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INELASTIC ELECTRON SCATTERING AND THE MULTI-REGGE MODEL †

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ABSTRACT

We investigate the multi-Regge model in the kinematic region where the four momentum of one initial particle is large and spacelike. These kinematics occur in inelastic electron scattering through single virtual photon exchange. We predict a rapid fall off in the cross section as the virtual particle becomes more spacelike, contradicting present data.

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After radiative corrections, inelastic electron scattering measures the total cross section for virtual photons as a function of the photon mass as well as its energy. This freedom of varying a particle mass is a new feature not yet available in other reactions; therefore, this process has recently attracted much theoretical interest. [1] [2] Denote the total cross sections for transverse and longitudinal virtual photons incident on spin averaged protons by σ_T and σ_L respectively. Let ν be the photon lab energy and let p_1^2 be the square of the photon four momentum. Bjorken [1] has shown that, as ν and p_1^2 became large with the ratio $\frac{\nu}{p_1^2}$ constant, it is likely that $\nu\sigma_T$ and $\nu\sigma_L$ have finite, as opposed to infinite, limits of value dependent on this ratio $\frac{\nu}{p_1^2}$. Current data seem to indicate that this limit is nonvanishing [6]. Most of the recent interest in the subject concerns the behavior of this limit as a function of $\frac{\nu}{p_1^2}$.

The multi-Regge model (MRM) has drawn interest as a possible description for highly inelastic hadronic collisions [3]. We wish to relate the MRM to inelastic electron scattering by discussing the behavior of the MRM as the four momentum p_1 of an initial particle becomes large and spacelike. In this way we hope to gain some insight into the behavior of Bjorken's limit functions. In particular, we ask if the above discussed behavior for $\nu\sigma_L$ and $\nu\sigma_T$ is consistent with the simple MRM.

Our calculation essentially follows the work of Halliday and Saunders [4], except we allow one initial mass to be variable. For simplicity we treat all particles as spinless. Figure 1. shows our kinematics. A virtual particle of momentum p_1 collides with a particle of momentum p_2 and unit mass ($p_2^2 = 1$). The final state consists of n identical particles labeled with momenta q_i, $i = 1, \cdots n$, and all of unit mass ($q_i^2 = 1$). Define invariants

$$s_i = (q_i + q_{i+1})^2 \tag{1}$$

$$t_i = (p_1 - \sum_{j=1}^{i} q_j)^2$$

$$s = (p_1 + p_2)^2 = (\text{center of mass energy})^2$$

$$\nu = p_1 \cdot p_2 = \tfrac{1}{2} (s - 1 - p_1^2) = \text{energy of } p_1 \text{ in the rest frame of } p_2$$

The simple form of the MRM which we shall use says that when all the s_i are large and the $|t_i|$ small the amplitude for the process is approximately of the form

$$T_n = G(p_1^2, t_1)\, G(t_1, t_2) - - - G(t_{n-1}, 1)\, s_1^{\alpha(t_1)} - - - s_{n-1}^{\alpha(t_{n-1})} \tag{2}$$

Here the G' s are unspecified vertex functions and $\alpha(t_i)$ is the trajectory function of the exchanged Reggeon. We consider only one type of Reggeon and take

$$\alpha(t_i) = j_o + j' t_i \qquad j' \neq 0 \tag{3}$$

For each of the n! orderings of the q_i there is a similar expression for T_n valid when the respective s_i are large and t_i small. It will be clear later that these n! kinematic regions are disjoint, giving no interference between them; thus, only one ordering need be considered. The factor of n! arising from these different orderings is canceled by the $\frac{1}{n!}$ occurring in the phase space for n identical particles.

The MRM contribution to the total cross section from n particle production is therefore given by

$$\sigma^n_{TOT} = \frac{1}{v_{rel}\, 2p_{1_o}\, 2p_{2_o}} \int \frac{d\,\Phi}{(2\pi)^{3n-4}} \,|T_n|^2 \tag{4}$$

where

$$d\,\Phi = \delta^4 (p_1 + p_2 - \sum_{i=1}^{n} q_i) \prod_{i=1}^{n} (\delta^+ (q_i^2 - 1)\, d^4 q_i) \tag{5}$$

$$\delta^+ (q^2 - 1) = \Theta\,(q_o)\quad \delta\,(q^2 - 1)$$

$$\Theta\,(x) = \begin{cases} 1 & x > o \\ 0 & x < o \end{cases}$$

v_{rel} = relative velocity of p_1 and p_2.

We work in a frame where $\vec{p}_1$ and $\vec{p}_2$ are parallel. In the limit $\nu \to \infty$ (note $s \geq n^2 \geq 1$ implies $- p_1^2 \leq 2\,\nu$)

$$\frac{1}{v_{rel} 2 p_{1_o}\; 2 p_{2_o}} \approx \frac{1}{4\nu}$$

To simplify the integral over phase space, let us change to a set of variables introduced by Sudakov [(5)] and apply them to this problem, following closely Halliday and Saunders [(4)]. To define these variables, introduce two new momenta

$$p'_1 = p_1 - (\nu - \sqrt{\nu^2 - p_1^2})\; p_2 \tag{6}$$
$$p'_2 = p_2 - \frac{1}{p_1^2}\; (\nu - \sqrt{\nu^2 - p_1^2}\,)\; p_1$$

These momenta have the useful property $p'^2_1 = p'^2_2 = 0$.

In the limit $\nu \to \infty$ we have

$$p'_1 \approx p_1 - \frac{p_1^2}{2\nu}\; p_2 \tag{7}$$
$$p'_2 \approx p_2 - \frac{1}{2\nu}\; p_1$$
$$p'_1 \cdot p'_2 \approx \nu$$

Now we define the Sudakov variables $\{\, \alpha_i, \beta_i, K_i \,\}$ by

$$q_i = \alpha_i p'_1 + \beta_i\, p'_2 + K_i \tag{8}$$

where K_i is the transverse part of q_i. A little algebra shows that in the limit $\nu \to \infty$ (allowing $- p_1^2$ comparable to ν)

$$d^4 q_i \approx \nu \, d\alpha_i \, d\beta_i \, d^2 K_i \tag{9}$$

$$\delta^+(q_1^2 - 1) \approx \frac{1}{2\nu} \, \Theta(\alpha_i) \, \delta\left((\alpha_i \beta_i - \frac{1 - K_i^2}{2\nu}\right) \tag{10}$$

$$\delta^4(p_1 + p_2 - \sum_{i=1}^{n} q_i) \approx \frac{1}{\nu} \, \delta^2(\sum_i K_i) \, \delta(\sum_i \alpha_i - 1) \, \delta\left(\sum_i \beta_i - 1 - \frac{p_1^2}{2\nu}\right) \tag{11}$$

$$s_i \approx 2\nu(\alpha_i + \alpha_{i+1})(\beta_i + \beta_{i+1}) + (K_i + K_{i+1})^2 \tag{12}$$

$$t_i \approx (\sum_{j=1}^{i} K_j)^2 - 2\nu \sum_{j=i+1}^{i} \alpha_j \sum_{j=1}^{i} \beta_j + p_1^2 \sum_{j=i+1}^{n} \alpha_j + \sum_{j=1}^{i} \beta_j - \frac{p_1^2}{2\nu} \tag{13}$$

Note that α_i and β_i are all positive because of $\delta^+(q_i^2 - 1)$.

In terms of these variables our integral becomes

$$\sigma_{TOT}^{n} = \frac{1}{2^{n+2}(2\pi)^{3n-4}\nu^2} \int \prod_{i=1}^{n} (d\alpha_i d\beta_i d^2K_i \, \Theta(\alpha_i) \, \delta(\alpha_i\beta_i - \frac{1-K_i^2}{2\nu})) \, \delta^2(\sum_i K_i)$$
$$\times \delta(\sum_i \alpha_i - 1) \, \delta(\sum_i \beta_i - 1 - \frac{p_1^2}{2\nu}) \, |G(p_1^2, t_1)|^2 |G(t_{n-1}, 1)|^2 \tag{14}$$
$$\times \prod_{i=1}^{n-2} |G(t_i, t_{i+1})|^2 \prod_i (s_i^{2j_o} s_i^{2j' t_i})$$

In the region where all the s_i are large, the factor $s_i^{2j' t_i}$ in (14) gives a rapid t_i dependence. If the G' s are smooth and polynomially bounded, this factor will cause small t_i to dominate the cross section. Furthermore, when the s_i are large enough the factor $s_i^{2j' t_i}$ will dominate all t_i dependence.

Restricting ourselves to this region of large s_i, we can immediately do the K_i integrals with the result

$$\sigma_{TOT}^{n} = \frac{1}{2^{n+2}(2\pi)^{3n-4}\nu^2} \left(\frac{\pi}{2j'}\right)^{n-1} \int \prod_{i=j}^{n} \left(d\alpha_i d\beta_i \Theta(\alpha_i) \quad \delta(\alpha_i\beta_i - \frac{1}{2\nu})\right) \tag{15}$$
$$\times \delta(\sum_i \alpha_i - 1) \, \delta(\sum_i \beta_i - 1 - \frac{p_1^2}{2\nu}) \, |G(p_1^2, t_i)|^2 |G(t_{n-1})1)|^2 \times$$

$$\times \prod_{i=2}^{n-2} |G(t_i, t_{i+1})|^2 \left(\prod_i s_i\right)^{2jo} \left(\prod_i s_i^{2j'} t_i\right) \frac{1}{\prod_i \log s_i}$$

where now

$$s_i \approx 2\nu\ (\alpha_i + \alpha_{i+1})\ \ (\beta_i + \beta_{i+1}) \tag{16}$$

$$t_i \approx -2\nu \left(\sum_{j=i+1}^{n} \alpha_j\right) \left(\sum_{j=1}^{i} \beta_j\right) + p_1^2 \sum_{j=i+1}^{n} \alpha_j + \sum_1^i \beta_j - \frac{p_1^2}{2\nu}$$

Since s_i is large and $\alpha_i \beta_i \approx \frac{1}{2\nu}$ we have

$$s_i \approx \frac{\alpha_i}{\alpha_{i+1}} + \frac{\alpha_{i+1}}{\alpha_i} >> 1 \tag{17}$$

However

$$\frac{p_1^2}{2\nu} - \sum_1^i \beta_j + t_i \leq -2\nu\ \alpha_{i+1}\ \beta_i \approx \frac{\alpha_{i+1}}{\alpha_i} \tag{18}$$

So to keep $| t_i |$ small we must have

$$s_i \approx \frac{\alpha_i}{\alpha_{i+1}} \approx \frac{\beta_{i+1}}{\beta_i} >> 1 \tag{19}$$

and

$$t_i \approx p_1^2 \quad \alpha_{i+1} - \frac{p_1^2}{2\nu} \tag{20}$$

This shows that the α_i are an increasing sequence and therefore demonstrates our earlier claim that other particle orderings have disjoint kinematic regions for the MRM. Remembering $\sum_i \alpha_i \approx 1$ we see from (19)

$$\alpha_1 \approx 1 \quad \beta_1 \approx \frac{1}{2\nu} \tag{21}$$

Similarly, since $\sum_i \beta_i = 1 + \frac{p_1^2}{2\nu}$,

$$\beta_n \approx 1 + \frac{p_1^2}{2\nu} \approx \frac{s}{2\nu} \qquad \alpha_n \approx \frac{1}{s} \tag{22}$$

From these relations clearly

$$\prod_i s_i \approx \frac{\alpha_1}{\alpha_n} \approx s \tag{23}$$

and

$$t_i \approx p_1^2 \prod_{j=1}^{i} s_j^{-1} - \frac{p_1^2}{2\nu} \tag{24}$$

Let us define y_i and γ by

$$s_i = s^{y_i} \qquad p_1^2 = s^{\gamma} \tag{25}$$

Since $\prod_i s_i = s$, we have $\sum_i y_i = 1$. We have required $s_i \to \infty$ so we must keep

$$y_i \geq \epsilon > 0 \tag{26}$$

where

$$\frac{1}{\epsilon} = o(\log s) \tag{27}$$

Furthermore $s_i \leq s$ implies $y_i \leq 1$. The variable γ can run from $-\infty$ to $+\infty$ as p_1^2 runs from zero to 2ν, although γ becomes large only very near these values of p_1^2. The usual limit $\frac{p_1^2}{\nu}$ = constant with $\nu \to \infty$ corresponds to $\gamma \to 1$.

In terms of these new variables

$$s_i^{2j' t_i} \approx \exp\left[- 2j' y_i \log s \; s^{\left(\gamma - \sum_{j=1}^{i} y_i\right)}\right] s_i^{(-2j' p_1^2/2\nu)} \tag{27}$$

$$\approx \Theta\left(\sum_{j=1}^{i} y_i - \gamma\right) s_i^{(-j' p_1^2/\nu)}$$

$$\prod_i s_i^{2j' t_i} \approx \Theta(y_1 - \gamma)\, s^{(-j' p_1^2/\nu)} \tag{28}$$

This Θ function represents a shrinkage with increasing γ of the kinematic region for the MRM. It means that in order to keep t_1 small s_1 must be larger than p_1^2.

Unless n = 2 we have $s_1 < s$ implying $y_1 < 1$. The $\Theta(y_1 - \gamma)$ reduces the multiplicity n as γ increases until for $\gamma = 1$ only n = 2 will contribute appreciably. Let us briefly discuss the case $\gamma < 1$ before going on to $\gamma = 1$ and the Bjorken limit.

We change integration variables from the α's to the y's using

$$y_i \approx \frac{\log \alpha_i}{\log s} - \frac{\log \alpha_{i+1}}{\log s} \tag{29}$$

This gives the result

$$\sigma^n_{TOT} = \frac{s^{2j_o - 2}}{\log s} \,|\, G(p_1^2, 0)\,|^2 \,|\, G(0,1)\,|^2 \left(\frac{|\, G(0,0)\,|^2}{32\pi^2 j'}\right)^{n-2} \frac{1}{32\pi j'} \; J_n(\epsilon, \gamma) \tag{30}$$

where

$$J_n(\epsilon, \gamma) = \int_\epsilon^1 \frac{dy_1 \ldots dy_{n-1}}{y_1 \ldots y_{n-1}} \; \delta\,(\Sigma y_i - 1)\; \Theta\,(y_1 - \gamma) \tag{31}$$

With $p_1^2 = 1$ this is just the result of Halliday and Saunders. Note that $J_n(\epsilon, \gamma) = 0$ unless $\gamma + (n-2)\epsilon < 1$. This means

$$n < 2 + \frac{1-\gamma}{\epsilon} = 2 + o\left(\log \frac{s}{-p_1^2}\right) \tag{32}$$

This shows the decrease in multiplicity mentioned above.

As we go to the Bjorken limit of $\nu \rightarrow \infty$ with $\frac{\nu}{p_1^2}$ fixed, the parameter γ goes to 1. In this limit we have the remarkable result that final states of two hadrons will dominate the cross section. This contribution from n = 2 is easily evaluated giving

$$\sigma_{TOT} = \sigma^2_{TOT} = \frac{1}{32\pi j'} \; \frac{s^{2j_o - 2}}{\log s} \; s^{\left(-\frac{j'}{\omega(2\omega - 1)}\right)} \left(\frac{2\omega - 1}{2\omega}\right) \left|\, G\left(p_1^2, -\frac{1}{2\omega(2\omega - 1)}\right)\right|^2 \left|\, G\left(\frac{-1}{2\omega(2\omega - 1)}, 1\right)\right|^2 \tag{33}$$

where

$$\omega = -\frac{\nu}{p_1^2} \tag{34}$$

The expression $\frac{1}{2\omega(2\omega-1)}$ occurring in (32) is the minimum value of $-t_1$ in the limit of large ν. Because it can approach this value only for n = 2, two final hadron states should dominate the cross section. Equation (33) is the prediction of the MRM for the cross section in the Bjorken limit with $\omega < \infty$.

If we put in a typical $j_0 = \frac{1}{2}$, then for fixed ω

$$\nu\sigma_{TOT} \sim \left(\frac{\left(p_1^2\right)^{\left(-\frac{j'}{\omega(2\omega-1)}\right)}}{\log p_1^2} \right) \times \left(\left| G\left(p_1^2, \frac{-1}{2\omega(2\omega-1)}\right)\right|^2 \right) \tag{35}$$

Bjorken predicts that this should go to a constant as $p_1^2 \to \infty$. Indeed the first factor is slowly varying for $\omega > 1$. However, from experience with elastic electromagnetic form factors, one might expect

$$\left| G\left(p_1^2, \frac{-1}{2\omega(2\omega-1)}\right)\right|^2 \sim \left(\frac{1}{p_1^2}\right)^4 \tag{36}$$

This fall off is quite rapid and no such effect has been seen as yet in the data. This indicates trouble with the model unless something drastic happens to the asymptotic behavior of $G(p_1^2, t)$ as t is varied from the physical mass of the exchanged particle to $t = -\frac{1}{2\omega(2\omega-1)}$. When $\omega \gtrsim 1$ this extrapolation is not large and we cannot theoretically justify such a change in behavior.

It may be that the data are not yet in the asymptotic region. In present experiments $-p_1^2$ is not large compared to the above mentioned extrapolation of t. Assuming this is the case and the model is still applicable, we can make a simple prediction on final momentum distributions.

If we define the final particle ordering for multiparticle production events by decreasing lab momentum (decreasing α_i) we should find most events with $s_1 = (q_1 + q_2)^2 \gtrsim -p_1^2$ whereas further $s_i = (q_i + q_{i+1})^2$ will tend to lower values. Furthermore as we increase the lab energy ν with fixed $\omega \neq 0$, the average multiplicity should decrease to two. These are effects that may begin to show up at nonasymptotic energies and should be looked for.

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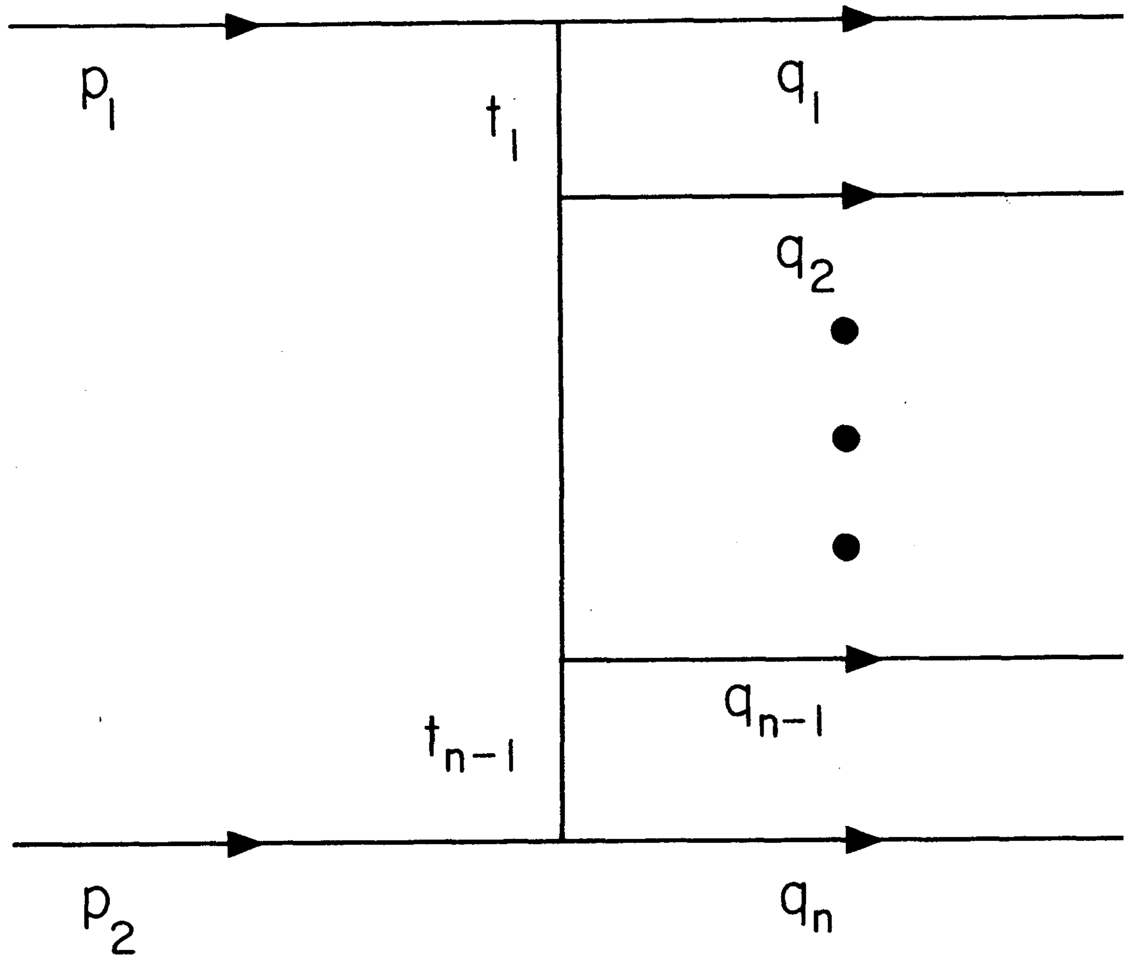

Fig. 1 Kinematics for the inelastic scattering.