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INELASTIC SHADOW EFFECTS IN NUCLEAR TOTAL CROSS SECTIONS*

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We have calculated the "shadowing" or double scattering effects in nucleon-nucleus scattering which are associated with inelastic intermediate states of the nucleon and find a significant decrease in the total cross section with increasing energy. The ordinary shadow effect, which can be calculated using the Glauber, or Eikonal, prescriptions, results from the semiclassical fact that when an object scatters on a composite or extended system, some parts of that system may eclipse other parts of it. Thus the cross section for scattering any hadron on a deuteron is expected to be less than the sum of its cross sections on a free neutron and proton, because one nucleon may hide behind the other. The ordinary shadow effect for scattering on a nucleus corresponds to diagrams in which the incident particle interacts with two nucleons, as depicted in Fig. 1b. Between the two interactions the particle is taken to be approximately or exactly on-mass-

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shell. The amplitude corresponding to Fig. 1b has opposite sign to that for Fig. 1a in which just one nucleon is struck (the dominant contribution at small momentum transfer), provided the amplitudes are mainly imaginary in phase, as elastic amplitudes are at high energy. It therefore indeed serves to reduce the total cross section. (The alternate in sign, can also be understood semiclassically.)

The elastic shadow effect given by Fig. 1b can be thought of as an increased transmission through the nucleus, resulting from some of the flux which is scattered out of the incident beam at x being scattered back into it at y. The inelastic shadow effects which we now propose to discuss[1] correspond to Fig. 1b, except that the system which propagates from x to y is no longer identical to the incident particle. In the case of proton-nucleus scattering, for example, it may be a resonance such at N^* (1470) or N^* (1688) or even a nonresonant set of particles such as a nucleon and several pions with low relative velocities mass near threshold. These inelastic states can increase the transmission of incident flux, i.e. decrease the total cross section, in the same way as the elastic ones, assuming that the amplitudes for the inelastic process are also predominantly imaginary.

We have used the missing mass experiment

$$p + p \rightarrow p + p^* \qquad (1)$$

of Anderson et al.[2] to calculate the shadowing effects assiciated with the inelastic intermediate states represented by p^*. We predict a significant increase at high energies in the "screening correction" to proton-deuteron scattering, $\sigma_n + \sigma_p - \sigma_d$, over and above the value given by the usual Glauber elastic calculation[3,4]. The increase amounts to a reduction of the deuteron total cross section by about 1.8 mb at 30 BeV. The elastic screening correction is expected to be about 4 - 5 mb; the total cross section is about 75 mb. (Experimental data indeed seem to show an increase in screening at high energies, but they are open to doubt because of possible systematic errors[5].) The predicted effects for heavy muclei are even greater: eg. the p-Pb total cross section may be reduced by 20% at 30 BeV. Because the inelastic shadow effect increases with nuclear mass number A, we predict a decrease at high energy in the power x which occurs in the approximate rule $\sigma \propto A^x$.[5] ($\sigma \propto A^{1.0}$ corresponds to no shadow effect; $\sigma \propto A^{2/3}$ corresponds to complete shadowing, in which interactions are confined to the nuclear surface.)

In order for an inelastic state of mass m^* to contribute significantly to the shadow effect, the three-momentum transfer $\Delta \cong (m^{*2} - m^2)/2q$ required to produce it must satisfy the coherence requirement $\Delta \cdot R \leq 1$, where R is the radius of the nucleus and q is the incident momentum in the laboratory frame. This is because the interactions at x and y must leave the nucleus in its ground state, which is only likely if Δ is small compared to the typical momentum of a nucleon in the nucleus, which equals $1/R$ by the uncertainty principle (we use $\hbar = c = 1$). The coherence requirement implies that inelastic intermediate states are just beginning to be important at energies of 5 - 10 BeV. The intire spectrum of m^* may contribute coherently at a few hundred BeV; if that spectrum remains concentrated near threshold, as predicted by diffraction-dissociation models.[6]

In order to estimate the inelastic shadow effects, we modify the standard Eikonal model.[7] The wave function of the incident particle inside a nucleus (taken for simplicity to be spherical and homogeneous) is given by

$$\psi^{(+)}(z,\rho) = e^{iqz} e^{-\frac{\lambda}{2} D},$$

where z and ρ are cylindrical coordinates, $D \equiv z+(R^2-\rho^2)^{\frac{1}{2}}$ is the depth penetrated, and

$$\frac{i\lambda}{2} = [\ 1-i\mathrm{Re}f(0)/\ \mathrm{Im}f(0)\ ]\ 3\sigma_N Ai/(8\pi\ R^3)$$

is the complex index of refraction. The wave function of an excited system of mass m^* is

$$\psi^{(+)}(z,\rho,m^*) = (\bar{\lambda}/2) \int_{-(R^2-\rho^2)^{\frac{1}{2}}}^{z} dz'\ e^{iqz'}\ e^{iq'(z-z)} e^{-\frac{\lambda}{2} D} \quad (4)$$

where $\bar{\lambda} = -3iAf(0,m^*)/(qR^3)$

and the cross section for the reaction (1) is given by $d^2\sigma/dtdm^* = |\ f(\theta,m^*)\ |^2$. We assume that the index of refraction for p* is the same as for p, and neglect the possible effects of spin flip and isospin dependence.

The forward elastic nucleon-nucleus amplitude in this model is

$$f_A(0)=f_N(0)(3A/4\pi\ R^3)\int e^{-\lambda\ D/2}\ dr$$
$$+\int dm\ (\frac{d^2\sigma}{dt\ dm*})_0\ (2q/\Delta)\ (\frac{1-i\rho}{1+i\rho})(3A/4\pi\ R^3)^2\int$$
$$(1-e^{-i\Delta D})e^{-\lambda\ D/2}\ dr$$

$$\rho = \mathrm{Re}\ f\ (0,m*)/\mathrm{Im}\ f\ (0,m^2)$$

using the approximate kinematics $q-q' = \Delta \cong (m*^2 - m^2)/2q$. The first term in Eq. (5) is the usual shadow effect; the second is the inelastic shadow effect, treated in lowest order. Performing the integrals and employing the optical theorem, we obtain

$$\sigma_A = \sigma_0 - \sigma_{\mathrm{inelastic}}$$

$$\sigma_0 = 4\pi\ \mathrm{Re}\ [\ \lambda\ \phi\ (\lambda)\ ]$$

$$\sigma_{\mathrm{inelastic}} = \int dm*\ (\frac{d\sigma}{dt}\ {}_{dm*})\ F(q,m*)$$

where $\sigma_{\mathrm{inelastic}}$ is the decrease in total cross section due to inelastic intermediate states, σ_0 is the total cross section with the usual elastic shadowing,

$$\phi(\lambda) \equiv [(1+\lambda\ R)e^{-\lambda R}-1+\lambda^2 R^2/2]\ /\ \lambda^3$$

and

$$F(q,m^*) = 18\, A^2 R^{-6} \Delta^{-1} \mathrm{Im}\ [\phi(\lambda) - \phi(\lambda + 2i\Delta)] \frac{1+i\rho}{1-i\rho}$$

Experimental values for $(d^2\sigma / dtdm^*)$ at 15 BeV are shown in Figs. 2 and 3. The cross section does not change greatly between these two energies, and is thus consistent with a large amount of diffraction dissociation or "Deck effect".[6] Integrating the cross section over m* we obtain, at 30 BeV,

$$\int_{\pi N\ \mathrm{threshold}}^{3.7\ \mathrm{BeV}} \left(\frac{d^2\sigma}{dtdm^*}\right)_0 dm^* = 3.8\mathrm{mb/BeV}^2$$

which is sizable compared to the forward elastic cross section $(d\sigma/dt)_0 = 80\ \mathrm{mb/BeV}^2$.

Figs. 2 and 3 also show the weight function $F(q,m^*)$ of Eq. (7) for various values of the mass number A. The weight functions indeed cut off around $\Delta \cdot R \approx 1$ because of the coherence requirement, which can be understood in the Eikonal picture as follows: $\Psi^{(+)}(z,\rho,m^*)$ in Eq.(4) can be large only if the contributions from various depths $z'+(R^2-\rho^2)^{\frac{1}{2}}$ can add in phase with each other, despite the fact that their momenta (wave numbers) differ by Δ.

Performing the integral in Eq.(6), we obtain the results shown in Table I. In calculating these results, we have used $R=r_0A^{1/3}$ with $r_0=1.3$ F, $\sigma_N=40$mb, and Re f(0)/Im f(0) = -.2. The results are not overly sensitive to these choices. We have extrapolated the spectrum of $(d^2\sigma/dtdm^*)_0$ to masses beyond those for which it was measured, in the manner shown in the figures. Our results are rather insensitive to this extrapolation, because of the coherence cutoff: eg. for $A = 9$ at 15 BeV the extrapolated region contributes only 25% of the inelastic effect; for $A = 207$ at 30 BeV, only 6%. Other reasonable extrapolations would therefore give similar results.

We have used the entire cross section $d\sigma/dtdm^*$ ($pp \to p^*p$) regardless of the stability of the p^*. At low energy for large nuclei, this approximation exaggerates $\sigma_{inelastic}$ because the state p^* will spread in time and not be absorbed on a nucleon with the same amplitude with which it was produced. For a medium mass nucleus the radius $R \approx 5$ Fermi. The approximation requires that the p^* spread $R/\gamma << 1$ F in traversing a distance R. So $\gamma >> 5$ implies $m^* << p_{lab}/s$ or $m^* << 3$ BeV for $p_{lab} = 15$ BeV/c, and $m^* << 6$ BeV for $p_{lab} = 30$ BeV/c. Referring to the figures and discussion of the extrapolation above, we see that the approximation is accurate except for medium nuclei below 15 BeV/c and heavy nuclei for $p_{lab} \approx 15$ BeV/c and below. It will be difficult to accurately calcualte $\sigma_{inelastic}$ at low energies. The method used here would give too large an effect so we can state that at, say 5 BeV, $\sigma_{inelastic}$ is much smaller than at 15-30 BeV. It is premature to compare our results with experiment[8] because of possible systematic experimental errors in the separate experiments at different energies and with different beams, and because of the crudity of our model of the mucleus. Our only real prediction is for the size of the inelastic effects. The most promising way to test those predictions is to measure the energy-dependence of proton-nucleus cross sections, extract the small variation due to the energy-dependence of σ_{pp}, and compare the remaining variation with our prediction.

To calculate the inelastic shadow effect in the case of light nuclei (small A), a homogeneous model for the nucleus is indaequate. A multiple-scattering approach such as the Glauber approximation is needed. As an example, we have calculated the inelastic correction to the proton-deuteron total cross section.

The contribution to the forward p-d amplitude from an inelastic state of mass m* is

$$f_d(0,m^*) = \frac{1}{4\pi^2|\vec{q}|}\int \frac{d\vec{k}\ \ S(\vec{k})\ f(k,m^*)\ f(-k,m^*)}{\sqrt{\vec{q}^2+m^2} - \sqrt{(\vec{q}-\vec{k})^2+m^{*2}} + i\epsilon}$$

where $f(k,m^*)$ is the amplitude for the reaction (1) at momentum transfer $\vec{k}$ and $S(\vec{k}) = \int e^{i\vec{k}\cdot\vec{r}}\ |\psi\ (\vec{r})|^2\ d\vec{r}$ is the nonrelativistic deuteron form factor. The energy denominator in Eq.(10) results from the propagation of the inelastic state and contains the coherence requirement discussed above. For simplicity, we make the approximations $f(k,m^*) = f(0,m^*)\ e^{-(\delta\vec{k}^2/2)}$ and $S(k) = e^{-(\alpha k^2/4)}$ we take $\delta = 5.\ \mathrm{BeV}^{-2}$ in rough accord with the data of Ref. 1, and $\alpha = 134\ \mathrm{BeV}^{-2}$ as in Ref. 3. Our results are insensitive to δ, so it is not even necessary to let it vary with m*.

Using the optical theorem we obtain

$$\sigma_{inelastic} = \int dm \left(\frac{d^2\sigma}{dt\,dm^*}\right)_0 \; F\,(q,m^*)$$

where

$$F\,(q,m^*) = \frac{8}{4\delta + \alpha} \quad \frac{1}{1+\rho^2} \left[(1-\rho^2)e^{-z^2} + \frac{4\rho}{\sqrt{\pi}} D\,(z) \right.$$

$$z = (\delta+\alpha/4)^{\frac{1}{2}} (m^2-m^{*2})/2q$$

$$D\,(z) = e^{-z^2} \int_0^z e^{t^2} dt$$

Performing the integral, we obtain $\sigma_{inelastic} = 1.3$mb at 15 BeV and 1.8mb at 30 BeV. It should be possible to measure these effects by the energy dependence of σ_{pd}, σ_{pp}, σ_{np}. Measurements at a single energy cannot establish the inelastic shadow effect, because of theoretical uncertainties in calculating the contribution of ordinary shadowing.

In π d total cross sections, an energy-dependent contribution to screening has apparently been observed.[5] Using data of Walker et al.[9] we estimate that effects of inelastic states containing three pions may be large enough to explain that result.

Another approach to detecting inelastic shadowing would be to measure dσ on deuterium at momentum transfers large enough that single-scattering is negligible. The momentum transfer dependence of the inelastic effect will be similar to the elastic, but its energy dependence will be different.

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7. Note that it has been suggested that "shadowing" may go to zero at high energy. This suggestion is based on studies (see, e.g., E. Abers *et al.*, Nuovo Cimento **42A**, 365 (1965)) of simple diagrams whose evaluation depends

on the off mass shell behavior of Feynman propagators. We do not yet know the correspondence between the Eikonal approximation which we believe relevant to high energy strong interactions, and Feynman diagrams. If a correspondence exists it may be either: (1) simple Feynman diagrams are relevant but we need a different mechanism for the exchange processes, e.g., fixed cuts (C. Risk, private communication); (2) complicated diagrams, e.g., Reggeons in the direct channel must be considered (see Henyey, Kane, Pumplin, and Ross, submitted to Phys. Rev.)

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9. W. D. Walker, private communication.

Figure Captions

1. (a) Scattering on single nucleons in the nucleus.

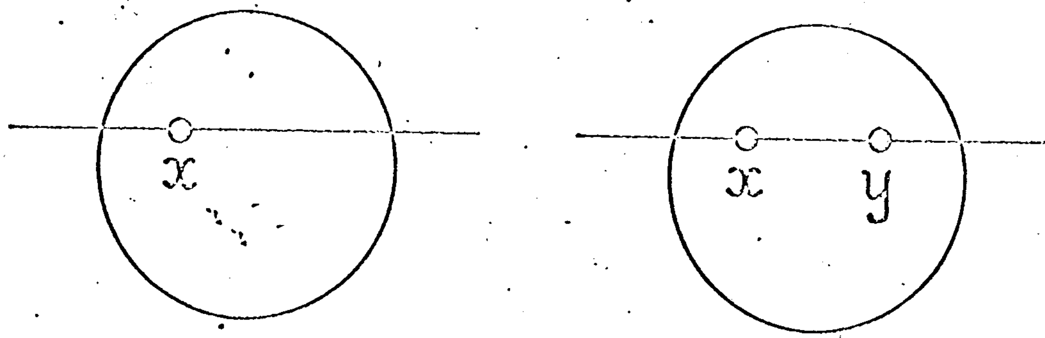
   (b) Double scattering processes (shadowing corrections) with elastic intermediate state p or inelastic intermediate state p^*.

2. Differential cross section at 0° and weight functions $F(m^*)$ for nuclei of mass number A and proton lab momentum 15 BeV/c. The heavy curve shows the extrapolated cross section assumed in numerical work.

TABLE I

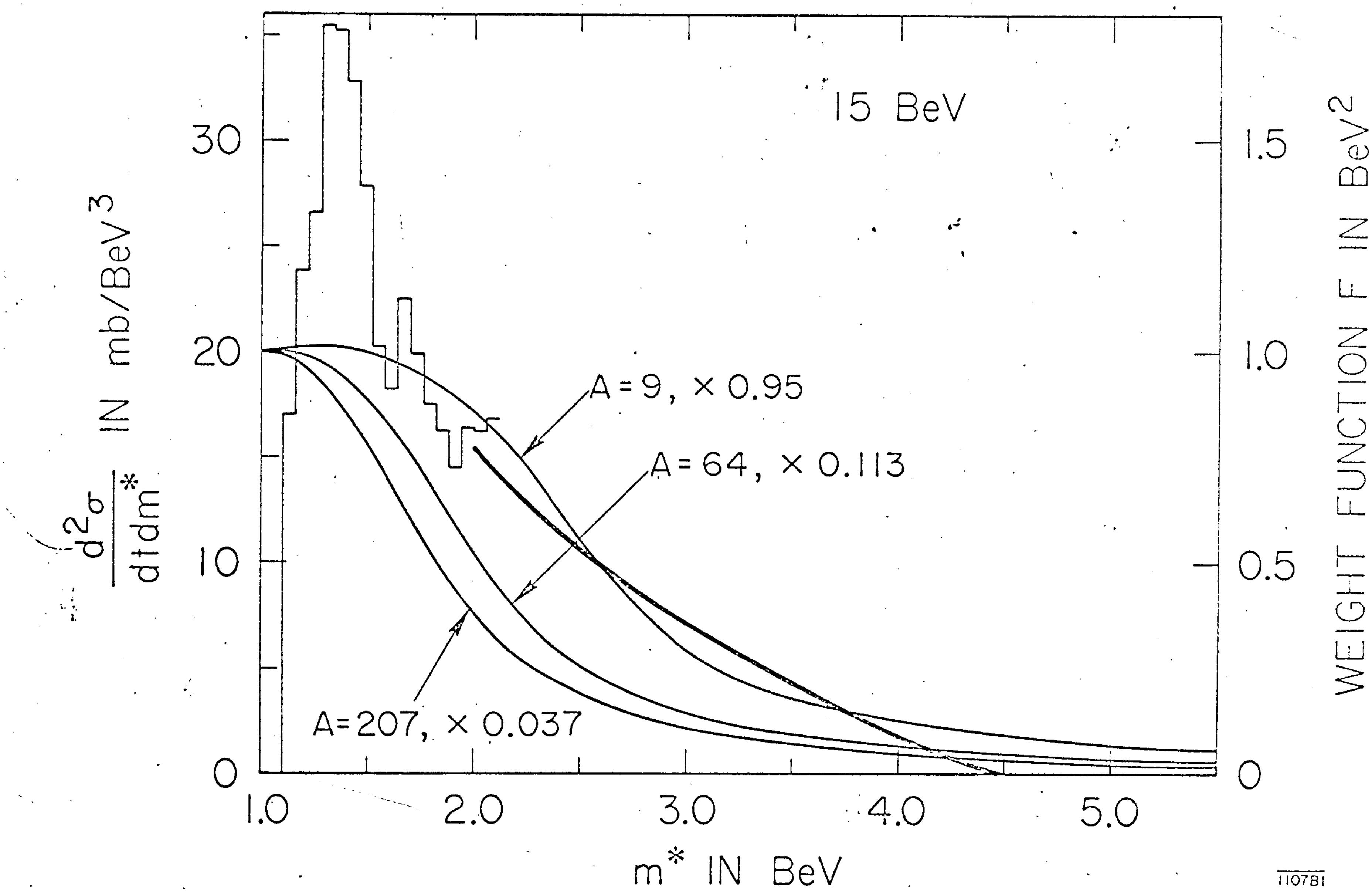
TOTAL CROSS SECTIONS AT HIGH ENERGY

	Energy	σ_o	$\sigma_{inelastic}$	σ_{net}	σ_{Jones}
A=9	15 BeV	243	28	215	
	30 BeV	243	38	205	250
A=64	15 BeV	1275	179	1096	
	30 BeV	1275	246	1029	1090
A=207	15 BeV	3185	466	2719	
	30 BeV	3185	655	2530	2630



(a) (b)

Figure 1



15 BeV
$\frac{d^2\sigma}{dt\,dm^*}$ IN mb/BeV^3
WEIGHT FUNCTION F IN BeV^2
A = 9, × 0.95
A = 64, × 0.113
A = 207, × 0.037
m* IN BeV
0
10
20
30
0
0.5
1.0
1.5
1.0
2.0
3.0
4.0
5.0

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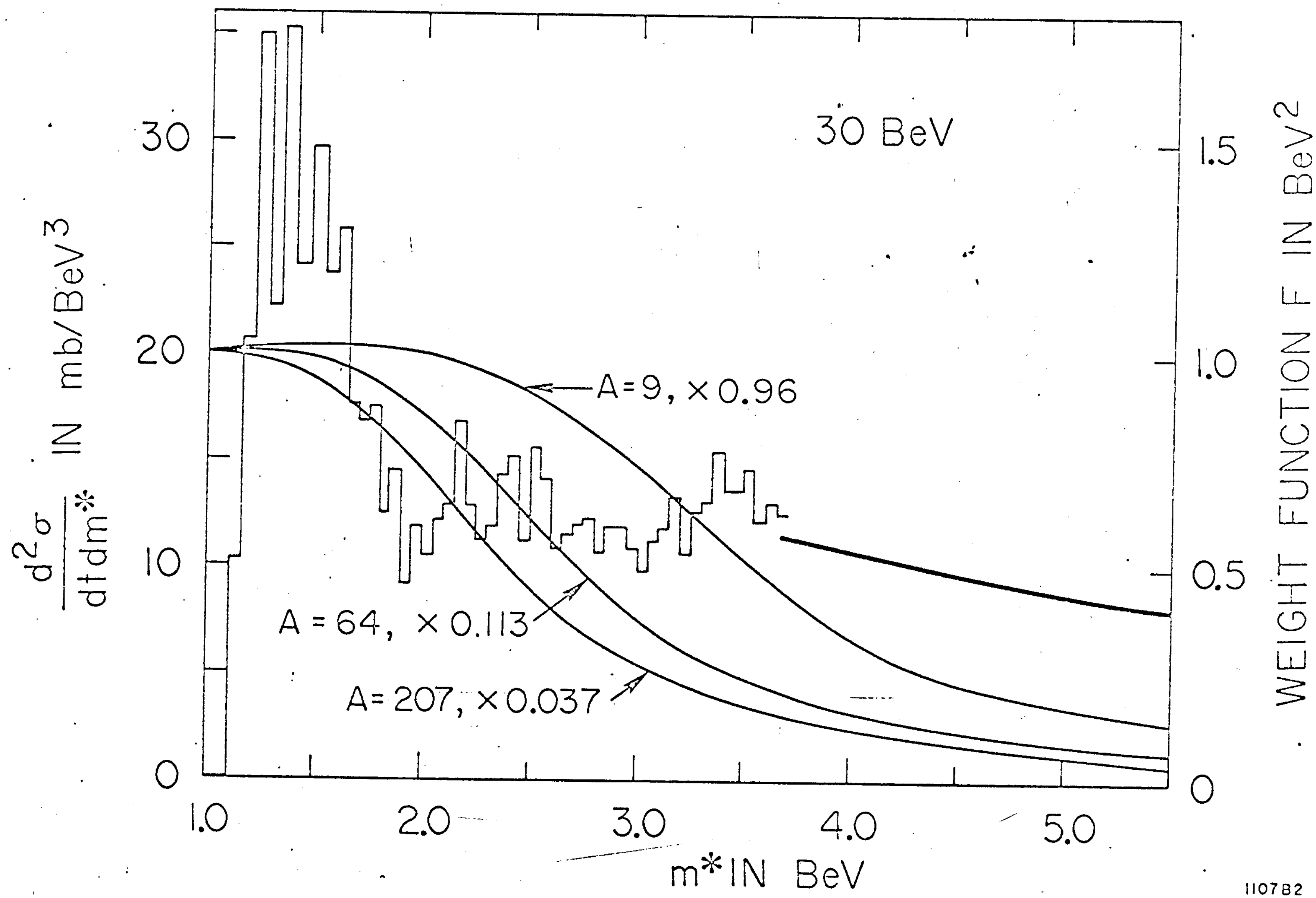

30 BeV
$\frac{d^2\sigma}{dt\,dm^*}$ IN mb/BeV^3
WEIGHT FUNCTION F IN BeV^2
A=9, ×0.96
A = 64, ×0.113
A= 207, ×0.037
0
10
20
30
0
0.5
1.0
1.5
1.0
2.0
3.0
4.0
5.0
m* IN BeV

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