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OLD FASHIONED PERTURBATION THEORY AND VECTOR MESON DOMINANCE*

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ABSTRACT

It is pointed out that the notion of vector meson dominance is best formulated in terms of old fashioned perturbation theory diagrams viewed in the Lorentz frame where the energy of the virtual photon q_0 is infinity. A more convincing way to derive Sakurai's result on ep inelastic scattering is presented. We give a simple way to understand why vector meson dominance does not give a correct transition form factor for the electro-excitation of N*(1238).

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Sakurai[1] predicted that the ratio of scalar to transverse $\gamma + p$ total cross section in the inelastic ep scattering to be $\sigma_S/\sigma_T \sim |q^2|/m_\rho^2$ in the deep inelastic region based on the vector meson dominance hypothesis. However, one feels somewhat uneasy about the way in which the result was obtained. Let us start out[2] with his Eq. (3)

$$\langle f|j_\mu|p\rangle = \left(em_\rho^2/f_\rho\right)\left(m_\rho^2-q^2\right)^{-1}\langle f|j_\mu^{(\rho)}|p\rangle \ . \tag{1}$$

Choosing the z axis along the momentum of the photon we obtain from the conservation of electromagnetic current j_μ, the relation

$$q_0\langle f|j_0^{(\rho)}|p\rangle = \left(q_0^2-q^2\right)^{\frac{1}{2}}\langle f|j_z^{(\rho)}|p\rangle \quad . \tag{2}$$

where q_0 is the time component of four momentum transfer q. This relation must hold no matter how the matrix element $\langle f|j_\mu^{(\rho)}|p\rangle$ is approximated as long as we assume Eq. (1) to hold. This seems to contradict with Sakurai's (on the mass shell) relation:

$$q_0\langle f|j_0^{(\rho)}|p\rangle_{\text{on-shell}} = \left(q_0^2-m_\rho^2\right)^{\frac{1}{2}}\langle f|j_z^{(\rho)}|p\rangle_{\text{on-shell}} \ . \tag{3}$$

If m_ρ^2 in Eq. (3) were replaced by q^2, we would have obtained $\sigma_S/\sigma_T \approx m^2/(-q^2)$ instead of Sakurai's $\sigma_S/\sigma_T \approx -q^2/m_\rho^2$.

We would like to present an argument to make Sakurai's result more convincing. Apparently Eq. (3) makes sense, whereas if m_ρ^2 is replaced by q^2 it does not make sense. Since current conservation for j_μ must be satisfied in order to write the inelastic ep cross section in terms of W_1 and W_2, the only way we can save the theory is to modify Eq. (1).

In actual applications of vector dominance theory, we are merely trying to relate the cross sections for the three processes: virtual $\gamma + p \rightarrow f$, real

$\gamma + p \to f$ and real $\rho + p \to f$ (or its inverse). In the last two processes γ and ρ are both real and moving in the positive time direction whereas the first process represented by a Feynman diagram (Fig. 1) actually consists of 3! = 6 old fashioned perturbation theory (OFPT) diagrams.[3] Only Fig. 2A corresponds to our notion that γ is changed into ρ and then ρ interacts with the target system. Hence we conclude that we should not use Eq. (1) which is represented by Fig. 1 but should use the matrix element given by the OFPT diagram Fig. 2A. Figure 2A has a denominator

$$\left[q^2\left(q^2-m_\rho^2\right)\right]^{-1} \frac{1}{4}\left(1 + q_0\left(q_0^2-q^2\right)^{-\frac{1}{2}}\right)\left(1 + q_0\left(q_0^2-q^2 + m_\rho^2\right)^{-\frac{1}{2}}\right) \qquad (4)$$

(q_0 is the time component of q) compared with the denominator for the Feynman diagram (Fig. 1), $\left[q^2\left(q^2-m_\rho^2\right)\right]^{-1}$. Only in the limit $q_0^2/\left(|q^2| + m_\rho^2\right) \to \infty$, Eq. (4) becomes covariant and reduces to the Feynman propagator. Because all physical processes must be Lorentz invariant, we conclude that the notion of the vector meson dominance can be formulated only in the Lorentz frame where $q_0 \to \infty$. This frame can be obtained from the laboratory system by moving the observer with a velocity $\to$ c against the direction of the incident photon. Let us denote $\underset{\sim}{Q}$ and $\underset{\sim}{P}$ as the momenta of the photon and proton respectively in the $q_0 \to \infty$ frame. The usual γ for the Lorentz transformation is given by $\gamma = (P^2 + M^2)^{1/2}/M$ and Q can be written in terms of q^2 and $\nu = (p \cdot q)/M$ as $Q \equiv \left(q_0^2 - q^2\right)^{1/2} = (\gamma^2-1)^{1/2}\nu + \gamma(\nu^2-q^2)^{1/2}$. In the $q_0 \to \infty$ frame both $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ are pointing toward the + z direction and their magnitudes become infinity as $\gamma \to \infty$. We recall that in OFPT all particles are on the mass shell and the momentum is conserved at each vertex but not the energy. Furthermore, in the frame $q_0 \to \infty$, even the energy is almost conserved at the vertex because $q_0 - \left(Q^2 + m_\rho^2\right)^{1/2} \approx \left(q^2 - m_\rho^2\right)/2q_0 \to 0$. Since the notion of vector meson dominance is valid only in a particular Lorentz frame we can modify Eq. (1) into

$$\langle f|j_\mu|p\rangle_{q_0\to\infty} = \left(em_\rho^2/f_\rho\right)\left(m_\rho^2-q^2\right)^{-1}\left(g_{\mu\alpha}-\rho_\mu\rho^\alpha m_\rho^{-2}\right)\left[\left(Q^2+m_\rho^2\right)^{\frac{1}{2}}/q_0\right]^{\delta_{\mu 0}}\langle f'|j_\alpha^{(\rho)}|p\rangle_{q_0\to\infty} \tag{5}$$

where $\rho_\mu = \left(\left(Q^2+m_\rho^2\right)^{1/2}, Q\hat{e}_z\right)$ and $\delta_{\mu 0} = 1$ if $\mu = 0$ and $\delta_{\mu 0} = 0$ if $\mu \neq 0$. Eq. (5) satisfies $q_0 j_0 = Qj_z$ and $\left(Q^2+m_\rho^2\right)^{1/2} j_0^{(\rho)} = Qj_z^{(\rho)}$. It does not violate any Lorentz invariance because we are comparing two matrix elements in a particular frame ($q_0\to\infty$). Because in ϘFPT the momentum is conserved but not the energy, we treated the three space components of $j_\mu^{(\rho)}$ on the same footing but not $j_0^{(\rho)}$. In the limit $q_0\to\infty$, the energy is conserved, so does the factor $\left(Q^2+m_\rho^2\right)^{1/2}/q_0$ become unity. In writing Eq. (5), we have assumed that $\langle f|j_\mu^{(\rho)}|p\rangle$ in Eq. (1) is approximated by a (real ρ) + p → f' amplitude obtained by assuming that in the $q_0\to\infty$ frame ρ has a momentum $\mathbf{Q} = \left(q_0^2-q^2\right)^{1/2}\hat{e}_z$ and energy $\left(Q^2+m_\rho^2\right)^{1/2}$. Notice that f and f' have the same momentum, and almost have the same energy (the difference in energy is equal to $\left(m_\rho^2-q^2\right)/2q_0$, but have a finite difference in the invariant masses, $M_{f'}^2-M_f^2 = \left(m_\rho^2-q^2\right)\left[1+M\left(\nu+(\nu^2-q^2)^{1/2}\right)^{-1}\right]$.

It is important to notice that this kinematical condition in the $q_0\to\infty$ frame, when transformed back to the laboratory systems of ep and ρp scatterings, actually becomes the condition that ρ has energy $\nu_\rho = \nu\left(1+M\left(\nu+(\nu^2-q^2)^{1/2}\right)^{-1}\right)$. Hence in the laboratory system where we actually compare the cross sections we must use the cross section for ρ + p → f' at energy ν_ρ, rather than $K = \nu - q^2/2M$ as was done by Sakurai. We could have imposed an alternative condition (as Sakurai did) that f and f', have the same invariant mass, but ρ and photon have different energies and momenta in the $q_0\to\infty$ frame. However this is not only very arbitrary but also causes unnecessary complications in kinematics when we try to transform these conditions back to the laboratory frame from the $q_0\to\infty$ frame. We therefore stick to our assumption and investigate its consequence.

There is one obvious question which comes to mind with regard to the applicability of vector meson dominance to the inelastic ep scattering. We know that vector meson dominance does not work for electro-excitation of N*(1238). The empirical form factor for this excitation,[4] good up to $-q^2 = 2.4$ GeV^2, is given by $[1 + 9(-q^2)^{1/2}]^{1/2} \exp[-3.15(-q^2)^{1/2}]$ which decreases much more rapidly than $\left(1-q^2/m_\rho^2\right)^{-1}$ predicted by a naive application of vector dominance theory. If the vector meson dominance does not work for the resonance excitation, why should one expect it to work for the continuum region? There might be some dynamical reasons why this is so, but the easiest way out is to notice that Eq. (5) fails if the cross section varies rapidly with M_f such as in the resonance region. Let us suppose that the cross section changes a considerable amount if we change M_f by a width Γ. Equation (5) fails unless $M_{f'}-M_f \ll \Gamma$, or

$$2M_f\Gamma \gg M_{f'}^2-M_f^2 = \left(m_\rho^2-q^2\right)\left(1 + M\left(\nu + (\nu^2-q^2)^{1/2}\right)^{-1}\right) . \qquad (6)$$

When $q^2 < 0$, this inequality can never be satisfied in the resonance region, where Γ is typically $0.1 \sim 0.2$ GeV and $M_f \sim 1$ to 2 GeV. In the deep inelastic region where the cross section is flat to within $\Gamma = 1 \sim 5$ GeV and $M_f > 2.5$ GeV this inequality can be satisfied easily as long as $-q^2$ is not very large. We believe this is a partial explanation of the failure of vector dominance theory when applied to calculate the form factor of the N*(1238) excitation. In addition to explaining why ρ dominance does not work for the 3-3 resonance, Eq. (6) has two pleasant features: (1) it is a covariant statement, and (2) it is always satisfied when $q^2 = m_\rho^2$.

As an illustration of how Eq. (5) can be used for an actual calculation, we relate in the following Drell and Walecka's[5] W_1 and W_2 to the total longitudinal and transverse $\rho + p$ cross sections, $\sigma_\parallel^\rho(\nu_\rho)$ and $\sigma_\perp^\rho(\nu_\rho)$. $W_1(\nu, q^2)$ and $W_2(\nu, q^2)$

are invariant functions of ν and q^2 defined by

$$W_{\mu\nu} \equiv M^{-2}\left(P_\mu - q_\mu (p\cdot q)/q^2\right)\left(p_\nu - q_\nu (p\cdot q)/q^2\right) W_2 - (g_{\mu\nu} - q_\mu q_\nu/q^2) W_1$$

$$\equiv P_0 M^{-1} \sum_f \langle p|j_\mu(0)|f\rangle\langle f|j_\nu(0)|p\rangle\ (2\pi)^3\ e^{-2}\delta^4(p+q-p_f) \quad . \qquad (7)$$

Since W_1 and W_2 are invariant we can evaluate them in any Lorentz frame. We evaluate them in our $q_0 \to \infty$ frame. From Eqs. (5) and (7), we have

$$W_1 = W_{xx} = \left(m_\rho^2/f_\rho\right)^2\left(m_\rho^2 - q^2\right)^{-2} \gamma\sum_f |\langle f'|j_x^{(\rho)}|p\rangle_{q_0\to\infty}|^2 \delta^4(p+\rho-p_{f'})(2\pi)^3 \qquad (8)$$

and

$$W_2 = \left[W_{zz} - (1 + Q^2/q^2)W_{xx}\right] M^2/\left(P - Q(p\cdot q)/q^2\right)^2 \qquad (9)$$

where W_{zz} is a similar expression as (8) with $j_x^{(\rho)}$ replaced by $j_z^{(\rho)}$

Since $\sigma_\parallel^\rho(\nu_\rho)$ and $\sigma_\perp^\rho(\nu_\rho)$ are also invariant (because our Lorentz transformation is along the direction of the incident beam) we calculate them in the $q_0 \to \infty$ frame.

$$\sigma_\perp^\rho(\nu_\rho) = (2\pi)^4 \gamma M^{\frac{1}{2}}\left[(p\cdot q^\rho)^2 - m_\rho^2 M^2\right]^{-\frac{1}{2}} \sum_{f'} |\langle f'|j_x^{(\rho)}|p\rangle_{q_0\to\infty}|^2 \delta^4(p+\rho-p_{f'}) \qquad (10)$$

$\sigma_\parallel^\rho(\nu_\rho)$ is a similar expression as (10) except that $j_x^{(\rho)}$ is replaced by $\epsilon^\ell \cdot j^{(\rho)} = -m_\rho\left(Q^2 + m_\rho^2\right)^{-1/2} j_3^{(\rho)}$, where $\epsilon^\ell = m_\rho^{-1}\left[Q, \left(Q^2 + m_\rho^2\right)^{1/2} e_z\right]$ and the current conservation has been used to eliminate $j_0^{(\rho)}$. Expressing W_{xx} and W_{zz} in terms of $\sigma_\perp^\rho(\nu_\rho)$ and $\sigma_\parallel^\rho(\nu_\rho)$ respectively, we obtain (for $\nu_\rho^2 \gg m_\rho^2$)

$$W_1 = \pi^{-1}\left(m_\rho^2/f_\rho\right)^2\left(m_\rho^2 - q^2\right)^{-2} \nu_\rho \sigma_\perp^\rho(\nu_\rho) \qquad (11)$$

and

$$W_2 = \pi^{-1}\left(m_\rho^2/f_\rho\right)^2\left(m_\rho^2-q^2\right)^{-2}\nu_\rho\left[\left(\sigma_{\parallel}^{\rho}/m_\rho^2\right)-\left(\sigma_{\perp}^{\rho}/q^2\right)\right]q^4\left(\nu^2-q^2\right)^{-1} \tag{12}$$

From W_1 and W_2 we can write down Hand's[1] σ_S and σ_T immediately and obtain

$$\sigma_T(q^2,\nu) = (e/f_\rho)^2\left[m^2/\left(m_\rho^2-q^2\right)\right]^2 \nu_\rho K^{-1}\sigma_{\perp}^{\rho}(\nu_\rho) \tag{13}$$

$$\sigma_S(q^2,\nu) = (e/f_\rho)^2\left[m_\rho^2/\left(m_\rho^2-q^2\right)\right]^2\left(-q^2/m_\rho^2\right)\nu_\rho K^{-1}\sigma_{\parallel}^{\rho}(\nu_\rho) \tag{14}$$

These two equations are essentially identical to Sakurai's Eq. (8) except for the factors such as $(K/\nu)^2$, (ν_ρ/K) and the energy used for the ρ cross sections. The difference can be traced back to whether one assumes the state f and f' to have the same momentum or the same invariant mass. We regard the difference between the two expressions simply represents the amount of uncertainty involved in using the vector meson dominance hypothesis.

The point of view expressed in this letter can also be applied to the peripheral mechanisms, where the virtual cross sections are also approximated by the real cross sections. It should be emphasized that the greatest advantage of using OFPT is that all particles are on the mass shell. This gives us an unambiguous way to define the density matrix[6] of the ρ hitting the target system. This fact will become important when one wants to apply vector meson dominance to investigate some specific channel of f, such as electroproduction of ρ.

In summary we have accomplished the following: (1) Constructed a vector meson dominance theory which satisfies both the current conservation with respect to $\langle f|j_\mu|p\rangle$ and current conservation with respect to $\langle f'|j_\mu^{(\rho)}|p\rangle_{\text{on-shell}}$ simultaneously. (2) We gave a simple criterion when we should not expect vector meson dominance to work. (3) We pointed out the advantages of using the concept of old fashioned perturbation theory in dealing with pole dominance types of mechanisms in general where questions of gauge invariance, current conservation and sum over the spin often cause many ambiguities when one tries to approximate a virtual amplitude with a physical amplitude. The author wishes to thank Dr. Tung-Mow Yan for discussions and comments.

REFERENCES

1. J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969).
2. Our metric is such that $q^2 < 0$ when the momentum q is spacelike. $\nu = (p \cdot q)/M$ is the laboratory energy of the virtual photon in the Feynman language (not in the old fashioned perturbation theory language).
3. See, for example, S. Weinberg, Phys. Rev. 150, 1313 (1966). Strictly speaking our argument is true only if we ignore the dynamics of $\rho + p$ scattering and assume it to occur at a definite time t. The author wishes to thank Dr. Tung-Mow Yan for this comment. Since we are not dealing with the dynamics of $\rho + p$ scattering our argument is justified.
4. A. J. Dufner and Y. S. Tsai, Phys. Rev. 168, 1801 (1968).
5. S. D. Drell and J. D. Walecka, Ann. Phys. (N. Y.) 28, 18 (1964).
6. The factor $\left(g_{\mu\alpha} - \rho_\mu \rho^\alpha m_\rho^{-2}\right)$ in Eq. (5) automatically guarantees that the density matrix constructed from our formalism will satisfy all the requirements for the density matrix of a spin 1 particle.

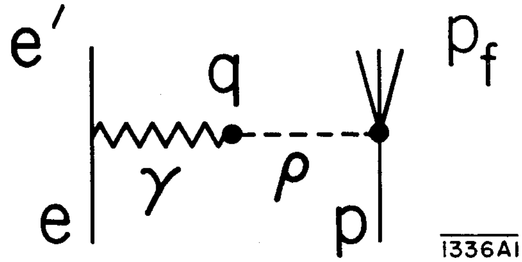

Fig. 1

Feynman diagram for inelastic ep scattering.

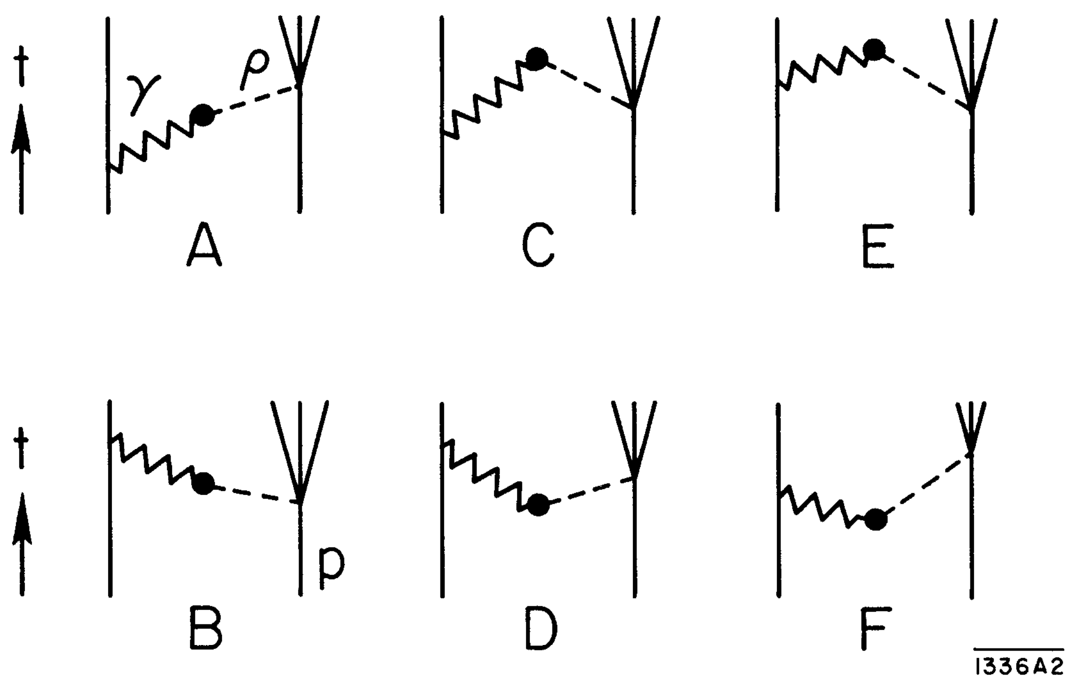

Fig. 2

Old fashioned perturbation theory diagrams. The direction of time is upward.