SLAC-PUB-608
June 1969
(TH) (EXP)

A SURFACE-WAVE DESCRIPTION OF π^+p BACKWARD SCATTERING*

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ABSTRACT

We report the results of fitting the Regge pole portion of the amplitude of H. M. Nussenzveig's impenetrable sphere solution to π^+p backward scattering measurements. The fits, which require determination of the location of a pole in the complex angular momentum plane (2 parameters), are made to data at 14 different energies from 2 to 17 GeV, and range in quality from good to indifferent. The locus of fitted poles on the angular momentum plane determines a trajectory similar to, but closer to the real axis than that of the lowest Nussenzveig-Regge surface pole. A study of fits at 5.2 and 6.9 GeV/c, in which the overall normalization of the data was varied, indicates that the magnitude of the backscattered pion cross section is near optimum for consistency with the surface-wave description.

(Submitted to Phys. Rev.)

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*Work supported by the U. S. Atomic Energy Commission.

**On leave from the University of New Mexico, Albuquerque.

I. INTRODUCTION

The striking similarity of backward-positive-pion-proton scattering above about 2 GeV to alpha scattering by light, spinless nuclei in the energy range from 20 to 50 MeV, and to the meteorological phenomenon of the glory, has been pointed out.[1] H. M. Nussenzveig has recently discussed[2] the scalar version of the meteorological glory (scattering of a plane wave by a real, square-well potential) and has shown that backward scattering is completely dominated by Regge poles which can be associated with surface waves in the high-frequency limit.

Surface waves (creep waves)[3] occur in scattering problems for which the index of refraction or potential changes within a distance small compared to the wavelength and can be thought of as a kind of diffraction in which the scattered flux appears to come from a damped traveling wave skirting around the scattering center in the neighborhood of its surface.

Probably the simplest scattering problem in which surface waves play an important role is that of scattering a scalar wave by an impenetrable sphere, which has been treated in great detail in the high frequency limit by H. M. Nussenzveig.[4] The scattering amplitude in the large angle regions breaks up naturally to two terms: one results from Regge poles and describes surface waves and the other comes from a line integral and can be shown to represent specular reflection in the high frequency limit.

II. A MODEL OF THE π^+-PROTON LARGE ANGLE ELASTIC SCATTERING

We conjecture that the large angle elastic scattering is due to a surface-wave type interaction having the functional form of the Regge-pole part of the impenetrable sphere problem. We neglect any contribution from specular reflections and allow the positions of the poles to be determined by the data.

III. NUSSENZVEIG'S SOLUTION

Nussenzveig's expression for the scattering amplitude of a scalar wave by an impenetrable sphere can be written (4, Eq. 9.54) for $\beta^{-1/3} \ll \theta \leq \pi$ as

$$f(k, \theta) = f_R + f_S \tag{1}$$

where f_R is the isotropic amplitude produced by specular reflection and f_S is the contribution from the Regge poles:

$$f_S = -e^{i\pi/3} a\left(\frac{\beta}{2}\right)^{1/3}\left(\frac{\pi-\theta}{\sin\theta}\right)^{1/2} \times \sum_{m=0}^{\infty} (-1)^m \sum_{n=1}^{\infty} \frac{\exp[i(2m+1)\pi\lambda n]}{[Ai'(-x_n)]^2} J_0[\lambda_n(\pi-\theta)] \tag{2}$$

where a is the radius of the sphere

β = ka, where k is 2π times the wave number

θ is the scattering angle

λ_n is the location on the complex angular momentum plane of the nth Regge pole

Ai' is the derivative of the Airy function

$-x_n$ is the nth zero of the Airy function

J_0 is the zero-order Bessel function of complex argument.

For the hard sphere, the Regge poles are located at (4, Eq. 3.5)

$$\lambda_n(\beta) = \beta + (\beta/a)^{1/3} x_n e^{i\pi/3} + O(\beta^{-1/3}) \tag{3}$$

for $\beta \gg 1$; $x_1 = 2.34$. The locations for arbitrary β can be determined from the zeros of the Hankel functions of the first kind, for which simple expressions exist.[5]

Consideration of the magnitudes of the terms in the f_S series leads to the neglect of all but the term m = 0, n = 1 of Eq. (2). To simplify somewhat our fitting procedure we write:

$$f_S \simeq -e^{i\pi/3}(\mathrm{Re}\lambda_1/k)\,(\mathrm{Re}\lambda_1/2)^{1/3}\left(\frac{\pi-\theta}{\sin\theta}\right)^{1/2}\frac{\exp[i\pi\lambda_1]}{A_i'(-x_n)^2}\,J_0[\lambda_1(\pi-\theta)] \tag{4}$$

giving us two parameters $\mathrm{Re}\lambda_1$ and $\mathrm{Im}\lambda_1$, the location on the complex plane of the position of the first surface-wave pole. We abandon the requirement that the pole satisfy Eq. (3).

IV. THE BESSEL FUNCTION

For our fits we write

$$J_0(Z) = \sum_{n=0}^{M}\frac{(-1)^n(z/2)^{2n}}{(n!)^2} \tag{5}$$

where $M = 20$ for our machine (single precision) calculations.

It is instructive to see the effect of small imaginary component of z on J_0. Putting $z = x + iy$, we get

$$J_0(z) = J_0(x) + i\,2y\sum_{n=0}^{\infty}\frac{n(-1)^n x^{2n-1}}{(n!)^2} + O(y^2). \tag{6}$$

Qualitatively, combining Eq. (6) with Eq. (4), we note the following features:

(1) The position of the dip in the π^+p cross section determines $\mathrm{Re}\lambda_1$. Note that $J_0(x) = 0$ at $x = 2.44$.

(2) The overall normalization is determined by $\mathrm{Im}\lambda_1$ in the exponential.

(3) The deviation from zero (filling in) at the dip of the amplitude is directly preporitional to $\mathrm{Im}\lambda_1$.

V. THE FITS

Table 1 summarizes our fits of Eq. 4 to 14 sets of π^+p data. We exclude points for which $\mathrm{Re}\lambda_1(\pi-\theta) < 6.0$. The positions of the pole on the complex plane are shown in Fig. 1. Plots of the data and the surface pole curves are shown in Fig. 2. The ordinates are normalized to 1 and the abscissae are $(\mathrm{Re}\lambda_1)(\pi-\theta)$. As a test of the uniqueness of the fits the overall normalizations for the data at 5.2 and 6.9 (Figs. 2i and 2k) were altered, and the data were then fitted. Figure 3 shows the χ^2 for the fit vs. the factor multiplying the cross sections and their errors. This clearly shows that the order of magnitudes of the cross sections are just about optimum for compatibility with the surface-wave formula, and tends to rule against the "accidental" success of these fits.

VI. DISCUSSION

(1) The fits appear to be comparable with those obtained using the Reggeized baryon exchange models. For example, K. P. Pretzl[6] finds a χ^2 of 155 for a six parameter fit to his data (5.2 and 6.9 GeV), whereas we obtain a χ^2 of 70.3 for the same data.

(2) The radius of the pion-proton interaction of about 0.9 F seems quite reasonable. Note that this is not an independent parameter, but is given by $\mathrm{Re}\lambda/k$.

(3) The overall normalization in most cases agrees well with the size of the cross section at the dip.

(4) The position of the pole on the complex angular momentum plane seems to follow a smooth trajectory for $p_\pi \gtrsim 3$ GeV. There seems to be some sort of threshold from 2 to 3 GeV.

(5) Deviations of the data from the model appear to be definitely present for large values $\mathrm{Re}\lambda_1(\pi-\theta)$. The data seem to be dropping off, and the model seems to be staying constant or increasing as this parameter increases above 6.

VII. CONCLUSION

In the region of backward scattering, the π^+p data seem to fit reasonably well a one-pole Regge-type model contained in Nussenzveig's solution to the problem of high frequency scattering by an impenetrable sphere. This pole, which can be associated semiclassically with surface waves, seems to be typical of interactions with rather abrupt transition regions.

If our interpretation is correct, the backward scattering of π^+ by protons must be due to a strongly absorptive interaction between the pion and the proton which sets in rather abruptly when the two particles are separated by a distance of about 1 Fermi. This mechanism seems to dominate above 2 or 3 GeV.

It remains to be seen if any connection can be established with the currently standard Reggeized-baryon-trajectory exchanges used to explain backward scattering, and if the model can be elaborated to include spin and to make predications for the other two pion charge states.

VIII. ACKNOWLEDGEMENTS

I am grateful to M. L. Perl, E. A. Paschos, and H. M. Nussenzveig for their helpful comments.

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TABLE 1: Summary of Surface-Pole Fits to Backward π^+p Scattering

P_π	Radius	$\frac{d\sigma}{du_\pi}$	Reλ	Imλ	# points	χ^2	Ref.
2.28 GeV/c	.783 Fermis	524. $(\mu b/GeV/c)^2$	3.73	.886	17	44.0	7
2.48	.881	873.	4.41	.844	14	39.6	7
2.58	.839	1011.	4.29	.795	15	47.0	7
2.78	.876	990.	4.68	.807	14	23.4	7
2.85	.953	547.	5.17	.935	9	7.8	9
2.93	.837	454.	4.61	.906	14	26.8	7
3.30	.995	160.	5.86	1.13	6	1.0	9
3.55	.938	143.	5.76	1.12	11	9.9	9
5.20	1.002	40.6	7.59	1.30	29	22.3	6
5.91	1.027	28.2	8.33	1.35	20	253.	8
6.90	1.037	15.7	9.14	1.43	19	48.0	6
9.85	.990	2.78	10.53	1.65	16	172.	8
13.73	.914	.46	11.56	1.87	10	105.	8
17.07	1.00*	1.81	14.14	1.66	2	1.2	8

*In this case the radius was set to 1 F, since there are only two points and they have very large errors.

FIGURE CAPTIONS

1. Positions of surface-wave Regge poles on the complex angular momentum plane for the fitted data.

2. Surface-wave fits to the backward angular distributions. All curves have been normalized to 1 at 180°.

3. χ^2 vs. factor multiplying the measured cross sections for data at 5.2 and 6.9 GeV. This demonstrates the sensitivity of the model to the overall normalization.

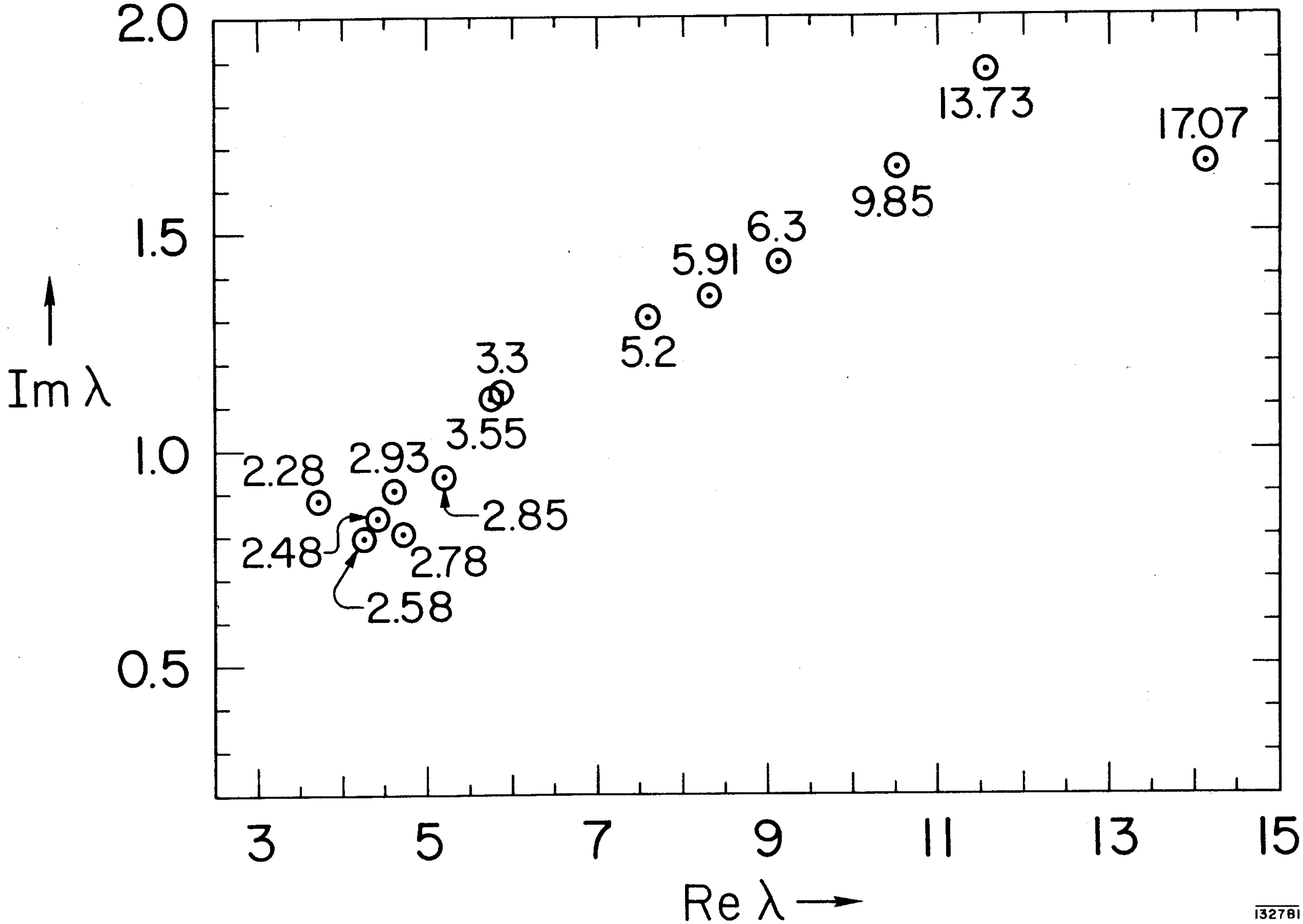
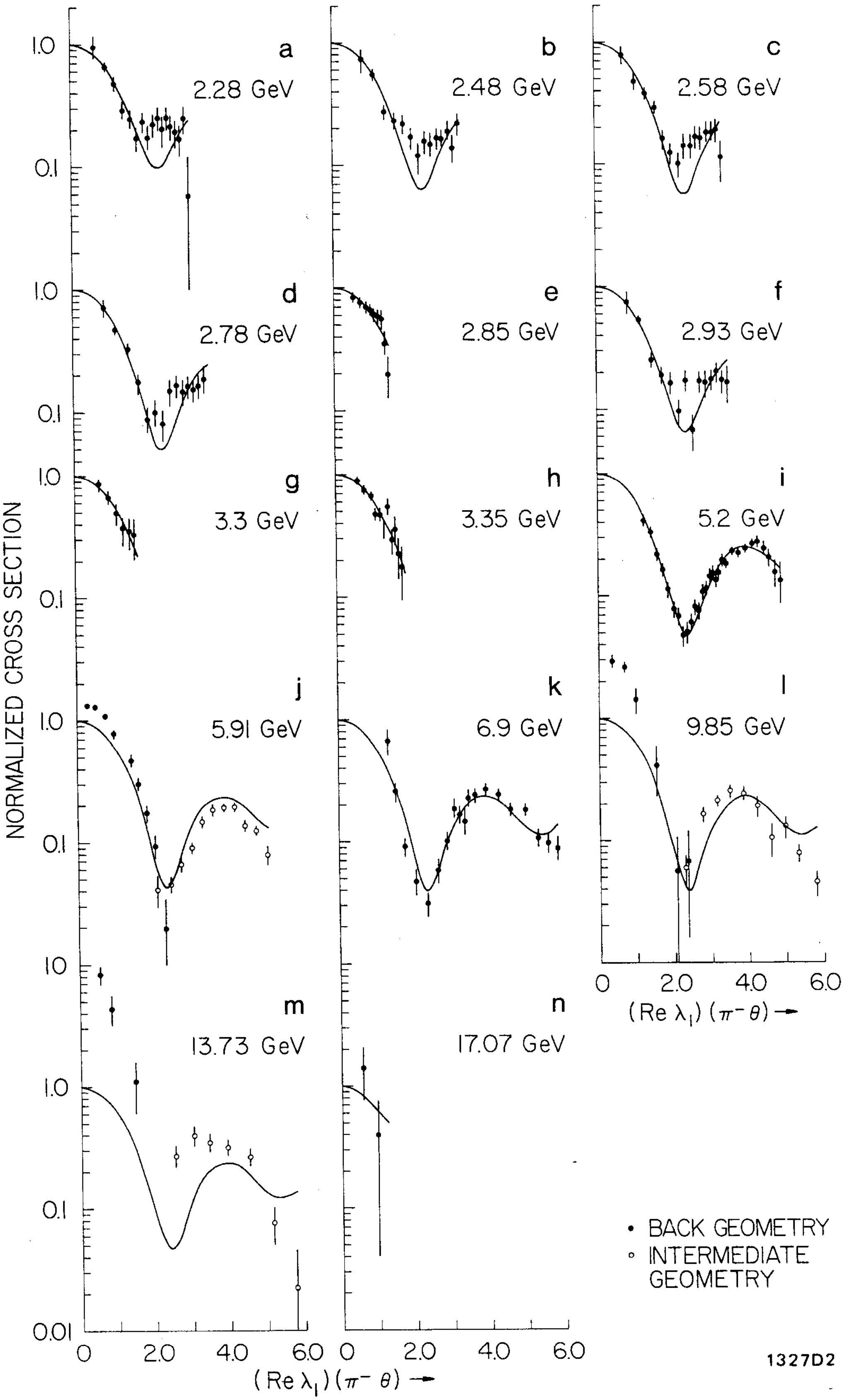
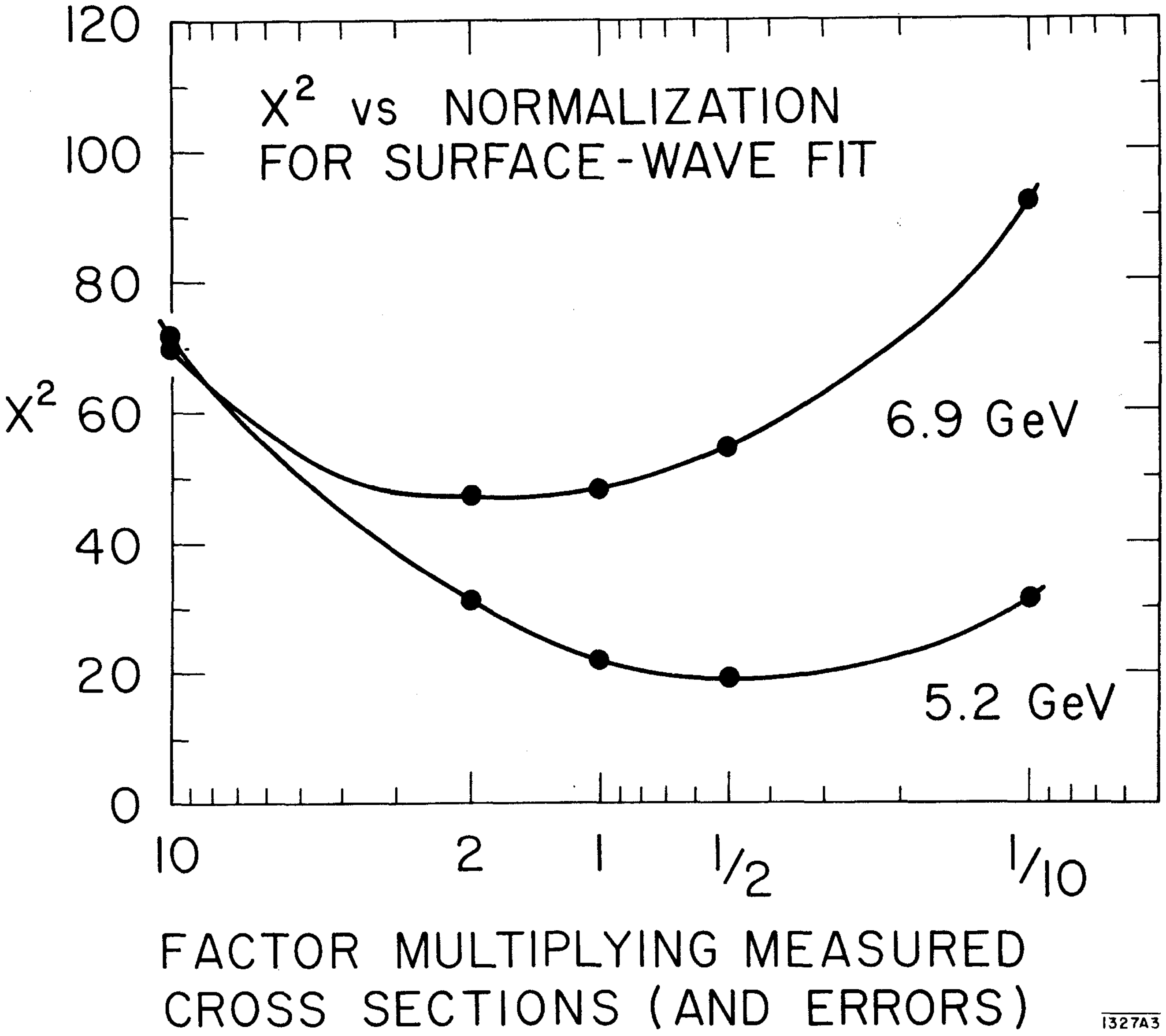

2.0
1.5
1.0
0.5
Im λ
3
5
7
9
11
13
15
Re λ
13.73
17.07
9.85
6.3
5.91
5.2
3.3
3.55
2.28
2.93
2.85
2.48
2.78
2.58
132781

Fig. 1



NORMALIZED CROSS SECTION
a
2.28 GeV
b
2.48 GeV
c
2.58 GeV
d
2.78 GeV
e
2.85 GeV
f
2.93 GeV
g
3.3 GeV
h
3.35 GeV
i
5.2 GeV
j
5.91 GeV
k
6.9 GeV
l
9.85 GeV
m
13.73 GeV
n
17.07 GeV
1.0
0.1
10
0.01
0
2.0
4.0
6.0
(Re λ_1) (π− θ) →
• BACK GEOMETRY
∘ INTERMEDIATE GEOMETRY
1327D2

Fig. 2



120
100
80
60
40
20
0
X^2 vs NORMALIZATION
FOR SURFACE-WAVE FIT
X^2
6.9 GeV
5.2 GeV
10
2
1
1/2
1/10
FACTOR MULTIPLYING MEASURED
CROSS SECTIONS (AND ERRORS)
1327A3

Fig. 3