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A PRACTICAL APPROACH TO POLE EXTRAPOLATION*

Günter Wolf
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

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I. INTRODUCTION

The Chew-Low extrapolation[1] to find on-mass-shell scattering cross sections from reactions expected to involve scattering of a virtual particle depends critically on what type of extrapolation function is used with a limited number of events (several thousand). The work of Baton et al.,[2] ($\pi^-\pi^o$ scattering), Ma et al.,[3a] (π^+p scattering), and Marateck et al.,[4] ($\pi^-\pi^o$ and $\pi^+\pi^-$ scattering), showed that linear or quadratic extrapolation functions[5] fit the data equally well but the extrapolated cross sections differ significantly: In general the higher order extrapolation leads to larger cross sections. However, neither of these simple functions may give the correct answer as was shown by Schlein and coworkers[3a-b] who used data on the reaction $pp \rightarrow p\pi^+n$ to determine the π^+p elastic scattering cross section in the Δ region ($\Delta = \Delta(1236)$) via the Chew-Low extrapolation. Since the π^+p elastic scattering cross section is well measured, this is a place where the extrapolation procedure can be checked. Both linearly and quadratically extrapolated cross section values did not reproduce the on-shell measurements. However, good agreement was found using the Dürr-Pilkuhn[6] (DP) parameterization which relates off-shell and on-shell scattering cross sections.

Recently a slightly different parameterization for off-shell scattering amplitudes has been proposed by Benecke and Dürr [7] (BD). As it turns out, their treatment has certain advantages over that of Dürr and Pilkuhn in that (a) it describes the data up to larger momentum transfers (up to ~ 1 GeV^2 as compared to ~ 0.5 GeV^2), and (b) no additional form factor is required by the data. At small t values ($|t| \leq 0.3$ GeV^2) the BD and DP parameterizations lead to identical results.[8a,b]

II. ONE-PION EXCHANGE FORMULAE

Before going into detail, some well known formulae are given for the sake of clarity.

Consider the OPE diagram (Fig. 1) for the reaction

$$\pi^- p \longrightarrow \pi^- \pi^+ n \quad . \tag{1}$$

The contribution of this diagram to the differential cross section for reaction (1) is given by

$$\frac{d^2\sigma(m,t)}{d|t|dm} = \frac{1}{4\pi p^{*2} s} m^2 q_t \sigma_{\pi^-\pi^+}(m,t) \frac{-t}{(t-\mu^2)^2} g^2 F^2_{NN\pi}(t) \tag{2}$$

where

- s square of total cm energy
- p^* cm momentum in the initial state
- μ pion mass
- t square of the four momentum transfer between incoming and outgoing nucleon
- m $\pi^-\pi^+$ rest mass
- q_t momentum of the exchanged pion in the $\pi^-\pi^+$ rest frame:

$$q_t^2 = (t-(m+\mu)^2)\,(t-(m-\mu)^2)/4m^2 \tag{3}$$

For $t = \mu^2$ q_t becomes the on-shell momentum q, $q^2 = \frac{m^2}{4} - \mu^2$

- g $NN\pi$ coupling constant; $g^2 = 2 \times 14.6$ in the case of reaction (1).

The unknown functions in Eq. (2) are $\sigma_{\pi^-\pi^+}(m,t)$, the cross section for the reaction $\pi^-\pi^+ \longrightarrow \pi^-\pi^+$ where one of the incoming pions is virtual with a mass squared of t, and $F_{NN\pi}(t)$, the pionic form factor of the nucleon. The determination of the on-shell $\pi^-\pi^+$ elastic scattering cross section $\sigma_{\pi^-\pi^+}(m)$ via Chew-Low extrapolation is usually done by extrapolating the experimentally measured quantity $\Sigma_{\pi^-\pi^+}(m,t)$ to the pion pole, $t = \mu^2$:

$$\Sigma_{\pi^-\pi^+}(m,t) = \frac{4\pi p^{*2} s}{m^2 q} \frac{1}{g^2} \frac{(t-\mu^2)^2}{-t} \frac{d^2\sigma(m,t)}{d|t|dm} \tag{3}$$

According to Eq. (2), $\Sigma_{\pi^-\pi^+}(m,t)$ in the OPE model is given by the following expression:

$$\Sigma_{\pi^-\pi^+}(m,t) = \left\{\frac{q_t}{q} \frac{\sigma_{\pi^-\pi^+}(m,t)}{\sigma_{\pi^-\pi^+}(m)} F^2_{NN\pi}(t)\right\} \sigma_{\pi^-\pi^+}(m) \tag{4}$$

which when evaluated at the pion pole, reduces to $\sigma_{\pi^-\pi^+}(m)$.

III. THE BENECKE DÜRR PARAMETERIZATION

In order to evaluate the OPE formulae (2), (4), we have to know $F_{NN\pi}(t)$ and the relationship between off-shell and on-shell scattering cross sections. Let us assume that the scattering at the $\pi\pi$ vertex proceeds through a given angular momentum state ℓ. Then one can replace $\sigma_{\pi^-\pi^+}(m,t)$ in Eqs. (2), and (4) by the partial cross section $\sigma^{\ell}_{\pi^-\pi^+}(m,t)$.

Benecke and Dürr have studied the connection between on-shell and off-shell scattering assuming that the elastic scattering of particles a, and b via

$$a + b = a' + b'$$

can be approximated by the exchange of a scalar particle x with mass m_x. (See Fig. 2.) Going off the mass shell, e.g., with particle a, the relation between the off-shell and the on-shell scattering amplitude is obtained as a function of the mass of the virtual particle a. The result for $\sigma^{\ell}_{\pi^-\pi^+}(m,t)$ is:

$$q_t \sigma^{\ell}_{\pi^-\pi^+}(m,t) = \frac{u_\ell(q_t R)}{u_\ell(qR)} q \sigma^{\ell}_{\pi^+\pi^-}(m) \tag{5}$$

where $1/m_x$ has been put equal to R,

$$u_\ell(x) = \frac{1}{2x^2} Q_\ell\left(1 + \frac{1}{2x^2}\right) \tag{6}$$

and $Q_\ell(z)$ are the Legendre functions of the second kind. For $\ell = 1$ for example, one finds

$$u_1(x) = \frac{1}{2x^2} \left\{\frac{2x^2 + 1}{4x^2} \ln (4x^2 + 1) - 1\right\} \tag{7}$$

The functions $u_\ell(x)$ have the following general properties

1. $u_\ell(x) \sim x^{2\ell}$ for $x \ll 1$.

Hence, near threshold one gets back the Born result:

$$q_t \sigma^\ell_{\pi^-\pi^+}(m,t) = \left\{\frac{q_t}{q}\right\}^{2\ell} q\sigma_{\pi^-\pi^+}(m) \quad (qR,\, q_t R \ll 1) \tag{8}$$

2. $u_\ell(x) \sim \frac{1}{x^2} \ln (4x^2)$ for $x \gg 1$.

i.e., for large t the off-shell cross section decreases like $|t|^{-3}$ in contrast to the Dürr-Pilkuhn parameterization which leads to a fall off proportional to $|t|^{-1}$.

An estimate of the parameter R may be found in the following way: The form factor $F_{abc}(t)$ associated with a vertex abc is defined as the ratio of the actual vertex function to the vertex function given by the Born result. In the BD parameterization this leads for the $\rho\pi\pi$ vertex to

$$F_{\rho\pi\pi}(t) = \frac{q}{q_t} \sqrt{\frac{u_\ell(q_t R_\rho)}{u_\ell(qR_\rho)}} \tag{9}$$

Interpreting F(t) as the Fourier transformation of a spherically symmetrical density distribution, the rms radius $\langle r^2\rangle^{1/2}$ can be calculated from[9]

$$\langle r^2\rangle = 6\frac{dF}{dt'} \text{ evaluated at } t' = 0 \tag{10}$$

where $t' = t - \mu^2$.

Formulae similar to Eq. (5) are obtained for πN off-shell scattering (for details see Ref. 8b). The BD parameterization has a free parameter the constant R. It will be assumed that there is one parameter R for each partial wave contributing to the off-shell scattering cross section; e.g., a R_ρ for the p-wave amplitude of the $\pi\pi$ system, à R_Δ for the (3/2, 3/2) partial wave of the πN system, etc.

For bound state scattering, as in the case of the $NN\pi$ vertex, the off-shell behavior will be calculatèd a la Dürr-Pilkuhn since the BD parameterization leads to complex expressions. Therefore,

$$F_{NN\pi}(t) = \sqrt{\frac{1 + R_N^2 Q^2}{1 + R_N^2 Q_t^2}} \tag{11}$$

with

$$Q^2 = \frac{-\mu^2(4m_N^2 - \mu^2)}{4m_N^2}$$

and

$$Q_t^2 = \frac{-t(4m_N^2 - t)}{4m_N^2}$$

where m_N nucleon mass; R_N is a parameter.

IV. FIT OF THE R PARAMETERS

The OPE model with the BD parameterization was applied to the reactions

$$\bar{p}p \rightarrow \bar{\Delta}^{--}\Delta^{++} \tag{12}$$

$$pp \rightarrow \Delta^{++}n \tag{13}$$

$$\pi^+p \rightarrow \Delta^{++}\rho^o \tag{14}$$

$$\pi^-p \rightarrow n\rho^o \tag{15}$$

There are three free parameters which enter into the OPE cross sections for reactions (12-15): R_Δ, R_N, and R_ρ. They were determined by fitting the OPE expressions to the differential cross sections $d\sigma/d|t|$ for the reactions (12-15) as measured for beam momenta between 1.6 and 10 GeV/c.[10-13] Figures 3 through 6 show the experimental data together with the OPE curves. The OPE curves give an excellent description of the differential cross sections up to $|t| \lesssim 1\ \mathrm{GeV}^2$ for all four reactions, and at all momenta. In a separate fit the OPE cross sections were multiplied with the square of a function G(t), which may be thought of as a correction to the pion propagator. The following ansatz was made for G(t):

$$G(t) = \frac{c-t}{c-\mu^2} \tag{16}$$

In addition to R_Δ, R_N, R_ρ the parameter c was varied. The fit gave $c = 140\ \mathrm{GeV}^2$ which shows that the data do not require an additional form factor.

Knowing the values of R_Δ, R_N, and R_ρ one can go on to fit the R parameters of other partial waves of the $\pi\pi$ and the πN systems. For instance with the assumption that the reactions

$$\pi^- p \rightarrow n f^0 \tag{17}$$

and

$$\pi^- p \rightarrow n g^0 \tag{18}$$

proceed via OPE the measured differential cross sections for (17) and (18) were used to fit the R parameters for the $f\pi\pi$ and $g\pi\pi$ vertices. The agreement between the data[14,15] and the OPE fit is good (see Fig. 7).

In Table I the values of the parameter R and the rms radii deduced from them are listed for various vertices. In some cases only estimated values are given due to limited statistics of the experimental data. The values of the rms radii range from 0.6 to 1.1f.

The pionic radius of the nucleon, $\langle r_N^2 \rangle^{1/2} = 1.06 \pm 0.04$ f, is larger than the electric charge radius of the proton $\langle r_p^2 \rangle^{1/2} = 0.83 \pm 0.02$f [16] by about 20 percent, but its value is close to the rms radius for πN interactions, $\langle r_{N\pi}^2 \rangle^{1/2} =$ 1.1f, obtained from an optical model analysis of πN diffraction scattering. The rms radius for the $\Delta N\pi$ vertex, $\langle r_{\Delta N\pi}^2 \rangle^{1/2} = 0.86 \pm 0.02$f agrees rather well with the value of 0.84f determined for the $\Delta N\gamma$ vertex from the M1 transition form factor measured in ep scattering.[17]

Table I suggests that for $\ell \geq 1$, the rms radii for the baryonic vertices are of the order of 0.8 - 1.0f and for the mesonic vertices 0.6 - 0.7f. Therefore, a good guess for the value of R and hence for the form factor can be obtained from Eq. (10) choosing a proper $\langle r^2 \rangle^{1/2}$ value, e.g., 0.9f for the baryonic vertices and 0.65f for pionic vertices. This can be a useful procedure in the case of small partial waves for which it is difficult to determine the R parameter from the t distributions. For s-wave states the value of R is practically zero, implying $u_\ell(q_t R)/u_\ell(qR) \simeq 1$ for these vertices.

V. IMPLICATIONS FOR THE EXTRAPOLATION FUNCTION

We are now in a position to calculate form factors and off-shell cross sections for various vertices in the BD parameterization. Figure 8 shows the square of the form factor as a function of t for the $NN\pi$, $\Delta N\pi$, $\rho\pi\pi$ and $f\pi\pi$ vertices. The fall-off with increasing -t is roughly the same in all four cases.

The behavior of the off-shell scattering cross sections can be read off from the t dependence of the quantities $\frac{-t}{\mu^2} F_{NN\pi}^2(t)$, $\frac{q_t \sigma_{\Delta N\pi}(m,t)}{q \sigma_{\Delta N\pi}(m)}$, and $\frac{q_t \sigma_{f\pi\pi}(m,t)}{q\, \sigma_{f\pi\pi}(m)}$ where q, q_t are the magnitudes of the three momenta of the on-shell and the off-shell pion respectively in the Δ, ρ and f rest frames. There is a striking difference in the behavior of the cross section ratios for the various cases: whereas the ratio is practically equal to unity for the $\rho\pi\pi$ and the $f\pi\pi$ vertices, in the case of the $NN\pi$ and $\Delta N\pi$ vertices a rapid increase with increasing $-t$ is observed. The explanation for the phenomenon is that in the case of the $NN\pi$ and $\Delta N\pi$ vertices one is close to threshold (q, $q_t \cong 0$), hence e.g.,

$$q_t\, \sigma_{\Delta N\pi}(m,t) \simeq \left(\frac{q_t}{q}\right)^2 q\sigma_{\Delta N\pi}(m) \tag{19}$$

For the $\rho\pi\pi$ and $f\pi\pi$ vertices, one is far from threshold ($m = 2\mu$), therefore (for $-t \lesssim 0.3\ \mathrm{GeV}^2$)

$$q_t\, \sigma_{\binom{\rho}{f}\pi\pi}(m,t) \simeq q\, \sigma_{\binom{\rho}{f}\pi\pi}(m) \tag{20}$$

The analysis indicates that also for other vertices, this type of correspondence holds when one is far from threshold and $|t|$ is small.

Let us now turn to the function $\Sigma_{\pi^-\pi^+}(m,t)$ defined in Eqs. (3), and (4). From the above discussion it is clear that in the ρ region and above the t dependence of $\Sigma_{\pi^-\pi^+}(m,t)$ is mainly that of $F^2_{NN\pi}(t)$. (Note: This is also true if the $\pi\pi$ s-waves are taken into account since $q_t\, \sigma^{\ell=0}(m,t) \simeq q\, \sigma^{\ell=0}(m)$.) Since $F^2_{NN\pi}(t)$ drops with increasing momentum transfer the function $\Sigma_{\pi^-\pi^+}(m,t)$ will rise when going to the pole.[18]

In Fig. 9, the values of $\Sigma_{\pi^-\pi^+}(m,t)$ as obtained by Marateck et al.,[4] for events selected from a 20 MeV band around the ρ^o mass are shown. The data came from a compilation of a large sample of events (14,890 events for reaction (1) at 1.9 GeV/c and 3.0 GeV/c incident momentum). Also shown in Fig. 9 are a linear extrapolation and one which uses the BD parameterization. Analogous to the situation in $pp \to pn\pi^+$ discussed by Prof. Schlein at this conference,

(a) the accuracy of the data does not allow a distinction between the two procedures, i.e., both fit the data equally well. As can be seen from

Fig. 9, this is due to the fact that the two curves depart from each other only at very small t values ($-t \lesssim 4\mu^2$).

(b) the extrapolated cross section values differ by 30-50 percent depending on which extrapolation function is used: the linear extrapolation leads to $\sigma_{\pi^-\pi^+}(m_\rho) = 85 \pm 15$ mb, the BD parameterization yields $\sigma_{\pi^-\pi^+}(m_\rho) = 124 \pm 4$ mb. From unitarity we expect the cross section to be either 116 mb or 132 mb depending on whether only the resonant ρ wave is considered or a resonating $T = 0$ s-wave is also taken into account. Hence the cross section value obtained using the BD parameterization agrees with the expected value whereas the linear extrapolation underestimates the cross section by ~40 percent.

VI. CONCLUSION

The number of events presently available does not allow a model independent determination of the $\pi\pi$ scattering cross sections via a Chew-Low extrapolation. Reliable results can be expected when the Benecke-Dürr (BD) parameterization is used for the pole extrapolation as was shown for reaction (1) in the ρ region. This conclusion is strengthened

(a) by the sucess of the OPE fits to the reactions (12-15);[8b]

(b) by the fact that in the case of the reaction $pp \rightarrow \Delta^{++}n$ the Chew-Low extrapolation lead to the correct values for the elastic π^+p scattering cross section in the Δ region when the Dürr-Pilkuhn (DP) parameterization was applied;[3a,b]

(c) by the good results obtained with the DP parameterization for reactions of the type $KN \rightarrow NK\pi\pi$.[3b,19] The last two points apply as well for the BD parameterization since at small t values (see Introduction) both procedures are equivalent.

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TABLE I: R parameters and rms radii $\langle r^2 \rangle^{1/2}$ as obtained from fits to the momentum transfer distributions.

	Vertex	R (GeV^{-1})	R (f)	$\langle r^2 \rangle^{1/2}$ (f)
R_N	$NN\pi$	2.86 ± 0.08	0.57 ± 0.02	1.06 ± 0.04
R_Δ	$\Delta N\pi$	1.76 ± 0.03	0.35 ± 0.01	0.86 ± 0.02
R_ρ	$\rho\pi\pi$	2.31 ± 0.19	0.46 ± 0.04	0.65 ± 0.05
R_{S11}	S11 $N\pi$	0.03	0.	0.
R_{P11}	P11 $N\pi$	0.34 fixed	0.1	0.14
R_{D13}	D13	4.5 fixed	0.9	0.9
R_{D15}	D15 $N\pi$	5.5		
R_{F15}	F15 $N\pi$	4.5 fixed	0.9	0.8
R_{F17}	F17 $N\pi$	4.5 fixed	0.9	
R_{F37}	F37 $N\pi$	4.5 fixed	0.9	0.7
R_{00}	$(\pi\pi)^{T=0}(\pi\pi)$	0.01	0.	0.
R_f	$f\ \pi\pi$	3.23 ± 1.46	$.65 \pm .29$	0.58 ± 0.26
R_g	$g\ \pi\pi$	4.5 fixed	0.9	0.5
R_{20}	$(\pi\pi)^{T=2,\ell=0}(\pi\pi)$	0.0 0.01	0.	0.
R_{22}	$(\pi\pi)^{T=2,\ell=2}(\pi\pi)$	3.59 ± 1.19	0.72 ± 0.24	0.72 ± 0.24
R_{24}	$(\pi\pi)^{T=2,\ell=4}(\pi\pi)$	4.5 fixed	0.9	0.6

REFERENCES

1. G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

2. J. P. Baton, G. Laurens, and J. Reignier, Phys. Letters 25B, 419 (1967).

3a. Z. Ming Ma, G. A. Smith, R. J. Sprafka, E. Colton, and P. E. Schlein, Report UCLA-1030 Rev., University of California, Los Angeles, (1968) (to be published);

b. P. E. Schlein, Report UCLA-1029 (1968), and review talk at The Informal Meeting on Experimental Meson Spectroscopy, University of Philadelphia, (April 26-27, 1968). Published in Meson Spectroscopy, Eds. C. Baltay and A. H. Rosenfeld (W. A. Benjamin, Inc., New York, 1968).

4. S. Marateck et al., Phys. Rev. Letters 21, 1613 (1968).

5. The quadratic and cubic extrapolations given in Ref. 3a should be called linear and quadratic extrapolations. For a discussion of the general extrapolation function see J. Naisse, and J. Reignier, Fortschritte der Physik 12, 523 (1964).

6. H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).

7. J. Benecke and H. P. Dürr, Nuovo Cimento 56, 269 (1968).

8a. G. Wolf, Phys. Rev. Letters 19, 925 (1967);

b. G. Wolf, Report No. SLAC-PUB-544 (1969) and Phys. Rev., to be published.

9. E. Clementel and C. Villi, Nuovo Cimento 4, 1207 (1956).

10. $\overline{p}p \to \overline{\Delta^{++}}\Delta^{++}$:

(a) H. C. Dehne, E. Raubold, P. Söding, M. W. Teucher, G. Wolf and E. Lohrmann, Phys. Letters 9, 185 (1964), and H. C. Dehne, University of Hamburg, Thesis, (1964) (unpublished), (3.6 GeV/c);

(b) Bonn-Hamburg-Milano collaboration, Phys. Letters 15, 356 (1966), (5.7 GeV/c);

c. V. Alles-Borelli, B. French, A. Frisk, and I. Michejda, Nuovo Cimento 48, 360 (1957), (5.7 GeV/c).

11. $pp \to \Delta^{++}n$:

(a) S. Coletti, J. Kidd, L. Mandelli, V. Pelosi, S. Ratti, V. Russo, L. Tallone, E. Zampieri, C. Caso, F. Conte, M. Dameri, C. Crosso, and G. Tomassini, Nuovo Cimento 49, 479 (1967), (4.0 GeV/c).

(b) H. C. Dehne, J. Diaz, K. Strömer, A. Schmitt, W. P. Swanson, and G. Wolf, Nuovo Cimento 53, 232 (1968), (10.0 GeV/c).

12. $\pi^+ p \to \Delta^{++}\rho^o$:

(a) N. Gelfand, Columbia University, NEVIS Report 137, (June 1965), (unpublished) (2.35 GeV/c); the data at 3-4 GeV/c and 6.95 GeV/c have been taken from

(b) D. Brown, G. Gidal, R. W. Birge, R. Bacastow, S. Yiu Fung, W. Jackson and R. Pu, Phys. Rev. Letters 19, 664 (1967);

(c) P. Slatterly, H. Kraybill, B. Froman, and T. Ferbel, Report UR-875-153, University of Rochester (July 1966), (unpublished) (6,95 GeV/c);

(d) Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I. C.)-München collaboration, Nuovo Cimento 35, 659 (1965); and

(e) Phys. Rev. 138, B897 (1965), (4.0 GeV/c);

(f) J. Ballam, A. Brody, G. Chadwick, Z.G.T. Guiragossián, W. B. Johnson, R. R. Larsen, D.W.G.S. Leith, and E. Pickup, private communication, (16.0 GeV/c).

13. $\pi^- p \to n\rho^o$:

(a) Saclay-Orsay-Bari-Bologna collaboration, Nuovo Cimento 29, 515 (1963), (1.59 GeV/c);

(b) preprint, (1964), and Nuovo Cimento 35, 713 (1965), (2.75 GeV/c);

(c) J. A. Poirier, N. N. Biswas, N. M. Cason; I. Derado, V. P. Kenney, W. D. Shephard, E. H. Synn, H. Yuta, W. Selove, R. Ehrlich, and A. L. Baker, Phys. Rev. 163, 1462 (1967), (8.0 GeV/c).

14. Aachen-Birmingham-Bonn-Hamburg-London (I. C.)-München collaboration Nuovo Cimento 31, 729 (1964).

15. J. A. Poirier, N. N. Biswas, N. M. Cason, I. Derado, V. P. Kenney, W. D. Shepard, E. H. Synn, H. Yuta, W. Selove, R. Ehrlich, A. L. Baker, Phys. Rev. 163, 1462 (1967).

16. This value of $\langle r_p^2 \rangle^{1/2}$ was obtained from a straight line fit to the values of the electric form factor of the proton as measured at momentum transfers below $1f^{-2}$ by B. Dudelzak, G. Sauvage, and P. Lehmann, Nuovo Cimento 28, 18 (1962), and T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. 142, 922 (1966).

17. W. Bartel, B. Dudelzak, H. Krehbiel, J. McElroy, U. Meyer-Berkhout, W. Schmidt, V. Walther, and G. Weber, Phys. Letters 27B, 660 (1968).

18. The behavior of the extrapolation function near the pole depends on the type of vertices involved. For example, for a determination of

$$\sigma_{\pi^+\pi^-}(m)$$

from the reaction

$$\pi^+ p \rightarrow \pi^+ \pi^- \Delta^{++}$$

the corresponding extrapolation function in the OPE model is

$$\left[\frac{q_t \sigma_{\pi^+\pi^-}(t)}{q\, \sigma_{\pi^+\pi^-}} \cdot \frac{Q_t \sigma_\Delta(t)}{Q\, \sigma_\Delta} \right] \sigma_{\pi^+\pi^-}$$

where Q, Q_t are the on-shell and off-shell pion momenta in the Δ rest system. Since

$$\frac{Q_t \sigma_\Delta(t)}{Q\, \sigma_\Delta}$$

is rising with increasing momentum transfer (see Fig. 8b) the extrapolation function will fall when going to the pole.

19. Th. G. Trippe, Chih-Yung Chien, E. Malamud, J. Melema, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Letters 28B, 203 (1968).

FIGURE CAPTIONS

1. OPE diagram for the reaction $\pi^- p \to \pi^- \pi^+ n$.

2. Approximation of the scattering process $a + b \to a' + b'$ by the exchange of a scalar particle x with mass m_x.

3. Differential cross sections $d\sigma/d|t|$ for events of reaction (12) in the $\overline{\Delta^{++}}$, Δ^{++} mass region. The curves give the result of the OPE fit.

   (a) at 3.6 GeV/c[10a] (1.13 GeV < $M_{\overline{\Delta}\Delta}$ < 1.33 GeV)

   (b) at 5.7 GeV/c[10c] (1.15 GeV < $M_{\overline{\Delta}\Delta}$ < 1.35 GeV).

4. Differential cross sections $d\sigma/d\,|t|$ for events of reaction (13) in the Δ^{++} region. The curves give the result of the OPE fit.

   (a) at 4 GeV/c[11a] (1.08 GeV < M < 1.40 GeV)

   (b) at 10 GeV/c[11b] (1.125 < M < 1.325 GeV).

5. Differential cross sections $d\sigma/d\,|t|$ for events of reaction (14) in the Δ^{++}, ρ^0 region. The curves in (a) - (d) give the result of the OPE fit. The curve for the 16 GeV/c case, (e), is a prediction of the OPE model.

   (a) at 2.35 GeV/c[12a] (0.675 GeV < m < 0.825 GeV, 1.185 GeV < M < 1.285 GeV).

   (b) at 3-4 GeV/c[12b] (0.68 GeV < m < 0.86 GeV, 1.12 GeV < M < 1.32 GeV).

   (c) at 4 GeV/c[12d] (0.66 GeV < m < 0.86 GeV, 1.12 GeV < M < 1.32 GeV).

   (d) at 6.95 GeV/c[12c] (0.64 GeV < m < 0.88 GeV, 1.12 GeV < M < 1.42 GeV).

   (e) at 16 GeV/c[12f] (01.68 GeV < m < 0.86 GeV, 1.12 GeV < M < 1.32 GeV).

6. The differential cross sections $d\sigma/d\,|t|$ for events of reaction (15) in the ρ^0 region. The curves give the results of the OPE fit.

   (a) at 1.59 GeV/c[13a] (0.616 GeV < m < 0.85 GeV)

   (b) at 2.75 GeV/c[13b] (0.65 GeV < m < 0.85 GeV)

   (c) at 8.0 GeV/c[13c] (0.675 GeV < m < 0.875 GeV).

7. The differential cross sections $d\sigma/d\,|t|$ for the events of the reaction $\pi^- p \rightarrow nf^o$. The curves give the result of the OPE fit.

(a) at 4 GeV/c[14] (1.16 GeV < m < 1.38 GeV)

(b) at 8 GeV/c[15] (1.17 GeV < m < 1.37 GeV)

The differential cross section $d\sigma/d\,|t|$ for the events of the reaction $\pi^- p \rightarrow ng^o$. The curve gives the result of the OPE fit.

(c) at 8 GeV/c[15] (1.60 GeV < m < 1.75 GeV).

8. The t dependence of the form factor and the off-shell cross section (see text) for the vertices

(a) $NN\pi$

(b) $\Delta N\pi$

(c) $\rho\pi\pi$

(d) $f\pi\pi$

9. The function $\Sigma_{\pi^-\pi^+}(m,t)$ as measured for events of the reaction

$$\pi^- p \rightarrow \pi^- \pi^+ n$$

with an effective mass of the $\pi^-\pi^+$ system between 0.76 GeV and 0.78 GeV[4]. Also shown are two different extrapolations of $\Sigma_{\pi^-\pi^+}(m,t)$ to the pion pole ($t = \mu^2$): one which assumes a linear t dependence for $\Sigma_{\pi^-\pi^+}(m,t)$, another which utilizes the Benecke-Dürr (BD) parameterization.

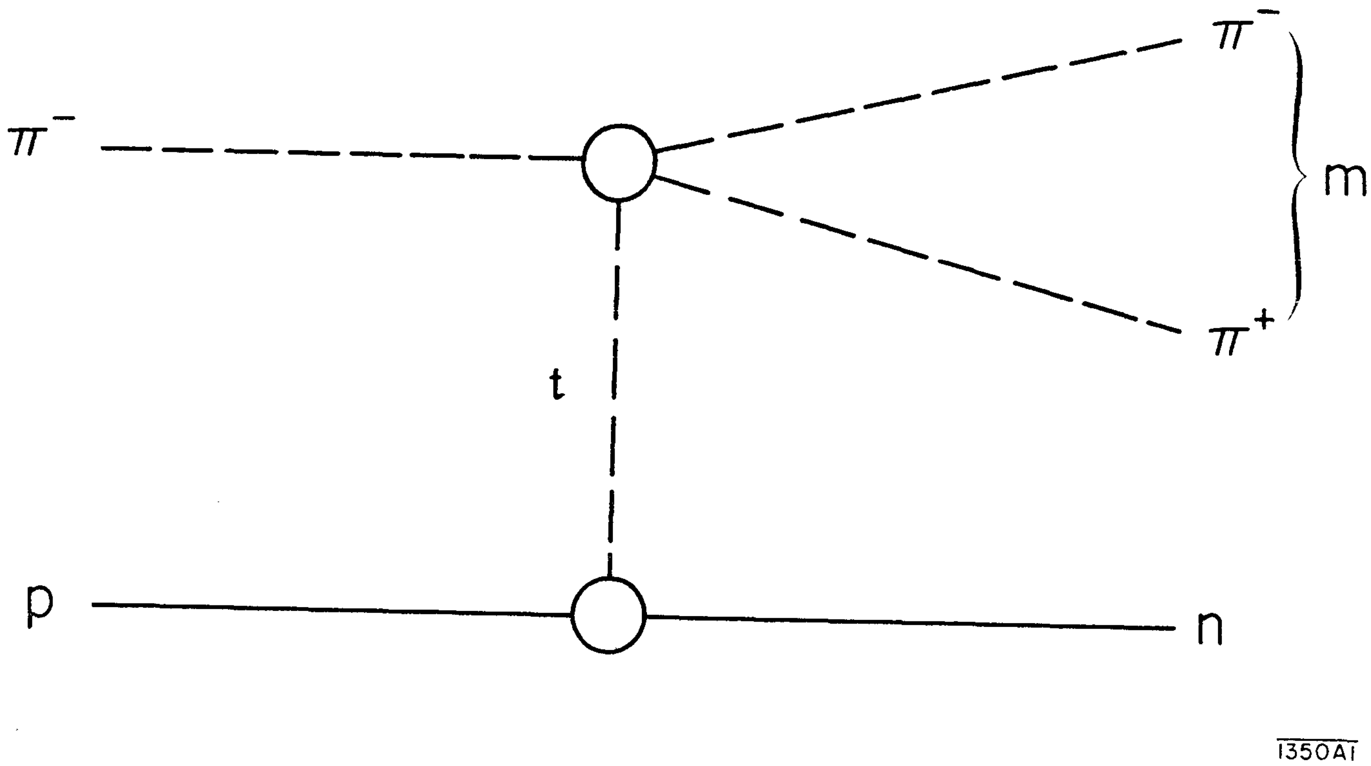


Fig. 1

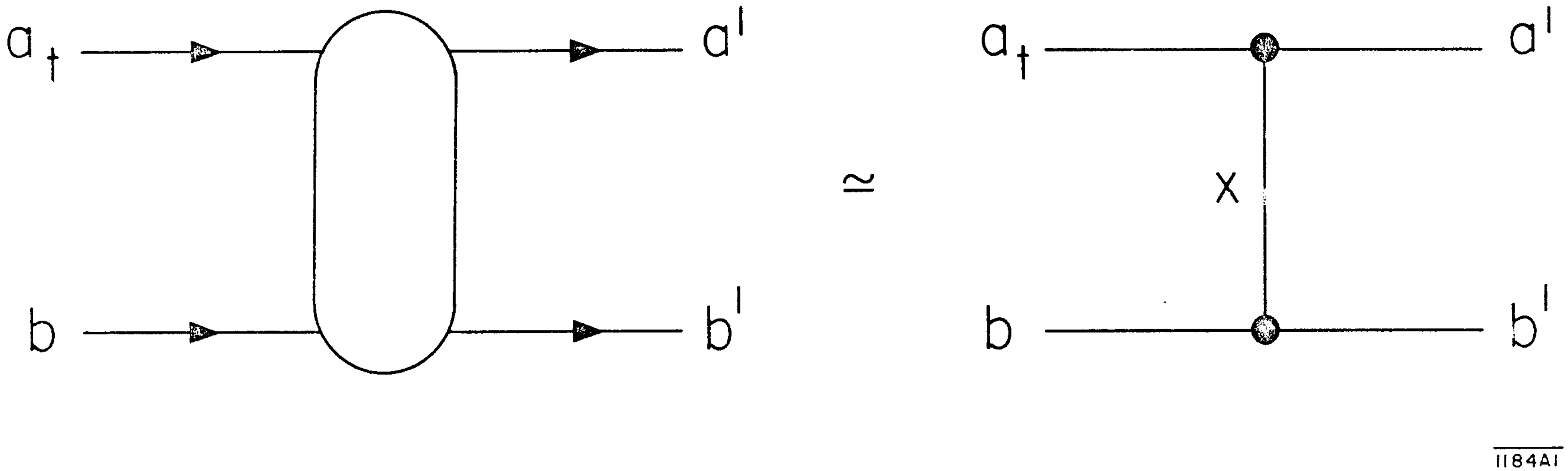
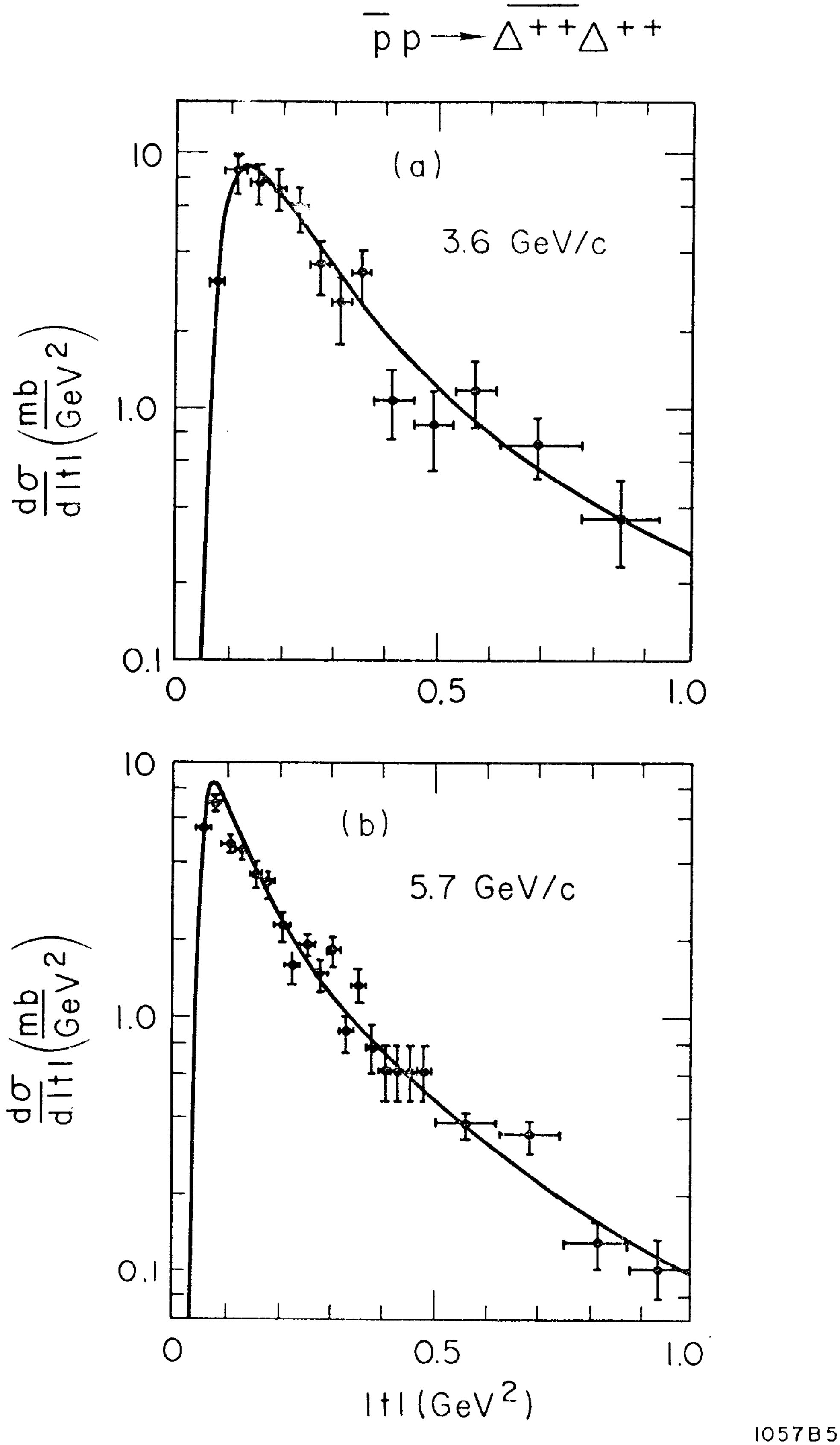


Fig. 2



$\bar{p}p \rightarrow \overline{\Delta^{++}}\Delta^{++}$
(a)
3.6 GeV/c
(b)
5.7 GeV/c
$\frac{d\sigma}{d|t|}\left(\frac{mb}{GeV^2}\right)$
10
1.0
0.1
0
0.5
1.0
$|t|\,(GeV^2)$
1057B5

Fig. 3

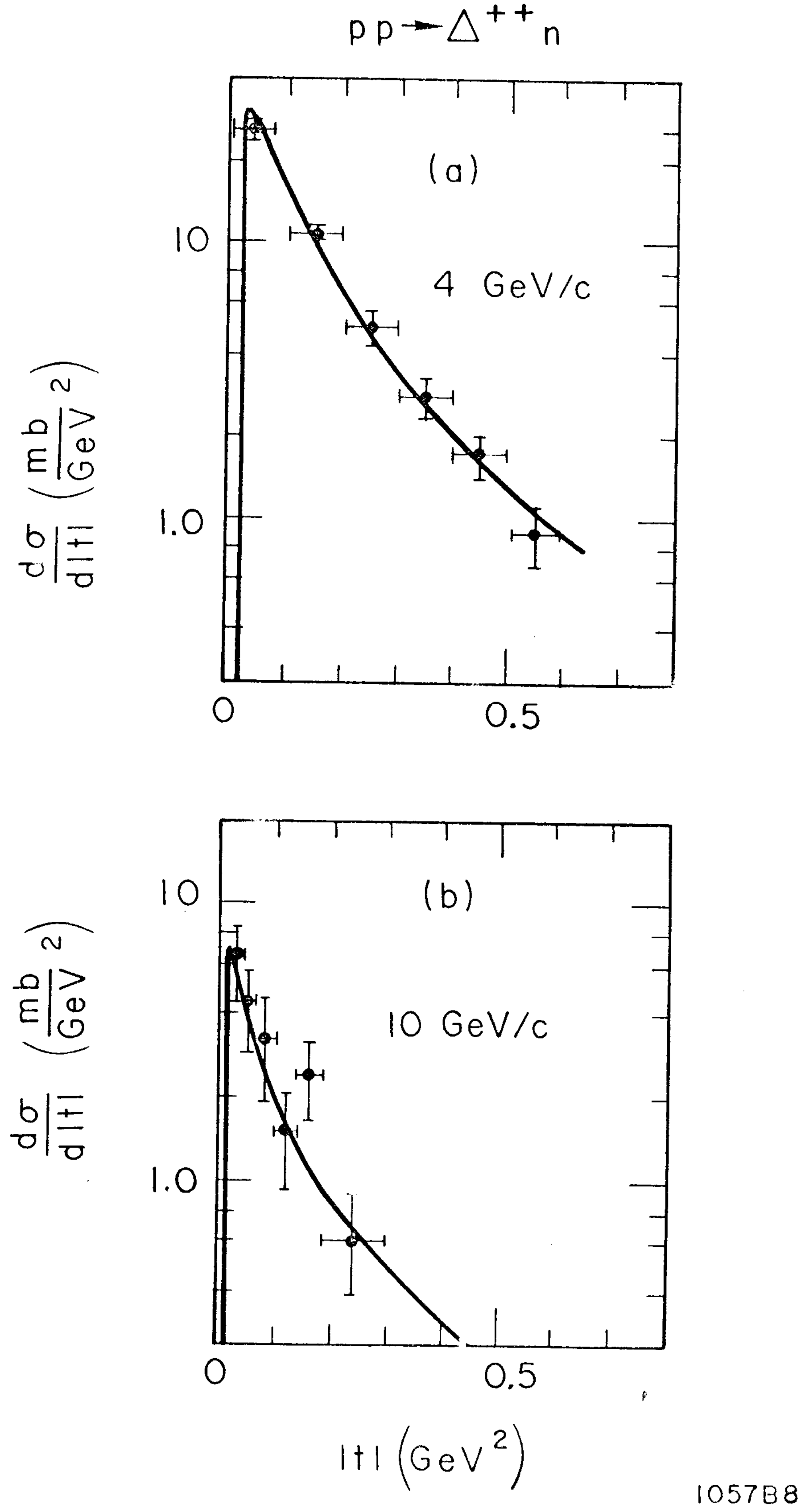
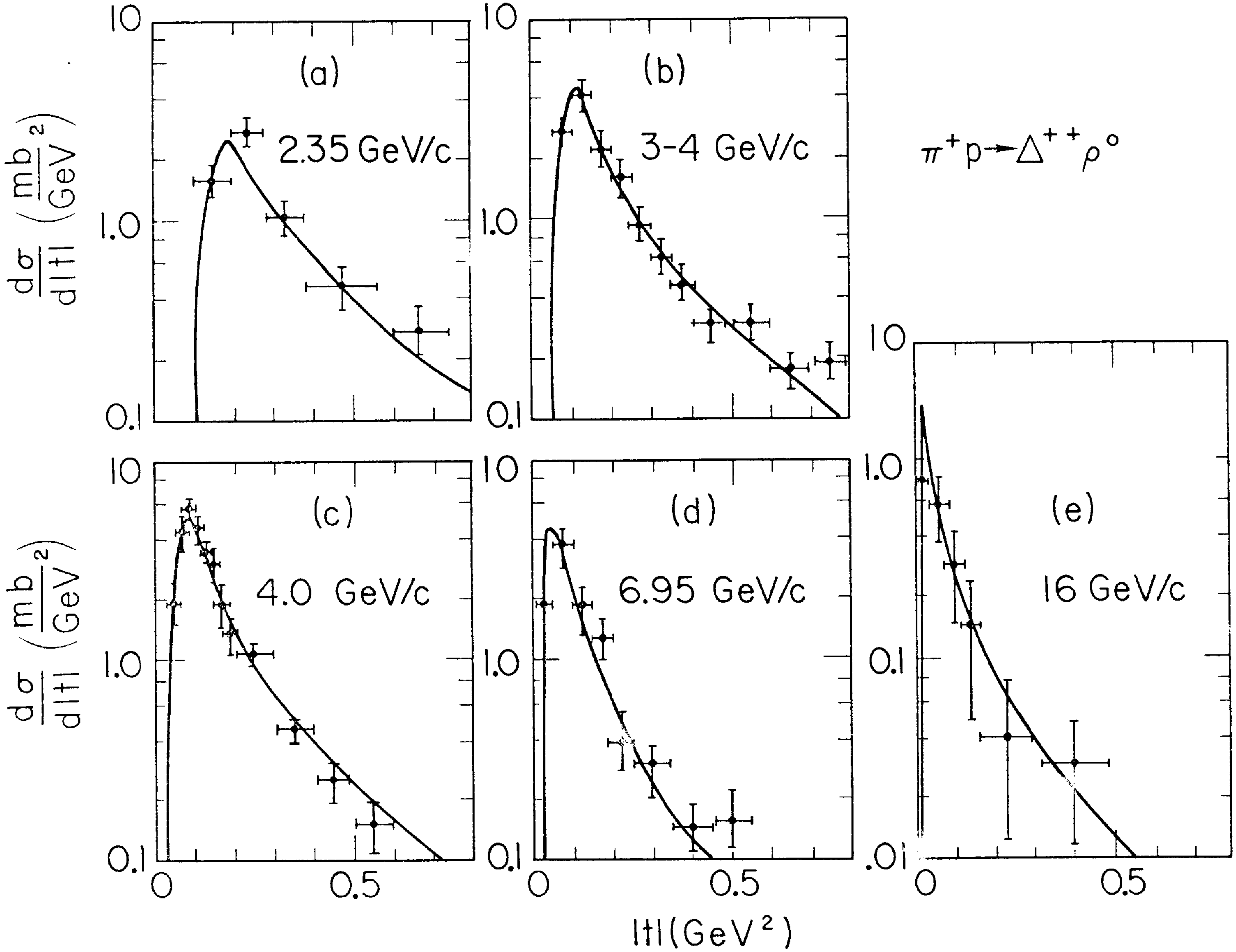

pp → Δ⁺⁺n
(a)
4 GeV/c
10
1.0
0
0.5
dσ/d|t| (mb/GeV²)
(b)
10 GeV/c
10
1.0
0
0.5
dσ/d|t| (mb/GeV²)
|t| (GeV²)
1057B8

Fig. 4



(a)
2.35 GeV/c
(b)
3-4 GeV/c
$\pi^+ p \rightarrow \Delta^{++} \rho^0$
(c)
4.0 GeV/c
(d)
6.95 GeV/c
(e)
16 GeV/c
$\frac{d\sigma}{d|t|} \left(\frac{mb}{GeV^2}\right)$
|t| (GeV²)
10
1.0
0.1
.01
0
0.5

1057C6

Fig. 5

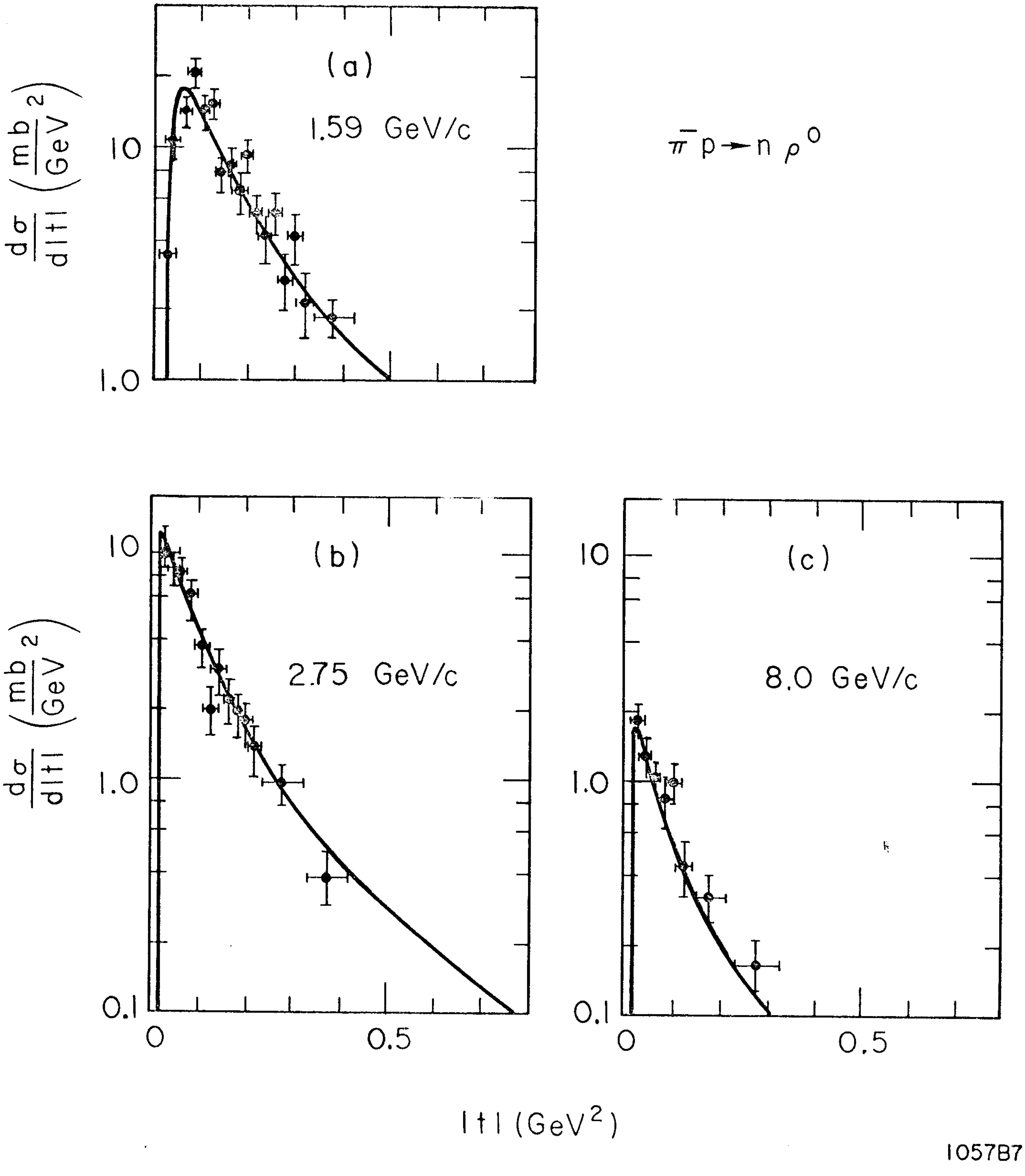
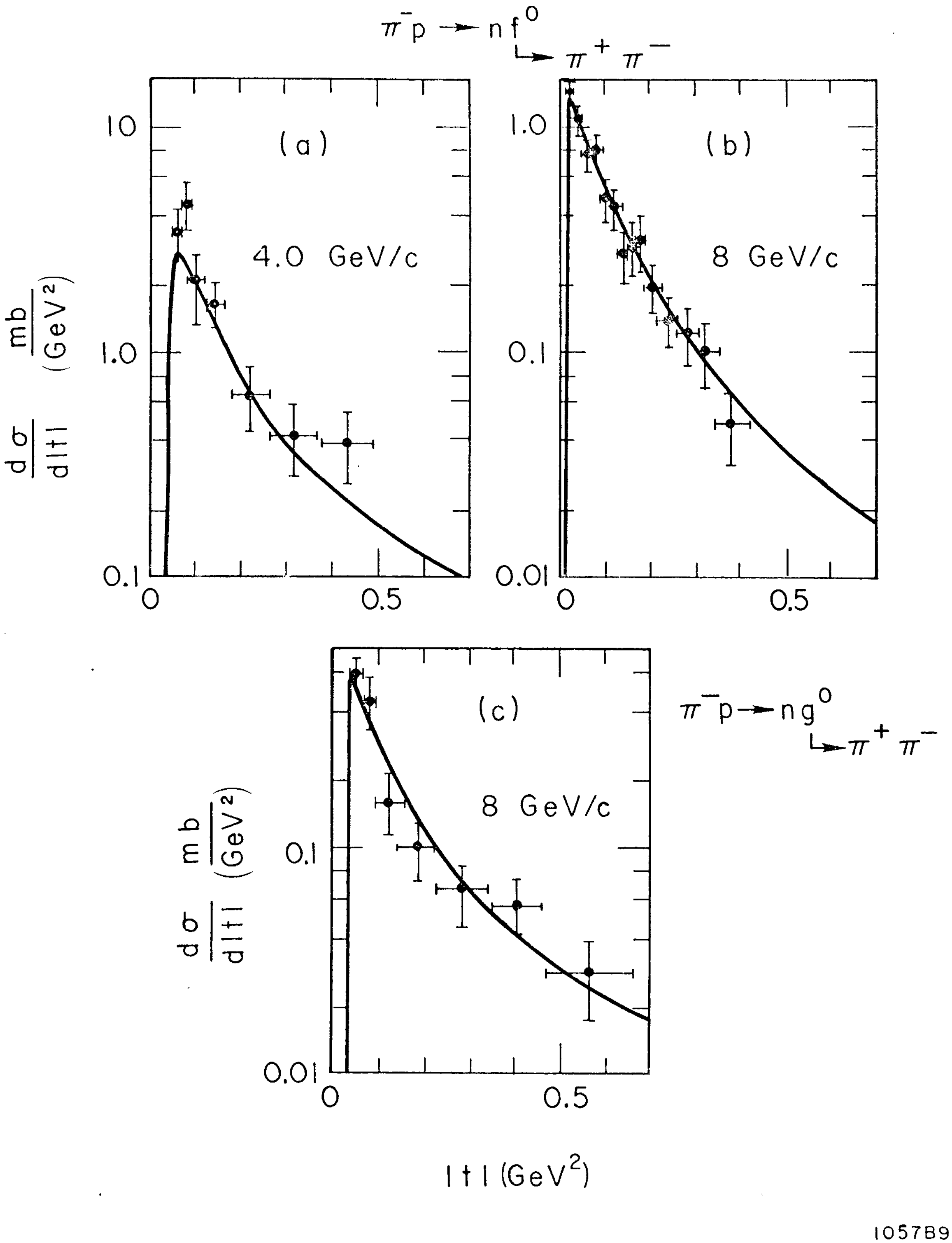
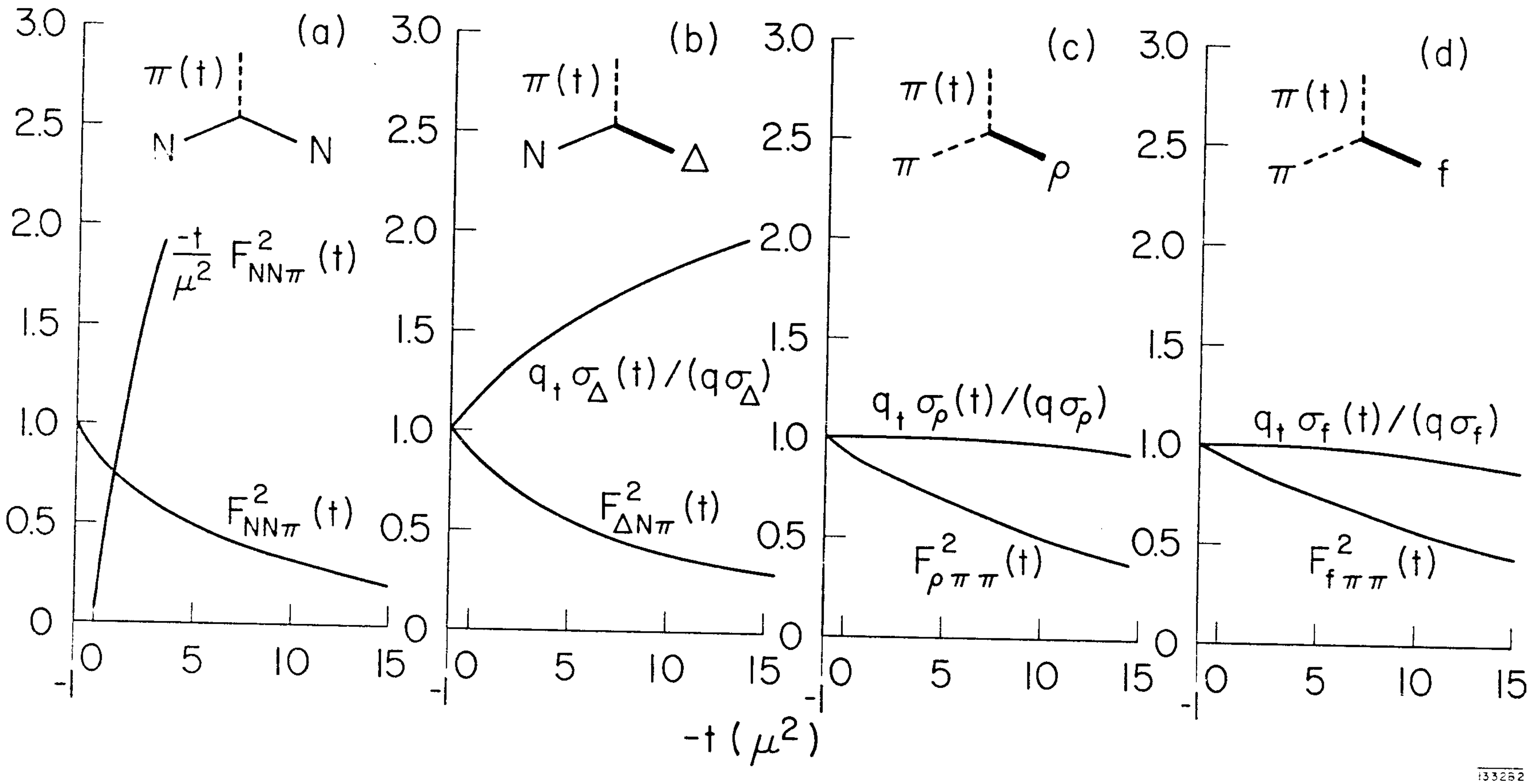

(a)
1.59 GeV/c
$\pi^- p \to n \rho^0$
$\frac{d\sigma}{d|t|} \left(\frac{mb}{GeV^2}\right)$
10
1.0
(b)
2.75 GeV/c
10
1.0
0.1
(c)
8.0 GeV/c
0
0.5
$|t|\,(GeV^2)$
1057B7

Fig. 6



$\pi^- p \to n f^0$
$\hookrightarrow \pi^+ \pi^-$
(a)
4.0 GeV/c
(b)
8 GeV/c
(c)
8 GeV/c
$\pi^- p \to n g^0$
$\hookrightarrow \pi^+ \pi^-$
$\frac{d\sigma}{d|t|}$ $\frac{mb}{(GeV^2)}$
$|t|$ (GeV^2)
10
1.0
0.1
0.01
0
0.5
1057B9

Fig. 7



(a)
(b)
(c)
(d)
π(t)
N
N
Δ
π
ρ
f
$\frac{-t}{\mu^2} F^2_{NN\pi}(t)$
$F^2_{NN\pi}(t)$
$q_t\,\sigma_\Delta(t)/(q\sigma_\Delta)$
$F^2_{\Delta N\pi}(t)$
$q_t\,\sigma_\rho(t)/(q\sigma_\rho)$
$F^2_{\rho\pi\pi}(t)$
$q_t\,\sigma_f(t)/(q\sigma_f)$
$F^2_{f\pi\pi}(t)$
$-t\ (\mu^2)$
1332B2

Fig. 8

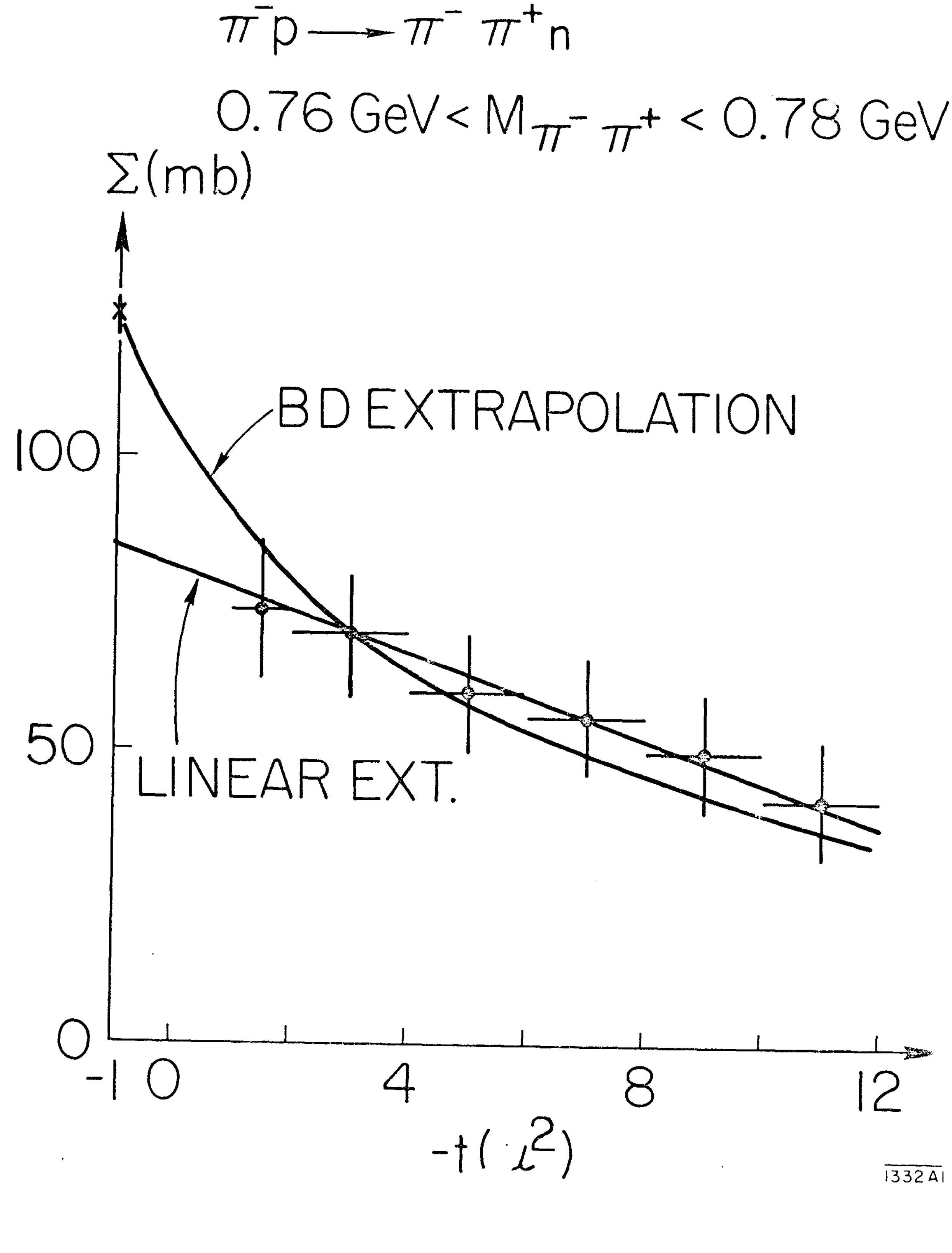

$\pi^- p \longrightarrow \pi^- \pi^+ n$
$0.76\ \mathrm{GeV} < M_{\pi^- \pi^+} < 0.78\ \mathrm{GeV}$
Σ(mb)
BD EXTRAPOLATION
LINEAR EXT.
100
50
0
-1 0
4
8
12
$-t(\mu^2)$
1332A1

Fig. 9