+$ can be explained[8] in terms of a duality argument by assuming almost degenerate N_α and N_γ trajectories.

(c) The π^+ and π^0 photoproduction data cannot be explained by a simple model of $(\Delta + N_\alpha)$. Phenomenological models[9] including the N_γ give a trajectory which is almost degenerate to the N_α in the region $-0.5 \lesssim u \lesssim 0.0$ $(\mathrm{BeV}/c)^2$.

(d) Figure 2 strongly suggests that in the region of $-0.2 < u < 0.0$ they are almost equal. These proposals can be checked by measuring the differential cross section for $\pi^- p \to \bar{p}D$ with the $\bar{p}$ moving in the direction of π^-. The two processes must satisfy

$$\frac{d\sigma}{du}(\pi^- p \to \bar{p}D) = 2\left(\frac{q_s}{q_t}\right)^2 \frac{d\sigma}{du}(pp \to D\pi^+) \tag{11}$$

only in the limited region of u described above.

The cross sections for $\pi^- p \rightarrow \bar{p}D$ predicted through (11) are large enough to be attractive to experimentalists. For example at $E_{lab} \approx 9$ BeV/c and $-2.5 < u < 0.0$, the differential cross section is $\sim 1\ \mu b/(BeV/c)^2$. In addition the final state has a well-defined signature, to make this a clean and interesting experiment.

A relation can also be obtained between $\gamma p \rightarrow n\pi^+$ and $\bar{p}n \rightarrow \pi^- \gamma$, provided that the Δ contribution is small. Such a relation is less rigorous and much more difficult to check experimentally.

REFERENCES

1. In the asymptotic limit, $s \to \infty$, $t = 0$ such results follow from more general considerations as has been stressed by L. Van Hove, Phys. Letters 5, 252 (1963). Specific applications in the asymptotic region have been discussed by:

   (a) A. A. Logunov et al., Phys. Letters 7, 69 (1963);

   (b) A. Białas and O. Cryzewski, Phys. Letters 13, 337 (1964);

   (c) V. Barger and D. Cline, Phys. Letters 25B, 415 (1967).

2. The residues corresponding to the amplitudes $A_i(u,s)$ still contain kinematic singularities which do not affect the argument of this paper.

3. Frederick J. Gilman, "Tests of exchange degeneracy by processes related by crossing," Report No. SLAC-PUB-576, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969).

4. Ibid, Refs. 1b, 1c.

5. E. W. Anderson et al., Phys. Rev. Letters 22, 1390 (1969).

6. R. L. Anderson et al., "High energy photoproduction of charged pions at backward angles," Report No. SLAC-PUB-631, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969).

7. J. E. Mandula, J. Weyers and G. Zweig, Report No. CALT-PREPRINT-68-193, California Institute of Technology, Pasadena, California (1968); V. Barger and C. Michael, Report No. COO-881-238, University of Wisconsin, Madison, Wisconsin (1969).

8. V. Barger and C. Michael, Phys. Rev. Letters 22, 1330 (1969).

9. D. Tompkins et al., "π^0 photoproduction from hydrogen at backward angles," Report No. SLAC-PUB-604, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969); Ibid, Ref. 6;

J. V. Beaupre and E. A. Paschos, "A phenomenological analysis of π^+ and π^0 photoproduction at backward angles," Report No. SLAC-PUB-632, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969);

V. Barger and P. Weiler, Report No. COO-251, University of Wisconsin, Madison, Wisconsin (1969).

LIST OF FIGURES

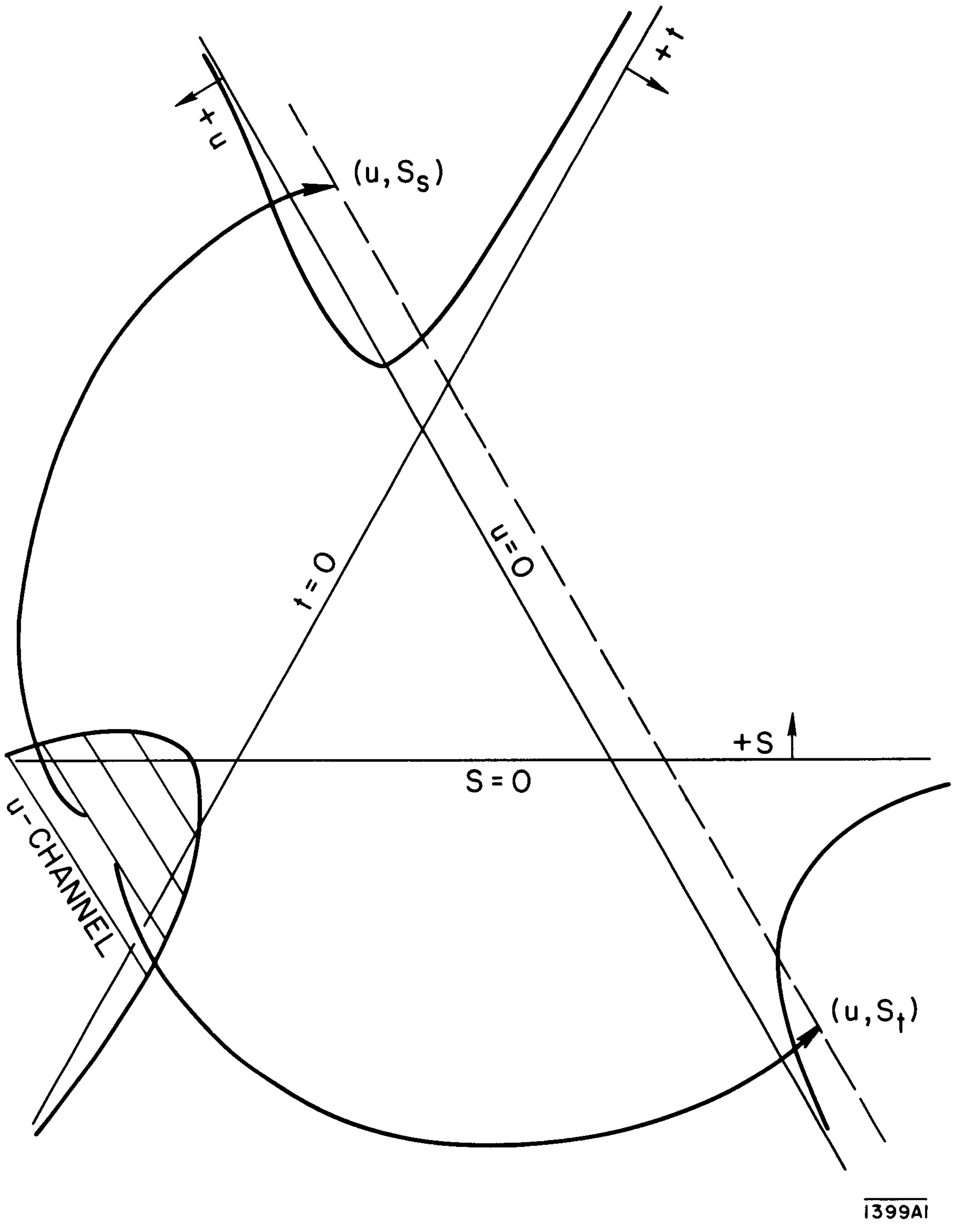

Fig. 1

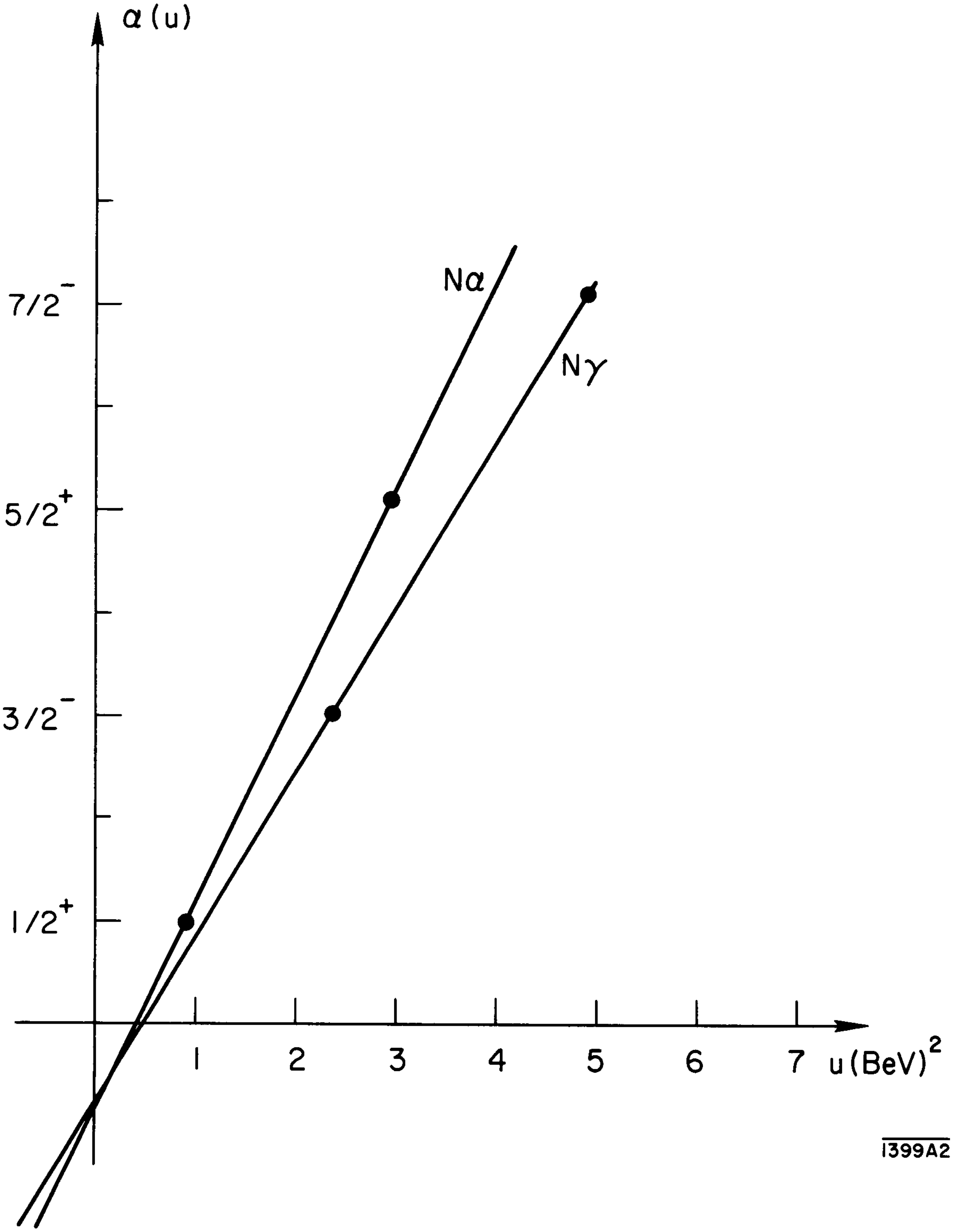

Fig. 2