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A SIMPLE DIODE MODEL INCLUDING CONDUCTIVITY MODULATION*

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SUMMARY

A simple lumped component diode model is presented including a representation of conductivity modulation, in addition to the usual characterization by diffusion capacitance, transition capacitance, and an ideal junction. It is shown that this model employed in a switching circuit exhibits the overshoot and oscillation characteristic of diodes at high forward currents, as well as the usual charge storage effects for the reverse transient. Using small-signal analysis it is shown that in some cases the model possesses an inductive impedance which is also characteristic of diodes at high forward currents.

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I. INTRODUCTION

The purpose of this paper is to present a simple junction diode model consisting of lumped components which is useful for intuitive and analytical circuit analysis and which is particularly appropriate for circuit analysis by computer. Using this model, analytical and computer analyses of diode switching transients demonstrate that the model exhibits many of the diode phenomena observed in switching circuits. Small-signal sinusoidal analysis shows the so-called "inductive effects" at high forward currents.[1]

Although lumped models are found in the literature[2-6] these models rarely include bulk resistance; when it is included, it is usually assumed constant, thus conductivity modulation is neglected. Without conductivity modulation, neither pulse overshoot (or forward recovery) nor small-signal inductive behavior in forward biased diodes can be accounted for.

The model presented in this paper includes a simple representation of conductivity modulation as a variable resistor based on the works of Ko[7] and Ladany,[1] who have analyzed the physics of forward conduction and conductivity modulation.

II. DEVELOPMENT OF THE SIMPLE MODEL

In the development of the model, the following guidelines were followed. (1) It should consist of conventional lumped components; (2) these components may be variable and dependent on internal diode parameters, if necessary; (3) the model need not be "exact" but it should be complete, that is, it should demonstrate commonly observed phenomena, and in particular the forward recovery transient; (4) the model should be simple, with each component relating to a physical process, if possible.

The complete model is shown in Fig. 1; a short discussion of the components in the model follows. Several parasitic elements such as surface leakage, lead inductance, and external capacitance have been omitted; these parameters are normally dominated by diode characteristics or other external circuit elements.

A. DC Characteristics of an Ideal Junction

It is generally agreed that the exponential form is a simple, but quite good description of the p-n junction conduction current characteristic:

$$i_j = I_0 \left(e^{v_j/V_T} - 1 \right) \tag{1}$$

or

$$v_j = V_T \ln \left(1 + \frac{i_j}{I_0} \right) \tag{2}$$

where I_0 is the reverse saturation current, v_j is the applied junction voltage, and V_T is $nkT/q \approx 26$ mV to 52 mV at room temperature for silicon. The junction can be represented by a variable resistor:

$$r_i = \left(\frac{di_j}{dv_j} \right)^{-1} = \frac{V_T}{i_j + I_0} \quad . \tag{3}$$

B. Diffusion Capacitance of the Junction

Since the junction voltage, and consequently the current, is associated with a buildup of excess minority carrier density, a related diffusion capacitance develops. Under the assumption of low level injection and exponential charge distribution, the minority carrier charge Q can be shown to be $Q = \tau_0 i_j$, and the incremental diffusion capacitance can be derived as

$$C_d \triangleq \frac{dQ}{dv_j} = \frac{\tau_0}{V_T} (i_j + I_0) \tag{4}$$

where τ_0 is the lifetime of the minority carriers.[9]

The diffusion capacitance shown in Fig. 1 is a function of the conduction current i_j, not of the terminal current, since the conduction current and the diffusion capacitance are both controlled by the same physical parameter: the excess minority carrier distribution. The representation of the diffusion capacitance as a single capacitor corresponds to the so-called single lump or charge storage model.

C. Transition Capacitance of the Junction

Another type of charge storage and related capacitance develops as a result of the changing thickness of the depletion region. It can be shown[10] that this incremental capacitance is of the form

$$C_t = \frac{C_0}{\left(1 - \frac{v_j}{V_0}\right)^m} \tag{5}$$

where C_0 and V_0 are constants of the physical construction and material, and m varies between 1/2 and 1/3 depending upon the type of junction.

D. Conductivity Modulation

The junction is connected to the terminals of the device by a finite amount of semiconductor material which possesses electrical resistance. Since minority carrier excess charges accumulate in some portion of this material, with resulting changes in the majority carrier density, the total resistance of the semiconductor bulk varies, although at low currents the change may be negligible. The phenomenon of variable bulk resistance is termed conductivity modulation and, although it is rarely discussed in basic semiconductor texts, there are several excellent papers on the subject.[1,7,11] Ko[7] has shown that for an ideal planar junction diode with zero p-region resistance and with low minority carrier density, the bulk resistance, assuming exponential charge distribution, is given by

$$r_s = R_0 \left[1 - \frac{1}{X_n} \ell n \; \frac{1 + K_r i_j}{1 + K_r i_j e^{-X_n}} \right] \tag{6}$$

where X_n is the ratio of the width of the n-region to the minority carrier diffusion length, R_0 is the intrinsic bulk resistance with no excess charge, and K_r is a constant for the diode materials, doping, and geometry. It is important to note that although Ko treats only the case of a step function change in forward current, Eq. (6) is more general, relating the bulk resistance to the conduction current or excess charge.

A considerable simplification of Eq. (6) may be achieved by assuming $X_n \ll 1$. Under this assumption, corresponding to a very narrow base diode in which the excess density is uniform, Eq. (6) can be reduced to the following simple form,

$$\frac{1}{r_s} = g_0 + \frac{i_j}{V_s} \tag{7}$$

where $g_0 = 1/R_0$ and $1/V_s$, defined as K_r/R_0, is the coefficient of conductivity modulation.

Although Eq. (7) does not express the conductivity modulation for a general diode, for simplicity this model will be used for the computation of diode transients. Later it will be shown that the transients obtained from this simpler model of the physical phenomena exhibit characteristics similar to those obtained from the more general and mathematically more complex resistance function described by Eq. (6).

Note that in either case, the diode model is essentially the same; only the particular function for r_s changes.

III. COMPUTATION OF DIODE SWITCHING TRANSIENTS WITH $C_t = 0$, $R_g \to \infty$

To demonstrate the switching characteristics of the diode model, diode transients will be analyzed for the circuit of Fig. 2 with $C_t = 0$, and $R_g \to \infty$.* These simplifications permit an explicit solution; the more general case is treated later. The driving source is a square wave current generator in which the durations of the positive and negative currents are much longer than the minority carrier lifetime, τ_0.

The terminal voltage of the diode can be written as

$$v_{in} = v_j + \frac{i_{in}}{g_0 + \frac{i_j}{V_s}} \quad . \tag{8}$$

Since $R_g \to \infty$, the current through it is $\ll |i_g|$, and one can write

$$i_g \cong i_j + i_c \quad . \tag{9}$$

Also,

$$i_c = C_d \frac{dv_j}{dt} = C_d \frac{dv_j}{di_j} \cdot \frac{di_j}{dt} = C_d r_i \frac{di_j}{dt} = \tau_0 \frac{di_j}{dt} \tag{10}$$

Substituting the result of Eq. (10) into Eq. (9) one obtains a linear differential equation:

$$i_g = i_j + \tau_0 \frac{di_j}{dt} \quad . \tag{11}$$

The solution of Eq. (11) yields the following turn-on transient;

$$v_{in} \approx V_T \left[\ell n \frac{I_{g+}}{I_0} + \ell n \left(1 - e^{-t/\tau_0}\right) + \frac{V_s/V_T}{\frac{g_0 V_s}{I_{g+}} + \left(1 - e^{-t/\tau_0}\right)} \right] \quad , \tag{12}$$

when the last term represents the voltage drop on the resistor, r_s.

It is of particular interest to examine the characteristics of the forward transient. Using the derivative of v_{in} to analyze the slope of the waveform, it can be shown that the nature of the forward transient is determined by the value of I_{g+}/g_0V_s. When $I_{g+}/g_0V_s < 4$, the terminal voltage monotonically increases, and there is no overshoot, as in the case of conventional diode analysis in which r_s is neglected. In the range $4 < I_{g+}/g_0V_s < 4.536$ the transient has a local maximum which is below the final value of the voltage. For larger values of current, when $I_{g+}/g_0V_s > 4.536$, the terminal voltage has an overshoot followed by an undershoot. As I_{g+} continues to increase beyond $4.536\ g_0V_s$, the overshoot further increases, but the magnitude of the undershoot quickly decreases and becomes insignificant. These possibilities are illustrated in Fig. 3b with practical forward currents, I_{g+} in the vicinity of $e^{20}I_0$ (e.g., if $I_0 = 1\text{nA}$, $I_{g+} \approx 5\text{mA}$) for a particular diode with $I_0 = 4\ e^{20} g_0 V_s$. This choice of I_0 is arbitrary; a different choice would only shift v_{in} by a constant (see Eq. (12)). These forward transient characteristics are in good qualitative agreement with those described in the literature.[7]

Again from Eq. (11) the <u>turn-off</u> transient of the diode, assuming that equilibrium conditions are established by the time t_{off}, is given for the asymptotic case of $R_g \to \infty$ by

$$v_{in} = V_T \ln\left\{1 + \frac{1}{I_0}\left[-I_{g-} + \left(I_{g+} + I_{g-}\right) e^{-(t-t_{off})/\tau_0}\right]\right\} - \frac{1}{\frac{g_0}{I_{g-}} + \frac{1}{V_s} e^{-(t-t_{off})/\tau_0}} \tag{13}$$

In addition, one can write a simple expression for the storage time t_s, defined as the turn-off delay when the stored charge is zero, corresponding to $i_j = 0$ or $v_j = 0$:

$$t_s = \tau_0 \ln\left(1 + \frac{I_{g+}}{I_{g-}}\right) \quad . \tag{14}$$

The transients given by Eqs. (12) and (13), plotted in Fig. 3, demonstrate the general effect of forward current on the overshoot and illustrate the storage time after the current reversal. The nature of the undershoot in the forward transient can be clearly seen in Fig. 3b where the scale of the terminal voltage is expanded. The reverse transient in Fig. 3a shows the instantaneous drop caused by the reversal of terminal current in r_s, followed by a delay t_s in the turn-off resulting from the stored charge in C_d. The storage time shown in Fig. 3 remains essentially constant since the ratio of I_{g+} to I_{g-} remains constant (cf. Eq. (14)).

IV. SWITCHING TRANSIENTS IN THE GENERAL CASE

If R_g is finite or C_t is not zero, the solution of the switching transient of the circuit in Fig. 2 becomes considerably more involved and solution is performed by digital computer. In addition to the model defining Eqs.(3), (4), (5), and (7), the following equations are used:

$$i_{in} = \frac{i_g R_g - v_j}{R_g + r_s} \tag{15}$$

$$v_{in} = (i_g - i_{in}) R_g \tag{16}$$

$$i_c = i_{in} - i_j \tag{17}$$

$$v_j = \int \frac{i_c}{C_d + C_t} dt \tag{18}$$

Equation (18) can be approximated by a finite sum:

$$v_j (t + \Delta t) = v_j(t) + \Delta v_j \tag{19}$$

where

$$\Delta v_j \approx \frac{i_c \Delta t}{C_d + C_t} \tag{20}$$

The above equations are then evaluated on a digital computer in a step-by-step fashion using the flowchart of Fig. 4(a); the computation of step size is a self-adjusting procedure within the main flowchart as detailed in Fig. 4(b) with $\Delta t_{max} = 0.1\ \tau_0$, $\Delta t_{min} = 10^{-6}\ \tau_0$, and $\Delta v_{max} = 0.1\ V_T$. The Fortran IV program implementing these flowcharts is shown in Fig. 5.

Representative computer generated transients for $R_g \to \infty$, but with finite C_t, C_d, and r_s, are shown in Fig. 6. These curves indicate much the same effects as shown previously in Fig. 3 except that a turn-on delay is now apparent, and the turn-off transients, in addition to the delay caused by the stored charge, now exhibit a slope due to the finite C_t. The most general case is shown in Fig. 7, which includes finite R_g as well as C_t, C_d, and r_s.

V. SWITCHING TRANSIENTS WITH THE GENERALIZED MODEL FOR CONDUCTIVITY MODULATION

Turn-on transients for the circuit of Fig. 2 will be computed with the generalized model for bulk resistance r_s as given by Eq. (5). For $R_g \to \infty$ and $C_t = 0$ the solution for the junction turn-on current is the same as for Eq. (11), which when substituted into Eq. (6) gives a new value for r_s. Then the diode terminal voltage can be written as the sum of the junction voltage and the bulk resistance drop:

$$v_{in} = v_j + i_g R_0 \left[1 - \frac{1}{X_n} \ln \frac{1 + K_r i_j}{1 + K_r i_j e^{-X_n}} \right] \tag{21}$$

where

$$K_r = \frac{1}{g_0 V_s} \quad .$$

Turn-on transients for $X_n = 1$ are plotted in Fig. 8 with an expanded scale for the terminal voltage. Generator currents and other diode parameters are the same as in Fig. 3(b). Qualitatively the transients of Fig. 8 are similar to those of Fig. 3(b) which are based upon the simpler version of conductivity modulation. Note that the amount of undershoot is reduced even further from the small undershoot seen in Fig. 3(b).

VI. SMALL-SIGNAL IMPEDANCE

It is possible to use the model developed in this paper to show that the small-signal impedance of the junction diode possesses an inductive component at high forward current levels. This effect has been analyzed[1] and measured.[12]

The model used in the analysis is the same as shown in Fig. 1 with $C_t = 0$, and $1/r_s = g_0 + i_j/V_s$. Using Eq. (11)

$$i_{in} = i_j + \tau_0 \frac{di_j}{dt} \quad . \tag{22}$$

The conduction current will be written $i_j = I^* + i$, where I^* is the dc part, and i is the small-signal part. (In the dc case $I^* = i_{in}$.) Therefore Eq. (22) becomes

$$i_{in} = I^* + i + \tau_0 \frac{di}{dt} \quad . \tag{23}$$

Using the complex exponential form for a small-signal sinusoidal i, $di/dt = j\omega i$, and Eq. (23) becomes

$$i_j = \frac{i_{in} + j\omega\tau_0 I^*}{1 + j\omega\tau_0} \tag{24}$$

Using Eq. (2) and (8) the diode terminal voltage can now be written in terms of i_{in}:

$$v_{in} = V_T \ln\left(1 + \frac{1}{I_0}\,\frac{i_{in} + j\omega\tau_0 I^*}{1 + j\omega\tau_0}\right) + \frac{i_{in}}{g_0 + \frac{1}{V_s}\,\frac{i_{in} + j\omega\tau_0 I^*}{1 + j\omega\tau_0}} \tag{25}$$

From this, the small signal impedance can be shown to be:

$$Z_{in} \triangleq \left.\frac{dv_{in}}{di_{in}}\right|_{i_{in}=I^*} = \frac{1}{\left(g_0 + \frac{I^*}{V_s}\right)^2}\left[g_0 + \frac{\frac{V_T}{I^* + I_0}\left(g_0 + \frac{I^*}{V_s}\right)^2 + \frac{I^*}{V_s}(\omega\tau_0)^2}{1 + (\omega\tau_0)^2} + j\omega\tau_0\,\frac{\frac{I^*}{V_s} - \frac{V_T}{I^* + I_0}\left(g_0 + \frac{I^*}{V_s}\right)^2}{1 + (\omega\tau_0)^2}\right] \tag{26}$$

Thus the small-signal impedance consists of resistive and reactive portions, both of which are dependent upon the dc current and upon the signal frequency, as well as upon the physical parameters of the diode. However, the sign of the reactance is not dependent on the frequency. It is seen from Eq. (26) that for negative I^* $(0 \geq I^* > -I_0)$ the junction is reverse-biased and the reactance is always capacitive.

However, for forward currents the reactance will be <u>inductive</u> if $V_s > V_T$ and

$$I^* \geq \frac{g_0 V_s \sqrt{V_T}}{\sqrt{V_s} - \sqrt{V_T}} \tag{27}$$

This result, that the reactance may be inductive in certain cases for high forward currents, agrees in general with published analytical results on diode impedance, except at very high frequencies; it is also in good agreement with measured characteristics of diode impedance at forward currents.[12]

VII. SUMMARY

A simple diode model has been presented which includes the effects of conductivity modulation. It has been shown that this model exhibits transient overshoot and oscillation, and small-signal inductive impedance at high forward currents. The model is useful for computer network analysis and synthesis, and for general qualitative investigations.

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LIST OF FOOTNOTES

Manuscript received________________;

revised_______________________________.

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* The resistor R_g can be arbitrarily large, but it cannot be omitted since the diode junction can support a maximum current of I_0 in the reverse direction; thus R_g is necessary to sink current when $-I_g < -I_0$.

LIST OF FIGURES

1. The diode model.

2. (a) The switching circuit to demonstrate forward and reverse transients.
   (b) The circuit of (a) incorporating the diode model of Fig. 1.
   (c) The current generator waveform.

3. (a) Transient response of a diode with $R_g \rightarrow \infty$, $C_t = 0$, and finite r_s, for different values of normalized generator current.
   (b) Same as 3(a) with expanded vertical scale.

4. (a) Flowchart of the computer program for computing the transient response of the diode.
   (b) Flow chart for calculation of step size.

5. Fortran IV computer program for the flowchart of Fig. 4.

6. Transient response of diode with $R_g \rightarrow \infty$, finite C_t, and finite r_s, for different values of normalized generator current.

7. Transient response of diode with finite R_g, finite C_t, and finite r_s, for different values of generator current.

8. Transient response of diode using "exact" resistance with $X_n = 1$, $R_g \rightarrow \infty$, and $C_t = 0$, for different values of normalized generator current.

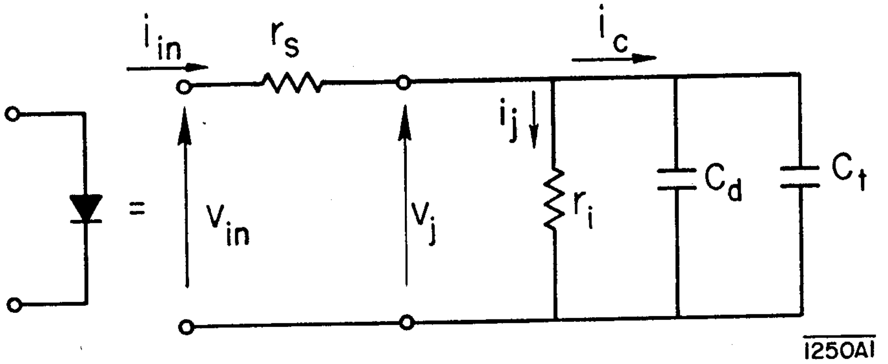

Fig. 1

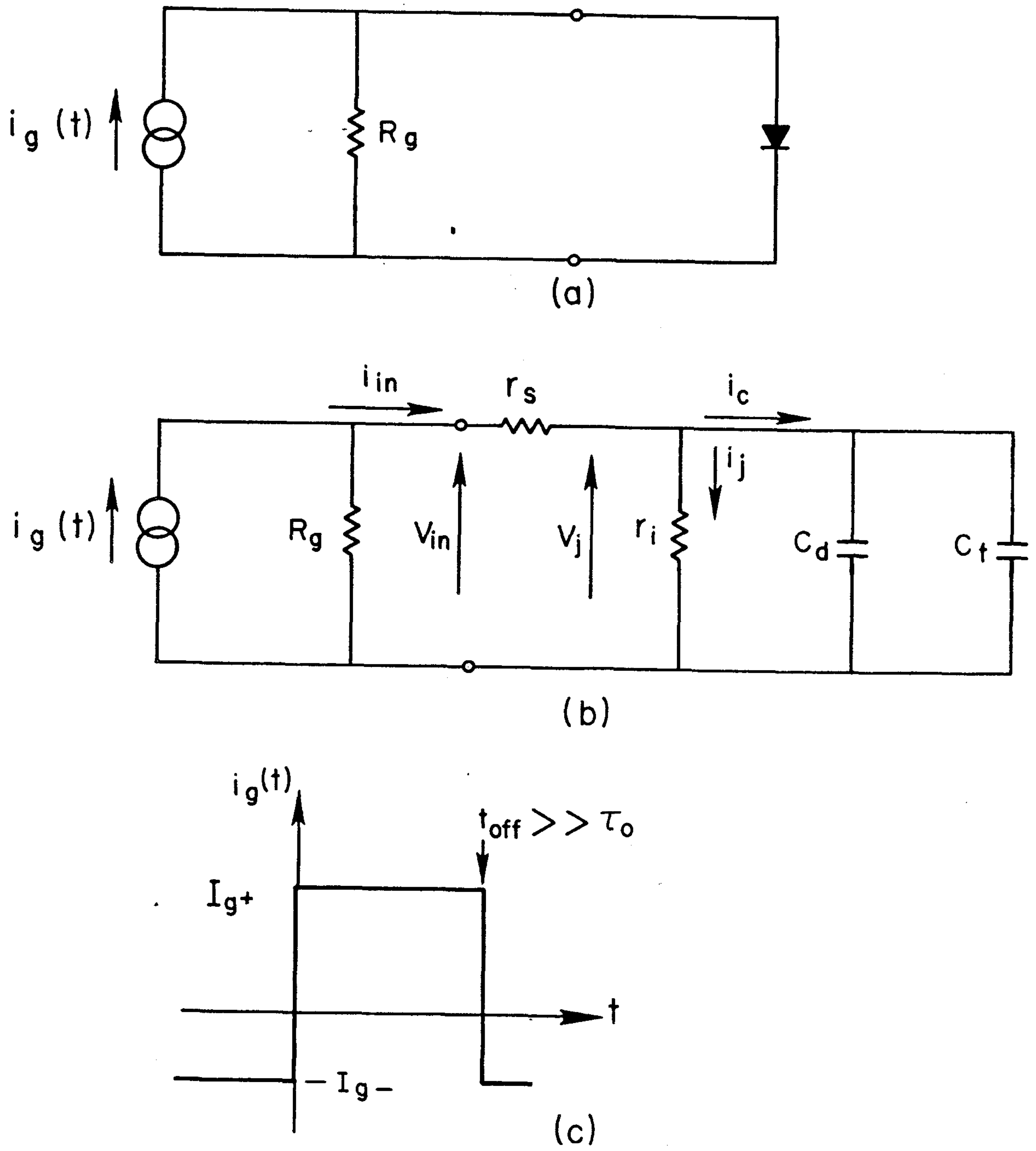

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Fig. 2

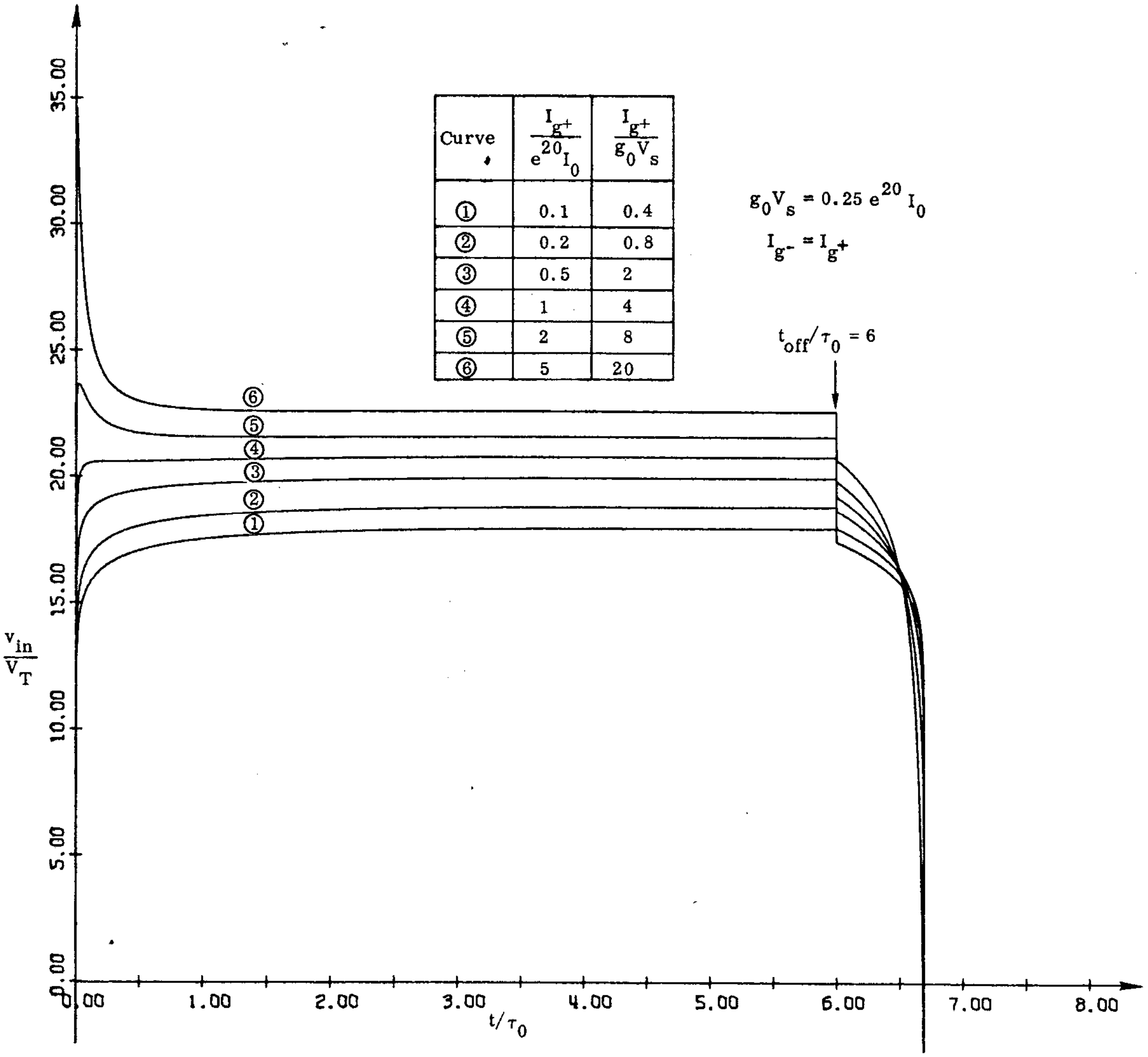
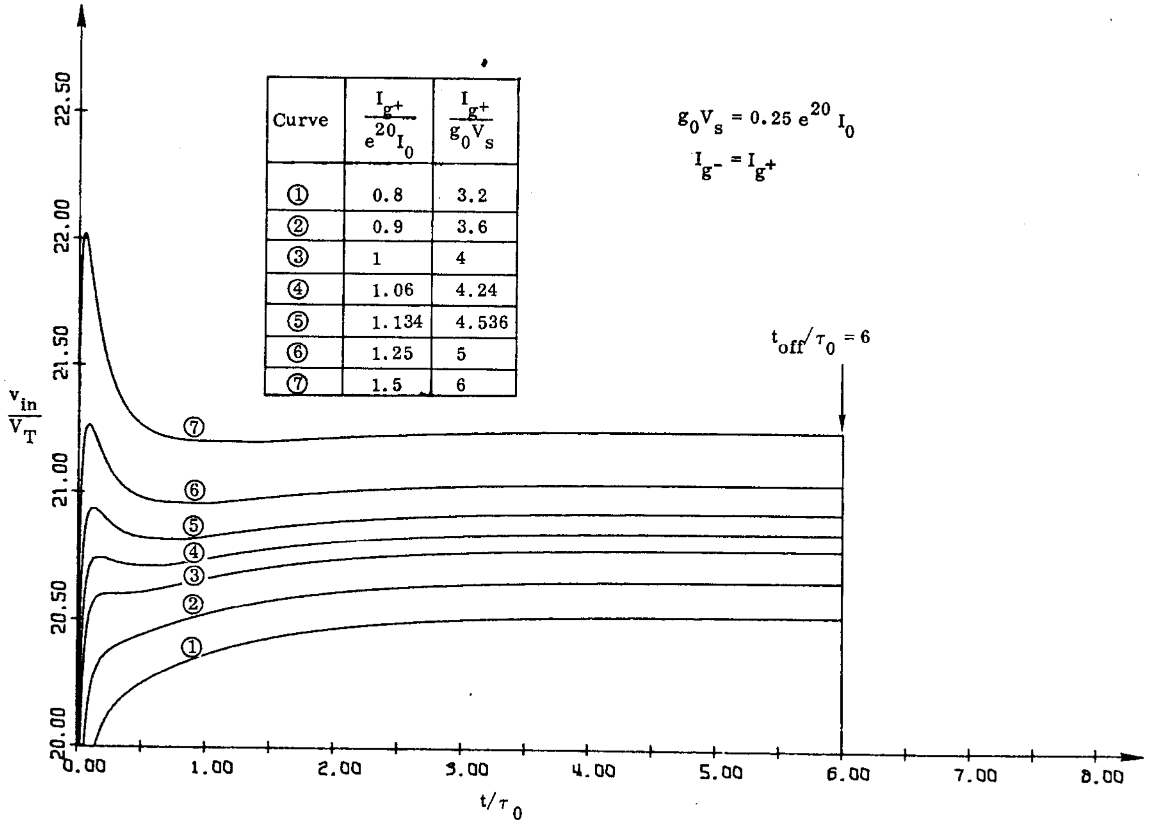

35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
$\frac{v_{in}}{V_T}$
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
t/τ_0
Curve #
$\frac{I_{g+}}{e^{20} I_0}$
$\frac{I_{g+}}{g_0 V_s}$
① 0.1 0.4
② 0.2 0.8
③ 0.5 2
④ 1 4
⑤ 2 8
⑥ 5 20
$g_0 V_s = 0.25\, e^{20}\, I_0$
$I_{g-} = I_{g+}$
$t_{off}/\tau_0 = 6$
⑥
⑤
④
③
②
①

Fig. 3a



Curve	$\frac{I_{g+}}{e^{20} I_0}$	$\frac{I_{g+}}{g_0 V_s}$
①	0.8	3.2
②	0.9	3.6
③	1	4
④	1.06	4.24
⑤	1.134	4.536
⑥	1.25	5
⑦	1.5	6

$g_0 V_s = 0.25\, e^{20}\, I_0$

$I_{g-} = I_{g+}$

Fig. 3b

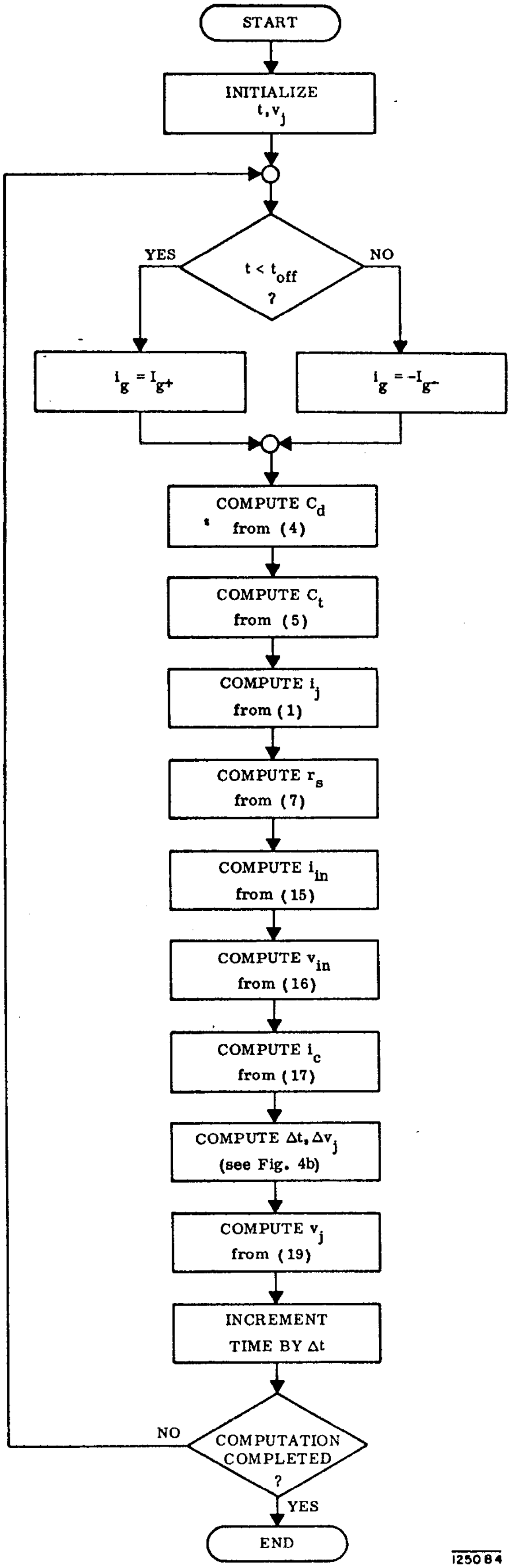

START
INITIALIZE
t, v_j
YES
t < t_off
?
NO
i_g = I_g+
i_g = -I_g-
COMPUTE C_d
from (4)
COMPUTE C_t
from (5)
COMPUTE i_j
from (1)
COMPUTE r_s
from (7)
COMPUTE i_in
from (15)
COMPUTE v_in
from (16)
COMPUTE i_c
from (17)
COMPUTE Δt, Δv_j
(see Fig. 4b)
COMPUTE v_j
from (19)
INCREMENT
TIME BY Δt
NO
COMPUTATION
COMPLETED
?
YES
END
125084

Fig. 4a

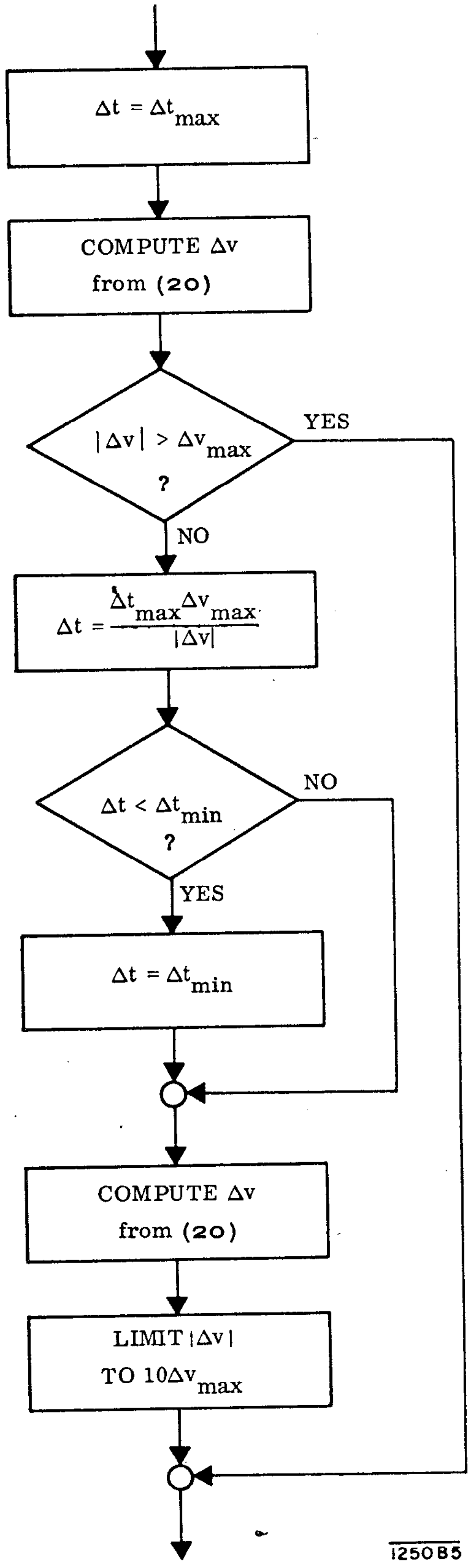

Δt = Δt_max
COMPUTE Δv from (20)
\|Δv\| > Δv_max ?
YES
NO
Δt = Δt_max Δv_max / \|Δv\|
Δt < Δt_min ?
NO
YES
Δt = Δt_min
COMPUTE Δv from (20)
LIMIT \|Δv\| TO 10Δv_max
125085

Fig. 4b

```
C      *** DIODE TRANSIENT***
C
       FUNCTION CQ(V)
       IF(V.LT.-100.0)V=-100.0
       IF(V.GT.+140.0)V=+140.0
       CQ=EXP(V-20.0)
       RETURN
       END

       FUNCTION DIODE(V)
       IF(V.LT.-100.0)V=-100.0
       IF(V.GT.+120.0)V=+120.0
       DIODE=EXP(V)-1.0
       RETURN
       END


       REAL ID,IC,IG,L
       DOUBLE PRECISION V
       CALL STRTP1(29)
       CALL PLOT1(0.0,-30.0,23)
       CALL PLOT1(0.0,0.5,23)
     1 FORMAT(6F10.4,I1)
     2 FORMAT(' ','RISETIME=',1PE10.3,'  TURNOFFTIME=',1PE10.3)
     3 FORMAT(' ',7F10.4,I1)
     4 FORMAT('1')
     5 FORMAT(' ')
     6 FORMAT(F10.4)
       ARG1=-20.0
       EXP1=EXP(ARG1)
     7 CONTINUE
       READ(5,6)CTR
       IF(CTR.LE.0.0)GO TO 100
       IF(CTR.LT.0.001)CTR=0.0
       WRITE(6,4)
    11 CONTINUE
       READ(5,1)G,GN,R1,DELT,L,C1,NEWPLT
       IF(G.LE.0.0)GO TO 7
       WRITE(6,5)
       WRITE(6,3)G,GN,R1,DELT,L,C1,CTR,NEWPLT
       IF(C1.GT.100.0)C1=1E20
       DELT2=1E-6
       CALL PLOT1(0.0,0.0,+3)
       IF(NEWPLT.EQ.0)GO TO 12
       CALL PLOT1(15.0,0.0,-3)
       CALL AXIS1(0.0,2.0,'T',-1,10.0,0.0,0.0,1.0,20.0)
       CALL AXIS1(0.0,0.0,'V',+1,10.0,90.0,-10.0,5.0,10.0)
    12 CONTINUE
       TRISE=0.0
       TTOFF=0.0
       XPLOTM=-0.02
       V=-GN
       VL=-GN
       IC=0.0
       T=0.0
       XPLOT=0.0
       YPLOT=2.0+0.2*VL
       IF(YPLOT.LE.0.0)YPLOT=0.0
       CALL PLOT1(XPLOT,YPLOT,+3)
    13 CONTINUE
       V1=V
       IF(CTR.LE.0.0)C=CQ(V1)
       IF(CTR.GT.0.0)C=CQ(V1)+CTR/SQRT(1.0-V1*0.04)
       IF(T.LT.6.0)VG=G
       IF(T.GE.6.0)VG=-GN
       ID=DIODE(V)*EXP1
       GS=C1+L*ID
       GS1=1.0/GS
       IG=(VG-V)/(GS1+R1)
       VL=VG-IG*R1
       IC=IG-ID
       DELT1=DELT
       DV=IC*DELT1/C
       ABSDV=ABS(DV)
       IF(ABSDV.LT.0.1)GO TO 16
       DELT1=0.1*DELT1/ABSDV
       IF(DELT1.LT.DELT2)DELT1=DELT2
       DV=IC*DELT1/C
       IF(DV.GT.0.1)DV=0.1
       IF(DV.LT.-0.1)DV=-0.1
    16 CONTINUE
       V=V+DV
       T=T+DELT1
       IF(VL.GT.0.0.AND.TRISE.EQ.0.0)TRISE=T
       IF(VL.LT.0.0.AND.TRISE.NE.0.0.AND.TTOFF.EQ.0.0)TTOFF=T-6.0
       XPLOT=T
       IF(XPLOT-XPLOTM.LT.0.01)GO TO 15
       XPLOTM=XPLOT
       YPLOT=2.0+0.2*VL
       IF(YPLOT.LT.0.0)YPLOT=0.0
       IF(YPLOT.GT.10.0)YPLOT=10.0
    14 CONTINUE
       CALL PLOT1(XPLOT,YPLOT,+2)
       IF(YPLOT.LE.0.0.AND.T.GT.6.0)GO TO 17
    15 CONTINUE
       IF(XPLOT.LE.10.)GO TO 13
    17 CONTINUE
       WRITE(6,2)TRISE,TTOFF
       WRITE(6,5)
       GO TO 11
   100 CONTINUE
       CALL PLOT1(15.0,0.0,-3)
       CALL ENDP1
       STOP
       END
```

Fig. 5

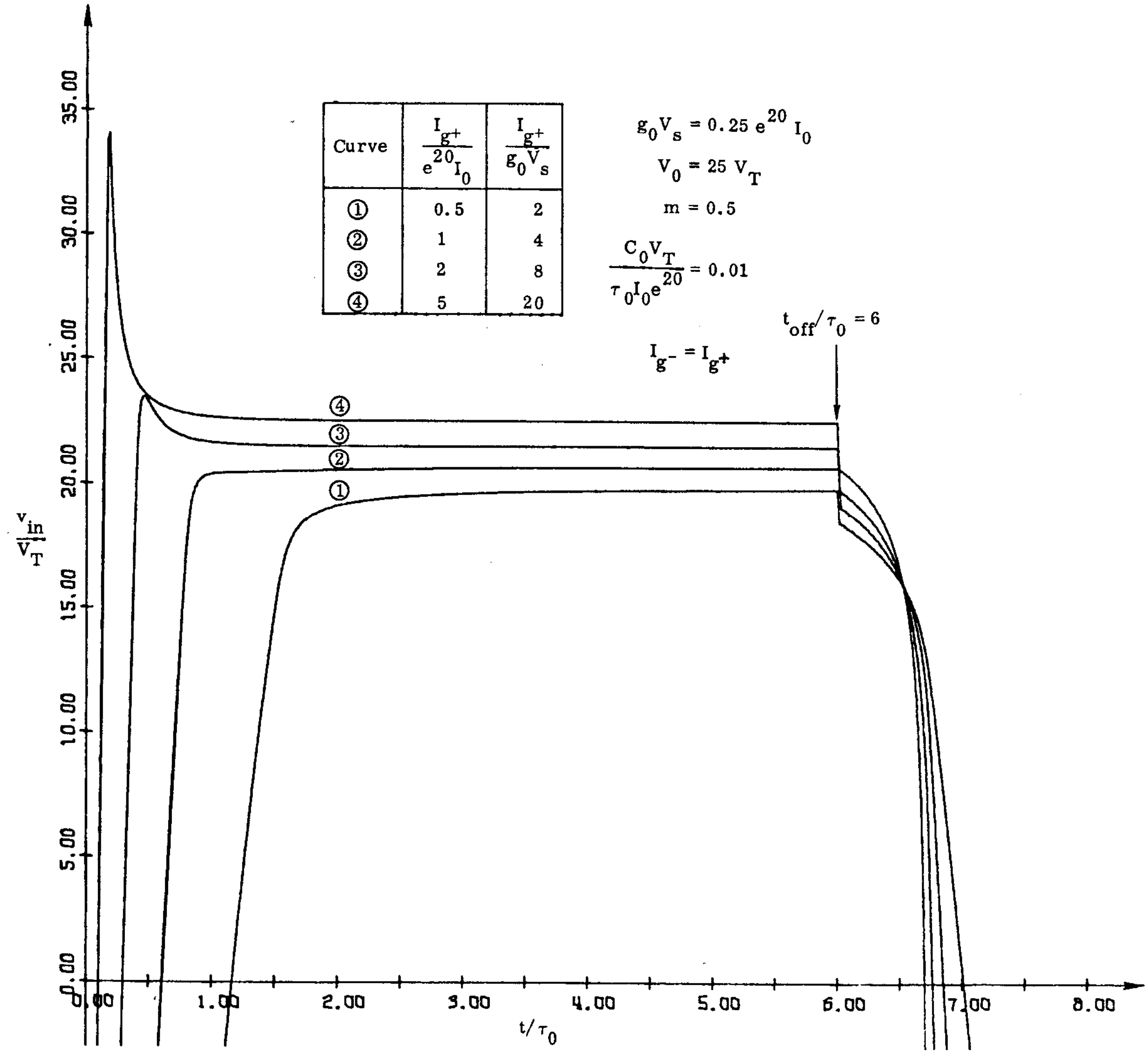

Curve
$\frac{I_{g+}}{e^{20} I_0}$
$\frac{I_{g+}}{g_0 V_s}$
① 0.5 2
② 1 4
③ 2 8
④ 5 20
$g_0 V_s = 0.25\, e^{20}\, I_0$
$V_0 = 25\, V_T$
$m = 0.5$
$\frac{C_0 V_T}{\tau_0 I_0 e^{20}} = 0.01$
$t_{off}/\tau_0 = 6$
$I_{g-} = I_{g+}$
④
③
②
①
$\frac{v_{in}}{V_T}$
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
t/τ_0

Fig. 6

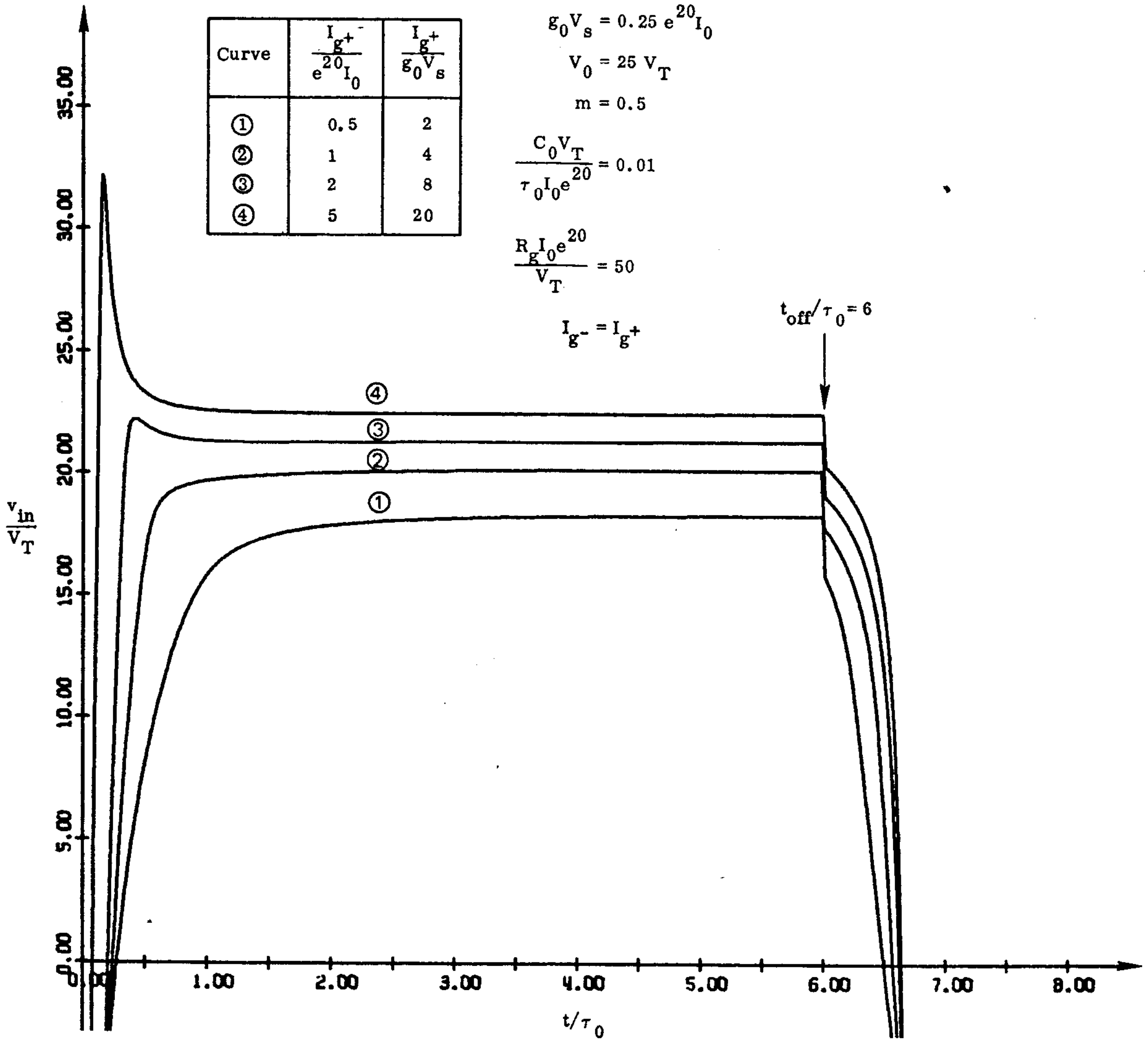
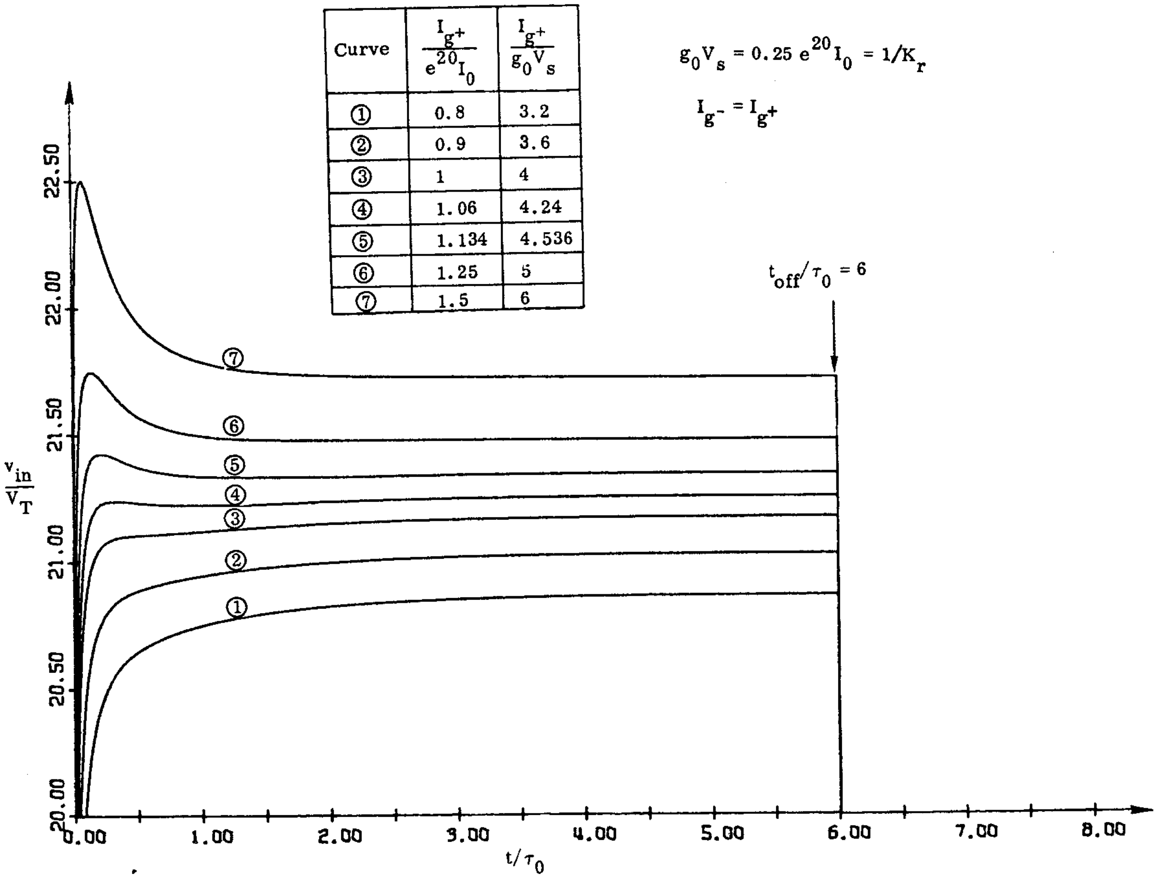

Curve
$\frac{I_{g+}^{-}}{e^{20} I_0}$
$\frac{I_{g+}}{g_0 V_s}$
① 0.5 2
② 1 4
③ 2 8
④ 5 20
$g_0 V_s = 0.25\, e^{20} I_0$
$V_0 = 25\, V_T$
$m = 0.5$
$\frac{C_0 V_T}{\tau_0 I_0 e^{20}} = 0.01$
$\frac{R_g I_0 e^{20}}{V_T} = 50$
$I_{g-} = I_{g+}$
$t_{off}/\tau_0 = 6$
④
③
②
①
$\frac{v_{in}}{V_T}$
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
t/τ_0

Fig. 7



Curve	$\frac{I_{g+}}{e^{20} I_0}$	$\frac{I_{g+}}{g_0 V_s}$
①	0.8	3.2
②	0.9	3.6
③	1	4
④	1.06	4.24
⑤	1.134	4.536
⑥	1.25	5
⑦	1.5	6

$g_0 V_s = 0.25\, e^{20} I_0 = 1/K_r$

$I_{g-} = I_{g+}$

Fig. 8