PRODUCTION OF STRONGLY INTERACTING W'S IN INELASTIC ELECTRON-NUCLEON COLLISIONS ${ }^{\dagger}$<br>John B. Kogut*<br>Stanford Linear Accelerator Center, Stanford University, Stanford, California


#### Abstract

We consider the possibility of extending the search for the intermediate vector boson by observing the products of inelastic electron-nucleon collisions. If the W has mass less than $5.1 \mathrm{BeV} / \mathrm{c}^{2}$ and can interact strongly, then experiments at SLAC, which look for muons with large transverse momentum, should provide a sensitive probe to its existence. If the W is not observed in the proposed experiments, we can deduce stringent upper bounds on the W-nucleon cross section or conclude that if a strongly interacting W exists, it must have a mass in excess of $5.1 \mathrm{BeV} / \mathrm{c}^{2}$.


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[^0]Recently increased interest has been directed toward the elusive W meson for several reasons. First, evidence has been accumulated that seems to indicate that muons detected far underground do not satisfy the $\sec \theta$ law ${ }^{1}$. Some authors feel that the existence of the W might account for this effect ${ }^{2}$. And second, with the advent of high flux neutrino beams it has become possible to search for the W in a relatively simple and systematic fashion ${ }^{3}$. However, the cosmic ray experiments suffer from very poor statistics and questionable interpretations. The neutrino experiments, on the other hand, are severely limited by available beam energies and have only been able to imply that the mass of the W is greater than about $2 \mathrm{BeV} / \mathrm{c}^{2}$ 。 In this paper we propose a search for the W using inelastic electron-nucleon scattering and consider, in particular, the experimental possibilities at the Stanford Linear Accelerator Center. Using a 20 BeV electron beam we can potentially create $W$ 's having mass as great as $5.1 \mathrm{BeV} / \mathrm{c}^{2}$. The W will probably decay weakly into a muon and neutrino, and impart a large transverse momentum to the $\mu$. Hence, even though W's may be produced with a small cross section, $\mu$ 's with large transverse momentum might be detectable over backgrounds. This is indeed found to be the case if, and only if, W's can interact strongly with nucleons.

Consider briefly the electromagnetic processes drawn in Figure la and lb. Using calculations and asymptotic formulae ${ }^{4}$ one can estimate that these diagrams lead to $W$ production cross sections of at best $10^{-37} \mathrm{~cm}^{2}$. Unfortunately, between the low muon counting rates that this cross section implies and the large muon background from $\pi$ and K decays that occur at SLAC, one cannot hope to detect the muons produced in this way. However,
there has recently been considerable speculation that the W might interact strongly ${ }^{5}$ as shown in Figure 2. It is this process that could lead to W production cross sections and $\mu$ counting rates which should be easily observable. The scattering matrix will be:

$$
\left.\mathrm{T}_{\mathrm{fi}}=\frac{\mathrm{g}}{\mathrm{~V}} \frac{1}{\sqrt{2 \mathrm{E}_{\mathrm{e}}}} \frac{1}{\sqrt{2 \mathrm{E}_{\nu}}} \overline{\mathrm{u}}_{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \mathrm{u}_{\mathrm{e}}\left[\frac{-\mathrm{g}_{\lambda \mu}+\frac{\mathrm{k}_{\lambda} \mathrm{k}_{\mu}}{\mathrm{m}_{\mathrm{W}}^{2}}}{\mathrm{k}^{2}-\mathrm{m}_{\mathrm{W}}^{2}}\right]<\mathrm{P}_{\mathrm{n}}\left|\mathrm{~g}^{\prime} \mathrm{W}_{(0)}^{\lambda}\right| \mathrm{P}\right\rangle
$$

where

$$
\begin{aligned}
\mathrm{P} & =\text { momentum of incoming nucleon } \\
\mathrm{P}_{\mathrm{n}} & =\text { momentum of outgoing hadron state } \\
\mathrm{W}^{\lambda} & =\mathrm{W} \text { field operator } \\
\mathrm{k} & =\text { momentum of internal } \mathrm{W} \text { line }
\end{aligned}
$$

We then calculate the cross section in the usual way:

$$
\frac{d \sigma}{\mathrm{~d} \Omega_{\nu} \mathrm{dE}}{ }_{\nu}=\frac{\mathrm{g}^{2}}{(2 \pi)^{3}} \frac{\mathrm{E}_{\nu}}{\mathrm{E}_{\mathrm{e}}} \frac{\mathrm{~m}_{\mathrm{p}}}{\mathrm{E}_{\mathrm{p}}}\left[\frac{1}{\mathrm{k}^{2}-\mathrm{m}_{\mathrm{W}}^{2}}\right]^{2}\left[\mathrm{p}_{\mu}^{\mathrm{e}} \mathrm{p}_{\lambda}^{\nu}+\mathrm{p}_{\lambda}^{\mathrm{e}} \mathrm{p}_{\mu}^{\nu}-\mathrm{p}^{\mathrm{e}} \cdot \mathrm{p}^{\nu} \mathrm{g}_{\mu \lambda}+\mathrm{i} \epsilon_{\mu \lambda \alpha \beta} \mathrm{p}_{\mathrm{e}}^{\alpha} \mathrm{p}_{\nu}^{\beta}\right] M^{\mu \lambda}
$$

where

$$
\left.\mathrm{M}^{\mu \nu}=\frac{\mathrm{E}_{\mathrm{p}}}{m_{\mathrm{p}}} \sum_{\mathrm{n}} \overline{\sum_{\text {spins }}}<\mathrm{P}\left|\mathrm{~g}^{\prime} \mathrm{W}^{\dagger \mu}(0)\right| P_{\mathrm{n}}\right\rangle\left\langle\mathrm{P}_{\mathrm{n}}\right| \mathrm{g}^{\prime} \mathrm{W}^{\lambda}(0)|\mathrm{P}\rangle(2 \pi)^{4} \delta\left(\mathrm{P}_{\mathrm{n}}-\mathrm{P}-\mathrm{k}\right)
$$

The general form of $\mathrm{M}^{\mu \lambda}$ reads:

$$
\mathrm{M}^{\mu \lambda}=\mathrm{P}^{\mu} \mathrm{P}^{\lambda} \mathrm{c}_{1}+\mathrm{P}^{\mu} \mathrm{k}^{\lambda} \mathrm{c}_{2}+\mathrm{P}^{\lambda} \mathrm{k}^{\mu} \mathrm{c}_{3}+\mathrm{k}^{\lambda} \mathrm{k}^{\mu} \mathrm{c}_{4}+\mathrm{g}^{\mu \lambda} \mathrm{c}_{5}+\mathrm{i} \epsilon^{\mu \lambda \alpha \beta} \mathrm{P}_{\alpha} \mathrm{k}_{\beta} \mathrm{c}_{6}
$$

However, since $\mathrm{k}_{\mu} \bar{u}_{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \mathrm{u}_{\mathrm{e}}=0$, taking lepton masses equal to zero throughout the calculation, only the first, fifth, and sixth terms contribute here. Now writing everything in lab coordinates and letting $\theta_{\nu}$ denote the angle between the $\nu$ and the beam we have:

$$
\frac{\mathrm{d} \sigma}{\mathrm{~d} \mathrm{E}_{\nu} \mathrm{d} \Omega_{\nu}}=\frac{4 \mathrm{~g}^{2}}{(2 \pi)^{3}} \mathrm{E}_{\nu}^{2} \mathrm{~m}_{\mathrm{p}}^{2}\left[\frac{1}{\mathrm{k}^{2}-\mathrm{m}_{\mathrm{W}}^{2}}\right]^{2}\left[\frac{1}{2} \cos ^{2}\left(\frac{{ }^{\theta}}{2}\right) \mathrm{c}_{1}-\frac{1}{\mathrm{~m}_{\mathrm{p}}^{2}} \sin ^{2}\left(\frac{{ }^{\theta}}{2}\right) \mathrm{c}_{5}-\frac{\left(\mathrm{E}_{\nu}+\mathrm{E}_{\mathrm{e}}\right)}{\mathrm{m}_{\mathrm{p}}} \sin ^{2}{ }^{2}\left(\frac{{ }^{\nu}}{2}\right) \mathrm{c}_{6}\right]
$$

We can complete the calculation be relating $\mathrm{c}_{1}, \mathrm{c}_{5}$, and $\mathrm{c}_{6}$ to the off-mass-shell scattering cross sections for polarized W's on nucleons (Figure 3). These cross sections are given in invariant form by

$$
\sigma\left(\mathrm{k}^{2}, \nu\right)=\frac{\mathrm{m}_{\mathrm{p}}}{2 \sqrt{(\mathrm{k} \cdot \mathrm{P})^{2}-\mathrm{m}_{\mathrm{W}}^{2} \mathrm{~m}_{\mathrm{p}}^{2}}} \epsilon_{\rho}^{*} \mathrm{M}^{\rho \lambda} \epsilon_{\lambda}
$$

where

$$
\begin{aligned}
& \epsilon=\text { polarization vector of the } \mathrm{W} \\
& \mathrm{k}=\text { momentum of the } \mathrm{W} \\
& \nu=\text { energy of the } \mathrm{W}
\end{aligned}
$$

Then extrapolating the kinematical factors such as $(k \cdot P)^{2}-m_{W}^{2} m_{p}^{2}$ into the region of spacelike $k$, we finally obtain the cross section for the desired process in terms of the variables $\mathrm{k}^{2}$ and $\nu$ :

$$
\begin{aligned}
\frac{\mathrm{d} \sigma}{\mathrm{dk}} \mathrm{~d}^{2} \nu & \left.=\frac{10^{-5} \mathrm{~m}_{\mathrm{W}}^{2}}{\sqrt{2\left(2 \pi^{2}\right) \mathrm{E}_{\mathrm{e}}}\left[\frac{1}{\mathrm{k}^{2}-\mathrm{m}_{\mathrm{W}}^{2}}\right]}\right]^{2}\left\{\frac{-\mathrm{k}^{2}\left(\mathrm{E}_{\mathrm{e}}-\nu\right)}{2 \mathrm{~m}_{\mathrm{p}}^{2 \sqrt{\nu^{2}-\mathrm{k}^{2}}}\left[2 \sigma_{\mathrm{L}}\left(\mathrm{k}^{2}, \nu\right)+\sigma_{\mathrm{t}}^{+}\left(\mathrm{k}^{2}, \nu\right)\right]}\right. \\
& -\frac{\left(\mathrm{k}^{2}\right)^{2}}{8 \mathrm{E}_{\mathrm{e}} \mathrm{~m}_{\mathrm{p}}^{2 \sqrt{\nu^{2}-\mathrm{k}^{2}}}}\left[2 \sigma_{\mathrm{L}}^{\left.\left(\mathrm{k}^{2}, \nu\right)+\sigma_{\mathrm{t}}^{+}\left(\mathrm{k}^{2}, \nu\right)\right]-\frac{\mathrm{k}^{2}}{4 \mathrm{E}_{\mathrm{e}} \mathrm{~m}_{\mathrm{p}}^{2}} \sqrt{\nu^{2}-\mathrm{k}^{2}} \sigma_{\mathrm{t}}^{+}\left(\mathrm{k}^{2}, \nu\right)}\right. \\
& \left.-\frac{\mathrm{k}^{2}\left(2 \mathrm{E}_{\mathrm{e}}-\nu\right)}{4 \mathrm{~m}_{\mathrm{p}}^{2} \mathrm{E}} \sigma_{\mathrm{t}}^{-}\left(\mathrm{k}^{2}, \nu\right)\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_{\mathrm{L}} & =\text { W-nucleon cross section for longitudinal W's. } \\
\sigma_{\mathrm{t}}^{+}\left(\sigma_{\mathrm{t}}^{-}\right) & =\begin{array}{l}
\text { sum (difference) of the W-nucleon cross sections } \\
\text { for left and right hand polarized W's. }
\end{array}
\end{aligned}
$$

In order to proceed we must make assumptions concerning $\sigma_{t}^{+}$, $\sigma_{t}^{-}$and $\sigma_{L}$. In the spirit of this calculation we make the simple assumption $\sigma_{L}=0, \sigma_{t}^{-}=0$, and $\sigma_{t}^{+}=$constant independent of $k^{2}$ and $\nu$. The integration over the allowed region of $\mathrm{k}^{2}-\nu$ space can now easily be done. The results are given in Table I. Although we might hope to say that $\sigma_{t}^{+}$is a sizeable strong interaction cross section, recent cosmic ray experiments ${ }^{6}$ have already yielded rather small upper bounds. However, we shall see that the proposed electron experiments could lower these upper bounds considerably and provide a more sensitive quest for the W .

According to recent cosmic ray calculations there exists a sea level neutrino flux in the horizontal direction which is approximately ${ }^{7}$

$$
\eta\left(\mathrm{E}_{\nu}\right)=\frac{0.029}{\mathrm{E}_{\nu}^{3}} \mathrm{BeV}^{-1} \mathrm{~cm}^{-2} \mathrm{sec}^{-1} \text { ster }^{-1}
$$

These neutrinos can then interact with nuclei in the earth and create W's as in Figure 4. The muon which accompanies this reaction will then quickly lose its energy while traveling through the nearby rock and perhaps pass through a detector. The resultant muon flux is then approximately

$$
\frac{\mathrm{dN}}{\left.\mathrm{dAd} \overline{\Omega \mathrm{dt}} \cong \int_{0}^{\infty} \mathrm{dx} \int_{0}^{\infty} \mathrm{dE}_{\mathrm{f}} \int_{\mathrm{E}_{\nu}^{\min }}^{\infty} \mathrm{dE}_{\nu}\left(\frac{0.029}{\mathrm{E}_{\nu}^{3}}\right)\left(\mathrm{N}_{\mathrm{T}}\right)\left(\frac{1}{2} \frac{\mathrm{~d} \sigma}{\mathrm{dE}}\right)\right), ~\left(\frac{1}{\mu}\right)}
$$

where

$$
\begin{aligned}
\frac{\mathrm{dN}}{\mathrm{dAd} \Omega \mathrm{dt}} & =\text { number of muons per } \mathrm{cm}^{2}-\mathrm{ster}-\mathrm{sec} \text { at detector } \\
\mathrm{N}_{\mathrm{T}} & =\text { Avogadro's number } \\
\mathrm{E}_{\mu} & =\text { energy of muon at creation (BeV) } \\
\mathrm{E}_{\mathrm{f}} & =\text { energy of muon at detector (BeV) } \\
\mathrm{x} & =\text { distance (gm/cm }) \text { muon travels in rock } \\
\mathrm{k} & =\text { energy loss ( } \mathrm{BeV} / \mathrm{gm}-\mathrm{cm}^{-2} \text { ) of muon }\left(\mathrm{E}_{\mathrm{f}}=\mathrm{E}_{\mu}-\mathrm{kx}\right) \\
\mathrm{E}_{\nu}^{\min } & =\text { threshold neutrino energy for this process }
\end{aligned}
$$

Taking $\mathrm{d} \sigma / \mathrm{dE}_{\mu}$ from our earlier calculations, we can do these integrals and obtain the estimate

$$
\frac{\mathrm{dN}}{\mathrm{dAd} \Omega \mathrm{dt}} \approx \frac{\mathrm{~N}_{\mathrm{T}}(0.029)}{6 \mathrm{k}\left(\mathrm{~m}_{\mathrm{W}}+\frac{\mathrm{m}_{\mathrm{W}}^{2}}{2 \mathrm{~m}_{\mathrm{p}}}\right)} \sigma\left(\mathrm{E}_{\nu}=20, \mathrm{~m}_{\mathrm{W}}\right)
$$

Demanding that this flux be less than the experimental value

$$
\frac{d N}{d A d \Omega d t}<6.10^{-13} \text { per } \mathrm{cm}^{2}-\text {-ster-sec. }
$$

we obtain the bounds on $\sigma\left(\mathrm{E}_{\nu}=20, \mathrm{~m}_{\mathrm{W}}\right)$ listed in Table II.
We now proceed to find the desired muon spectrum for

$$
\mathrm{e}+\mathrm{p} \rightarrow \nu+\mathrm{W}+\text { "stuff" } \rightarrow \nu+\mu+\nu+\text { "stuff" }
$$

assuming that the W produced at the W -nucleon vertex emerges in the forward direction with an energy distribution essentially flat and extending from $\frac{1}{2} E_{e}$ to the kinematic extreme. These approximations, especially thefirst, are motivated by the detailed calculations ${ }^{4}$. Finally, the physical $W$ decays and we compute the angular distribution of the muons produced in the process $W \rightarrow \mu \nu$. This then gives us the differential cross section $d \sigma / d E_{\mu} d \cos \theta_{\mu}$ for Figure 4 (cf. Graph 1). In order to relate this to an actual experimental situation we consider an 0.3 r .1 . Be target and compute a yield (number muons/electron-ster-GeV/c), and compare with the SLAC background yields ${ }^{8}$ (cf. Graph 2). For a given $\mathrm{m}_{\mathrm{W}}$ and $\mathrm{E}_{\mu}$ simple kinematics cuts off the theoretical curves at various maximum muon angles as shown. Actually these curves will be smeared out since the $W$ will be produced with some transverse momentum. However, we see that if we look for energetic muons at large angles the experiment will be most sensitive to the existence of the W . In fact the $\mathrm{m}_{\mathrm{W}}=5$, $\mathrm{E}_{\mu}=16$ curve exceeds the expected background by several orders of magnitude. Also, since a yield of about $10^{-10} /$ electron-ster $-\mathrm{GeV} / \mathrm{c}$ corresponds to a counting rate of about one per second, the experimentalist should have no difficulty with absolute rates.

Up to this point we have made several simplifications which should be pointed out. First, we have not multiplied our cross sections by the branching ratio for $\mathrm{W} \rightarrow \mu \nu$. The rate for $\mathrm{W} \rightarrow \mathrm{e} \nu$ is essentially identical to
the $W \rightarrow \mu \nu$ rate, so we should at least divide our results by a factor of two. Finally, we have not taken into account the fact that the electron beam loses energy as it passes through the 0.3 r .1 . Be target. This effect reduces the number of very energetic W's produced which then reduces the number of muons produced at large angles for given $\mathrm{m}_{\mathrm{W}}$ and $\mathrm{E}_{\mu}$. However, since the target is thin this effect is not severe and should not amount to a reduction in muon intensity in excess of a few percent.

In conclusion, inelastic electron-nucleon scattering could easily and profitably be employed in the search of the W . In fact, we expect that the background $\mu$ flux should increase slowly with beam energy and maintain its fast exponential decline in scattering angle. However, the muon flux resulting from $W$ production at larger $\mathrm{m}_{\mathrm{W}}$ and beam energy could certainly increase in intensity and maintain its unique angular dependence without conflicting with present cosmic ray experiments. So, a much more decisive search for W's could be made once higher energy electron beams become available. Background muon intensities from $\pi$ and K decays might also be greatly reduced by placing lead absorber behind the target. The experiment's sensitivity to the existence of the W could also be improved by several orders of magnitude in such a way. This idea has recently been used in an experiment at $\mathrm{BNL}^{9}$ which looked for wide angle muons emerging from proton-nucleon collisions. The experiment we propose, however, is preferable to its Brookhaven counterpart because electron-nucleon collisions are simpler and better understood theoretically than nucleon-nucleon collisions. The major drawback of all these experiments is, however, than even if wide angle muons were found it would not unambiguously imply the existence of the W . The very discovery of wide angle muons would in itself, however, be very important.

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Fig. la

$\overline{1283 A 1}$
Fig. 1b

$\overline{1283 A 2}$

Fig. 2


Fig. 3

$\overline{1283 A 4}$

Fig. 4

$\overline{1283 A 5}$

Fig. 5

$\overline{1283 A 6}$

Fig. 6


Fig. 7

$$
\begin{array}{cc}
\mathrm{m}_{\mathrm{W}}\left(\mathrm{BeV} / \mathrm{c}^{2}\right) & \sigma\left(\mathrm{cm}^{2}\right) \\
3 & \left(4.0 \cdot 10^{-7}\right) \sigma_{\mathrm{t}}^{+} \\
4 & \left(5.4 \cdot 10^{-8}\right) \sigma_{\mathrm{t}}^{+} \\
5 & \left(1.3 \cdot 10^{-9}\right) \sigma_{\mathrm{t}}^{+}
\end{array}
$$

Table I

| $\mathrm{m}_{\mathrm{W}}\left(\mathrm{BeV} / \mathrm{c}^{2}\right)$ | $\mathrm{E}_{\mathrm{e}}(\mathrm{BeV})$ | $\sigma($ barns $)$ |
| :---: | :---: | :---: |
| 3 | 20 | $<3.75 \cdot 10^{-12}$ |
| 4 | $\sigma_{\mathrm{t}}{ }^{+}($barns $)$ |  |
| 4 | $<6.0 \cdot 10^{-12}$ | $<9.5 \cdot 10^{-6}$ |
| 5 | 20 | $<8.75 \cdot 10^{-12}$ |

Table II


[^0]:    $\dagger$ Work supported by the U. S. Atomic Energy Commission.

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