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Lamb Shift and Validity of Quantum Electrodynamics

Quantum electrodynamics (QED) has a simple conceptual basis. It is constructed very simply by applying the ordinary quantum rules to the electromagnetic field amplitude, for which the space-time development is determined by Maxwell's equations, and to the electron field amplitude satisfying the Dirac equation. From this program there emerges the quantum theory of photons and electrons. Since this procedure starts with fields satisfying wave equations, it leads to a theory in which the fields are continuous functions of continuous space-time coordinate parameters x and t, and changes in the fields at x are determined by properties of the fields infinitesimally close to x. In other words, quantum electrodynamics takes over the notion of local point interactions from classical field theory.

It is not surprising then that the divergence difficulties of classical field theories with point interactions appear in the quantum theory. What is remarkable is that it has been possible to shunt aside all those difficulties via the renormalization program. By straightforward, unambiguous calculations unique and finite answers can be obtained and checked against experimental observations, and over a very broad scale of phenomena and to a very high precision the theory of QED is in full agreement with experiment.

Spurred by the impressive successes of QED we seek new and exacting challenges for the theory. We may wonder, for example, whether the notion of point interactions is exact or perhaps only an idealization that can be adopted in the sense of a correspondence principle. It may be a sufficiently precise description when the theory is being tested by low-resolution probes that "see" the average behavior over a volume of linear dimensions of the electron Compton wavelength, $\sim\hbar/m_ec \sim 3.8 \times 10^{-11}$ cm. However, if we look with a higher resolution microscope at dimensions comparable to, say, the nucleon Compton wavelength, $\hbar/Mc \sim 2 \times 10^{-14}$ cm, an elementary space-time structure or granularity may reveal itself. This is precisely what occurs for most physical systems. Sound waves or vibrating membranes, for example, are described by wave fields. Such a description is an idealization valid only for distances larger than a characteristic length that measures the structure of the medium (the interatomic separation $\sim 1Å$ or 10^{-8} cm). At smaller distances there are profound modifications in these theories.

On the scale of atomic dimensions no comparable granularity is observed for the electromagnetic field. Indeed it was just the absence of any evidence for the existence of an "ether" or of any need for a mechanistic interpretation of the radiation field that led to Einstein's theory of special relativity. Now we take it for granted that photon and electron fields, too, satisfy differential wave equations and exhibit local interactions. We should recognize, however, the enormity of the extrapolation of this concept from atomic ($\sim 10^{-8}$ cm) to electron ($\sim 10^{-11}$ cm) and eventually to nuclear ($\sim 10^{-14}$ cm) dimensions, and we must ask whether this description may falter ever so slightly along the way. For example, if we make perturbation calculations we are led to divergent expressions for the magnitude of the electron's charge and mass renormalization in QED. Is this a result of the inadequate mathematics of perturbation theory, or is it indicative of the failure of the field description of local action at arbitrarily small distances? If it is the latter suppose there is some fundamental length (or high momentum cutoff) defining a region over which the local field description of interactions at a point must be smoothed out. The very existence of such a length will affect QED predictions for processes probing small distances (or high momenta) comparable with it.

How can we find out what is going on in this region? One way is to take the high-energy-road of experiments with very large momentum transfers q that probe distances R of the order of $R \sim \hbar/q \sim 10^{-14}$ cm for $q \sim 2$ GeV/c to an accuracy of several percent; examples of such experiments are colliding beam electron-electron scattering or electron-positron scattering or two-quantum annihilation, or wide-angle electron (or muon) pair photoproduction. An alternate route is along the low-energy road of very high precision atomic and resonance experiments; examples here are Lamb shift, hyperfine structure, and free electron (or muon) gyromagnetic ratio measurements to higher accuracies.

There has been extensive work and progress in both of these directions in recent months. Among the most significant have been new experimental results and theoretical refinements on the Lamb shift in hydrogen. Recall that according to the Dirac theory of the motion of one electron in a pure Coulomb potential two levels of the same principal quantum number n and total angular momentum j but of different orbital angular momentum l are degenerate. The historic measurements of Lamb in 1947 showed, however, that the $2p_{1/2}$ and $2s_{1/2}$ levels in hydrogen were split with the s level lying higher by ~ 1050 mHz $\sim 4 \times 10^{-6}$ eV. This observation posed the first major quantitative challenge to the develop-

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ment of a working theory of QED. To meet this challenge the renormalization theory was developed, and it became possible to actually compute and hence include the interaction of the electron in the hydrogen atom with the zero-point fluctuations of the quantized radiation field in addition to its Coulomb attraction to the proton. These fluctuations perturbed the Coulomb orbits of the Dirac electron and led to a quantitative agreement with very precise measurements to an accuracy of $\sim 10^{-9}$ eV.

This is how things stood last year after almost 20 years of noble effort:

$$\mathcal{S} \equiv E_{2s_{1/2}} - E_{2p_{1/2}} = \begin{array}{ll} 1057.77 \pm 0.10 \text{ mHz} & \text{(experiment)}, \\ 1057.64 \pm 0.21 \text{ mHz} & \text{(theory)}. \end{array} \quad (1)$$

These numbers are for the hydrogen atom. Reduced mass corrections alter them very slightly in deuterium and the agreement is still excellent —as it is for ionized He^+ with $Z = 2$. The experimental number was obtained by directly inducing the radio-frequency transition ($\sim$30-cm radiation) between the two levels and measuring their separation to an accuracy of $\approx 0.1\%$ of the linewidth. The quoted error is approximately the three standard deviations figure. The theoretical number includes all Coulomb binding corrections through order α^2 ($\alpha = e^2/\hbar c$) relative to the dominant term, plus an estimate of further terms reduced by $\alpha^2/\ln\alpha$ as well as nuclear size corrections $\sim$0.12 mHz and nuclear recoil contributions $\sim m/M$ which add 0.36 mHz to the splitting. However, higher order effects of the radiation field of relative order $\alpha/\ln\alpha$ are only approximately evaluated and are the greatest source of uncertainty in the above figure.

This year we now have cause for serious worry. Two new measurements of the Lamb shift in hydrogen have been made by inducing a level crossing of two different hyperfine lines of the $2s_{1/2}$ and $2p_{1/2}$ levels in a magnetic field whose strength is measured by observing the nuclear magnetic resonance of protons in water. Correcting back to the splitting in a zero magnetic field by the Breit-Rabi formula, these give individually for the Lamb shift

$$\mathcal{S} = \begin{array}{l} 1058.07 \pm 0.10 \text{ mHz}, \\ 1058.05 \pm 0.10 \text{ mHz}. \end{array} \quad (2)$$

The quoted errors are approximately three standard deviations. With the inclusion of *all* α^2 corrections the latest theoretical value for the Lamb shift in hydrogen is now reduced to

$$\mathcal{S}_{\text{th}} = 1057.50 \pm 0.11 \text{ mHz}. \quad (3)$$

In view of the liberal allowance of error made to both the experimental and theoretical numbers it would appear to be difficult to harmonize them.

What we have here may be no more than a disagreement between experiments which will dissolve of its own accord in some modification of the theory of the experiments; time will tell. If, however, it is the new results of Eq. (2) that supplant the earlier ones of Eq. (1), it will be difficult to harmonize observation with the refined theoretical prediction of Eq. (3). Then we will have to turn to a more fundamental theoretical level for what may be wrong. There is not enough freedom in the values of the natural constants to alter this comparison significantly. Whatever we come up with must not disturb the very precise agreement between theory and experiment on the electron and positron $g-2$ values which are presently given by

$$[(g-2)/2]_{e^-} = \alpha/2\pi - (0.327 \pm 0.005)\,\alpha^2/\pi^2,$$
$$[(g-2)/2]_{e^+} = \alpha/2\pi + (1.2 \pm 2)\,\alpha^2/\pi^2,$$
$$[(g-2)/2]_{th} = \alpha/2\pi - 0.328\,\alpha^2/\pi^2.$$

The electron and positron g values must be equal according to the *TCP* theorem and the agreement between theory and experiment is within a factor of five of the uncertainty in our knowledge of the fine-structure constant. Further progress in the interpretation of the electron $g-2$ value will require the complete sixth-order calculation of the moment, a more precise value of α from fine-structure studies, as well as improvements in the $g-2$ measurements themselves.

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