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ANGULAR CORRELATIONS IN ELECTROPRODUCTION OF RHO MESONS*

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ABSTRACT

Electroproduction of rho mesons followed by decay into two pions is studied. Angular correlations between the pions and electrons are explicitly evaluated and shown to allow a separation of the longitudinal and transverse photon cross sections.

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I. INTRODUCTION

Various models of the electromagnetic interactions of hadrons make predictions about the interactions of scalar (or longitudinal) photons with nucleons. However, very little information exists to test this aspect of the models. An experiment designed to detect the electroproduction of rho mesons and their decay into two pions would greatly increase the information on this topic.[1] The reaction can be treated as a triple scattering experiment in which the electron scatters producing a polarized photon, which collides with a target producing a rho meson. The polarization of the rho meson is analyzed by its decay distribution into a two-pion final state. The angular correlations of the electron and pions make it possible to separate the longitudinal and transverse cross sections as well as interference terms between the various amplitudes.

In addition to being of intrinsic interest, this particular process can be thought of as "virtual photoproduction of the rho mesons" in analogy to the reaction $\gamma p \rightarrow \rho p$. Models using the idea of "vector meson dominance" predict large cross sections for the production of rho meson by longitudinal photons. The reaction discussed here contains all the relevant information needed to determine this longitudinal cross section.

Sections II and III reiterate the existing formalism[2] for such calculations and put it into a useful form for calculations of angular correlations. In Section IV, angular distributions are calculated and discussed, while Section V shows how the process relates to vector dominance. Appendix A discusses heavy targets, Appendix B illustrates how the longitudinal and transverse amplitudes can be separated. Appendix C gives the angular correlations for rho photoproduction ($q^2 = 0$) with linearly polarized photons.

II. AMPLITUDE

The diagram for electroproduction of rho's is shown in Fig. 1. Figure 2 shows the coordinate systems chosen for the calculations.

Single photon exchange is assumed to be the only contribution to the amplitude, which can be written,[2]

$$T(ep \to ep\rho) = \langle k|j_\mu|k'\rangle \frac{e^\lambda_\mu e^\lambda_\nu}{q^2} \langle p|J_\nu|p'\sigma\rangle \tag{1}$$

where

j_μ = electron current

J_ν = nucleon current

λ = index to specify the photon spin state = 1, 2, 3, 4

σ = helicity state of the rho = $\pm$ 1, 0

and electron and target spins will be summed and averaged over in obtaining cross sections and, therefore, are not explicitly shown.

Different notations will be used in various places to make the formulae simpler in appearance.

$$\langle k|j_\mu|k'\rangle \; e^\lambda_\mu = j \cdot e^\lambda = j^\lambda$$

$$\langle p|J_\mu|p'\sigma\rangle \, e^\lambda_\mu = J \cdot e^\lambda = J^\lambda$$

Another way to write the amplitude is then

$$T = \frac{1}{q^2} \, j^\lambda J^\lambda .$$

Finally, the state normalization, coupling constant and metric are

$$\langle A|A\rangle = 2E_A$$

$$e^2/4\pi = \alpha = \frac{1}{137}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & +1 \end{pmatrix}$$

so that $q^2 = q_4^2 - |\vec{q}|^2 \leq 0$ for this paper. $q_4 = q_0$ is the time component of q.

The virtual photon states are quantized along the virtual photon 3-momentum $\vec{q}$ so that the 3-axis is parallel to $\vec{q}$ and the 1-axis is in the plane determined by $\vec{p}$ and $\vec{q}$.

Current conservation can be used to combine the effects of the scalar and longitudinal photons into one term. With the coordinate system used here and e_μ^λ transverse for $\lambda = 1, 2$ current conservation makes the following restriction

$$q_\mu j_\mu = 0 = -q_3 j_3 + q_4 j_4 \quad .$$

The same restriction is applied to the nuclear current

$$q_\mu J_\mu = 0 = -q_3 J_3 + q_4 J_4 \quad .$$

Using these conditions the amplitude can be written,[3]

$$T = \frac{-1}{q^2} \sum_{\alpha=1,2} j^\alpha J^\alpha - \frac{1}{q_4^2} j^3 J^3 \quad .$$

The last term contains the sum of the scalar and longitudinal contributions to the amplitude and the minus sign comes from the metric.

In order to write T in a symmetric form, orthogonal but not orthonormal basis vectors are used. These vectors are eigenstates of the Z-component of the photon angular momentum and differ from the basis vectors used in Eq. (1).

$$\epsilon_\mu^{1,2} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mp 1 \\ -i \\ 0 \\ 0 \end{pmatrix} \qquad \text{helicity } 1, -1$$

$$\epsilon_\mu^{3} = \frac{\sqrt{-q^2}}{q_4} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{helicity } 0$$

The amplitude can now be written as the sum of three terms. The first two ($\lambda = 1, 2$) contain the amplitudes for transverse photons and the third ($\lambda = 3$)

contains the combined contributions of the longitudinal and scalar currents.

$$T = \frac{-1}{q^2} \sum_{\lambda=1}^{3} j \cdot \epsilon^\lambda J \cdot \epsilon^\lambda, \quad \text{or}$$

$$T = \frac{-1}{q^2} \sum_{\lambda=1}^{3} (j \cdot \epsilon)^\lambda T_\sigma^\lambda \tag{2}$$

where

$$T_\sigma^\lambda = \langle p | J_\mu | p' \sigma \rangle \, \epsilon_\mu^\lambda = J \cdot \epsilon^\lambda \ .$$

The amplitude (2) is now in a form similar to a double scattering amplitude; namely a virtual photon is produced and then scatters inelastically with a helicity amplitude[4] T_σ^λ .

If the amplitude for subsequent decay of the rho into an angle θ, ϕ in its rest system is $D_\sigma(\theta, \phi)$, the chain of reactions has an amplitude:

$$T(\vartheta, \phi; ep \rightarrow ep\rho^0, \rho^0 \rightarrow \pi^+\pi^-) = \frac{-1}{q^2} j \cdot \epsilon^\lambda T_\sigma^\lambda D_\sigma(\theta, \phi) \ . \tag{3}$$

III. CROSS SECTION

The cross section for production and decay is proportional to the amplitudes (3) squared[5]:

$$\overline{\sum} |T|^2 = \frac{1}{q^4} \sum_{\substack{\lambda,\lambda'=1 \\ \sigma,\sigma'}}^{3} \widetilde{B}^{\lambda\lambda'} \, \widetilde{\rho}_{\sigma\sigma'}^{\lambda\lambda'} A_{\sigma\sigma'} \tag{4}$$

where

$\tilde{B}^{\lambda\lambda'} = (j \cdot \epsilon^{\lambda})(j \cdot \epsilon^{\lambda'})$ is the "modified photon density matrix" which is

$\lambda, \lambda' = 1, 3$ unchanged by a Lorentz transformation along q.

This is due to the definition of ϵ^{λ}.

$\tilde{\rho}^{\lambda\lambda'}_{\sigma\sigma'} = T^{\lambda}_{\sigma} T^{\lambda' *}_{\sigma'}$ production matrix

$A_{\sigma\sigma'} = D_{\sigma} D^{*}_{\sigma'}$ decay matrix

and

$\overline{\sum}$ = average over initial and sum over the final spin states of the electron and target.

If the rho decay distribution is averaged over θ and ϕ,

$$\overline{\sum} |T|^2 = \frac{1}{q^4} \overline{\sum_{\substack{\lambda,\lambda' \\ \sigma}}} \tilde{B}^{\lambda\lambda'} \tilde{\rho}^{\lambda\lambda'}_{\sigma\sigma}$$

The production cross section can be written:

$$d_{ep} = \frac{1}{2^5 (2\pi)^5} \frac{1}{\sqrt{(k \cdot p)^2 - m^2 M^2}} \overline{\sum} |T|^2 \frac{d^3 p'}{p'_0} \frac{d^3 \rho}{\rho_0} \frac{d^3 k'}{k'_0} \delta^4 \left(\sum p_i - \sum p_f\right) \tag{5}$$

where p_i and p_f are the 4-momenta of the initial and final particles.

The electron kinematics can be evaluated in the laboratory and the virtual photon scattering in the center of momentum system (C.M.S.) of the incident γ-ray and proton.

$$\begin{aligned} d\sigma_{ep} &= \frac{1}{32(2\pi)^5} \left(\frac{1}{Mk} \frac{d^3 k'}{k'_0}\right)_{lab} \left(\overline{\sum} |T|^2 \frac{d^3 p'}{p'_0} \frac{d^3 \rho}{\rho_0}\right)_{c.m.s.} \delta^4 (\Sigma p_i - \Sigma p_f) \\ &= \frac{1}{32(2\pi)^5} \left(\frac{k'}{k} dk'_0 \, d\Omega_{k'}\right) \left(\frac{1}{M} \overline{\sum} |T|^2 \frac{p'^*}{W} d\Omega^*_{p'}\right) \end{aligned} \tag{6}$$

where $W = (p+q)^2$ = invariant mass of γ-p system.

Following various authors[2] an analogy can be made with the cross section for real photons ($|\vec{q}| = q_4$)

$$d\sigma_\gamma^{real}(q^2{=}0) = \frac{1}{2^4(2\pi)^2} \frac{p'^*}{Mq_4W} \overline{\sum}|N|^2 d\Omega^*_{p'},$$

where $\overline{\sum}|N|^2$ = transverse matrix elements squared of the nuclear current and the (*) denotes the center of momentum. Similarly, for virtual photons

$$d\sigma_\gamma(q^2{\neq}0) = \frac{1}{2^4(2\pi)^2} \frac{p'^*}{Mq_4W} d\Omega^*_{p'} \frac{1}{\tilde{B}_T} \overline{\sum_\sigma} \tilde{B}^{\lambda\lambda'} \tilde{\rho}^{\lambda\lambda'}_{\sigma\sigma} \tag{7}$$

where $\tilde{B}_T = \tilde{B}^{11} + \tilde{B}^{-1-1} \propto$ transverse photon flux.

Combining (6) and (7):

$$d\sigma_{ep} = \frac{1}{2(2\pi)^3} \left(\frac{k'}{k} dk'_0 \, d\Omega_0\right) \frac{K}{q^4} \tilde{B}_T d\sigma_\gamma \tag{8}$$

where a somewhat arbitrary choice[6] is made in defining $K = q_4 + \frac{q^2}{2M}$. This is the lab energy for a virtual photon reaction which has the same invariant mass as a reaction with a real photon would have when the same final state is produced.

Kinematical terms in (8) are combined to obtain:

$$\frac{d\sigma_{ep}}{d\Omega_{k'} \, dk'_0} = \tilde{\Gamma}_T d\sigma_\gamma \tag{9}$$

so that after averaging over all angles the Hand[2] expression is obtained

$$\left\langle \frac{d^2\sigma}{d\Omega_{k'} \, dk'_0} \right\rangle_{av} = \tilde{\Gamma}_T \int d\sigma_\gamma = \tilde{\Gamma}_T(\sigma_T + \epsilon\sigma_L)$$

where $\tilde{\Gamma}_{T,L}$ = transverse, longitudinal photon "flux"

$$\epsilon = \tilde{\Gamma}_L / \tilde{\Gamma}_T$$

$$\tilde{\Gamma}_T = \frac{1}{2(2\pi)^3} \frac{k'}{k} \frac{K}{q^4} \tilde{B}_T \ .$$

To get an expression for $\tilde{\Gamma}_T$ the photon density matrix is evaluated by taking the electron spin sums:

$$\tilde{B}^{\lambda\lambda'} = 2e^2 \left(g_{\mu\nu}\, q^2/2 + k_\mu k'_\nu + k_\nu k'_\mu \right) \epsilon^\lambda_\mu \epsilon^{\lambda'*}_\nu$$

$$1/2\tilde{B}_T = \tilde{B}^{11} = \tilde{B}^{-1-1} = -q^2 e^2 \left(1 + \frac{2m^2}{q^2} + \frac{2k_0 k'_0 + q^2/2}{|\vec{q}|^2} \right)$$

$$\tilde{B}_L = \tilde{B}^{00} = -2e^2 \frac{q^2}{|\vec{q}|^2} \left(2k_0 k'_0 + q^2/2 \right)$$

$$\tilde{B}^{1-1} = -\left\{ 2e^2 (q^2/2) + \tilde{B}_{11} \right\} e^{2i\phi_e} = -2e^2 k_T^2\, e^{2i\phi_e}$$

where

$$k_T^2 = k_1^2 + k_2^2 = k'^2_1 + k'^2_2$$

and ϕ_e = angle between electron plane and production plane

$$\tilde{B}^{10} = 2e^2 \left(\frac{k_T}{\sqrt{2}} \frac{\sqrt{-q^2}}{|\vec{q}|} (k_0 + k'_0) \right) e^{i\phi_e} \;.$$

Using $\tilde{B}_{T,L}$ the "photon flux" becomes:

$$\tilde{\Gamma}_T = \frac{-\alpha}{2\pi^2} \frac{k'}{k} \left(1 + \frac{2m^2}{q^2} + \frac{2k_0 k'_0 + q^2/2}{|\vec{q}|^2} \right) \frac{K}{q^2} \tag{10}$$

$$\epsilon = \tilde{\Gamma}_L / \tilde{\Gamma}_T = \tilde{B}^{00}/2\tilde{B}_{11} = \left(\frac{2k_0 k'_0 + q^2/2}{|\vec{q}|^2} \right) \Big/ \left(1 + \frac{2m^2}{q^2} + \frac{2k_0 k'_0 + q^2/2}{|\vec{q}|^2} \right)$$

Normalizing $\tilde{B}^{\lambda\lambda'}$ and $\tilde{\rho}^{\lambda\lambda'}_{\sigma\sigma'}$ so that $d\sigma_\gamma$ for rho production can be written as an angular factor times the usual cross section factor gives:

$$d\sigma_\gamma = \frac{d\sigma_\gamma^T}{dt} \sum_{\substack{\lambda\lambda' \\ \sigma}} B^{\lambda\lambda'} \rho^{\lambda\lambda'}_{\sigma\sigma}\, dt \frac{d\phi_e}{2\pi} \tag{11}$$

where

$$B^{\lambda\lambda'} = \widetilde{B}^{\lambda\lambda'}/\widetilde{B}_T$$

$$\rho^{\lambda\lambda'}_{\sigma\sigma'} = \widetilde{\rho}^{\lambda\lambda'}_{\sigma\sigma'}/\widetilde{\rho}_T$$

$$\widetilde{\rho}_T = \widetilde{\rho}^{11}_{00} + \widetilde{\rho}^{11}_{11} + \widetilde{\rho}^{11}_{-1-1}$$

= transverse part of production matrix

and

$$\frac{d\sigma^T_\gamma}{dt} = \text{cross section for the transverse virtual photons.}$$

Averaging (11) over all angular variables gives the usual Hand-type formula:

$$\frac{d\sigma_\gamma}{dt} = \frac{d\sigma^T_\gamma}{dt}\left(1 + \epsilon\,\frac{\dfrac{d\sigma^L_\gamma}{dt}}{\dfrac{d\sigma^T_\gamma}{dt}}\right) = \frac{d\sigma^T_\gamma}{dt} + \epsilon\,\frac{d\sigma^L_\gamma}{dt}$$

Using (11), (9), and the rho decay-matrix $A_{\sigma\sigma'}$ the formula for rho production followed by decay becomes:

$$\frac{d\sigma_{ep}}{d\Omega_{k'}\,dk'_0\,dt} = \widetilde{\Gamma}_T\,\frac{d\sigma^T}{dt}\left(\sum_{\substack{\sigma\lambda\\ \sigma'\lambda'}=1}^{3} B^{\lambda\lambda'}\rho^{\lambda\lambda'}_{\sigma\sigma'}A_{\sigma\sigma'}\right)\frac{d\phi_e}{2\pi}\,\frac{d\Omega_\pi}{4\pi} \tag{12}$$

IV. ANGULAR DISTRIBUTION — RHO PRODUCTION AND DECAY

The remainder of this paper will focus on the angular correlations contained in the bracketed part of Eq. (12). This term is defined to be:

$$W\left(k_0, K, q^2, t, \phi_e, \phi, \theta\right) = \sum_{\substack{\lambda\lambda'\\ \sigma\sigma'}} B^{\lambda\lambda'}\left(k_0, K, q^2, \phi_e\right)\rho^{\lambda\lambda'}_{\sigma\sigma'}(K, q^2, t)A_{\sigma\sigma'}(\theta, \phi) \tag{13}$$

where

$$\int W \frac{d\phi_e}{2\pi} \frac{d\Omega_\pi}{4\pi} = 1 + \epsilon \left(\frac{d\sigma_\gamma^L}{dt} \Big/ \frac{d\sigma_\gamma^T}{dt} \right)$$

In general, only the variables of interest will be explicitly written for W, B, ρ, and A.

The summation over indices in (13) has 81 terms which can be reduced to 25 by the following relations resulting from parity conservation and hermiticity.[5]

Parity	Hermiticity
$\rho^{-\lambda-\lambda'}_{-\sigma-\sigma'} = (-1)^{\lambda-\lambda'+\sigma-\sigma'} \rho^{\lambda\lambda'}_{\sigma\sigma'}$	$\rho^{\lambda\lambda'}_{\sigma\sigma'} = \rho^{\lambda'\lambda *}_{\sigma'\sigma}$
$B^{-\lambda-\lambda'} = (-1)^{\lambda-\lambda'} B^{\lambda\lambda' *}$	$B^{\lambda\lambda'} = B^{\lambda'\lambda *}$ (14)
$A_{-\sigma-\sigma'} = (-1)^{\sigma-\sigma'} A^*_{\sigma\sigma'}$	$A_{\sigma\sigma'} = A^*_{\sigma'\sigma}$

so

$$B^{10} = -B^{10*} = -B^{0-1} = B^{01*}, \quad \text{etc.}$$

The symbol (*) means complex conjugate.

If the target has spin zero, an additional restriction follows from parity:

$$\rho^{\lambda-\lambda'}_{\sigma-\sigma'} = (-1)^{\lambda'-\sigma'} \rho^{\lambda\lambda'}_{\sigma\sigma'} \quad \text{spin 0 target.} \tag{15}$$

If the target were a heavy nucleus with spin $\neq 0$, the above restriction might be true. However, it is not necessary to make this approximation, since an angular correlation measurement between the electron and pions will give information on this point.

The sum over indices is shown in Eq. (16). Formula (15) has not been used.

$$
\begin{array}{lll}
W = & & \\
\rho^{00}_{00}B^{00}A_{00} & +2B^{00}\left(\rho^{00}_{11}A_{11}+\rho^{00}_{1-1}\mathrm{re}A_{1-1}\right) & +4\mathrm{re}\,\rho^{00}_{10}B^{00}\mathrm{re}A_{10} \\
+2A_{00}\left(\rho^{11}_{00}B^{11}+\rho^{1-1}_{00}\mathrm{re}B^{1-1}\right) & +2\left(\rho^{11}_{11}B^{11}A_{11}+\rho^{1-1}_{1-1}\mathrm{re}\left(B^{1-1}A_{1-1}\right)\right) & +4\left(\mathrm{re}\,\rho^{11}_{10}B^{11}\mathrm{re}A_{10}+\mathrm{re}\,\rho^{1-1}_{10}\mathrm{re}\left(A_{10}B^{1-1}\right)\right) \\
 & +2\left(\rho^{11}_{-1-1}B^{11}A_{11}+\rho^{1-1}_{-11}\mathrm{re}\left(B^{1-1}A^{*}_{1-1}\right)\right) & -4\left(\mathrm{re}\,\rho^{11}_{-10}B^{11}\mathrm{re}A_{10}+\mathrm{re}\,\rho^{1-1}_{-10}\mathrm{re}\left(A^{*}_{10}B^{1-1}\right)\right) \\
 & +4\left(\mathrm{re}\,\rho^{11}_{1-1}B^{11}\mathrm{re}A_{1-1}+\mathrm{re}\,\rho^{1-1}_{11}A_{11}\mathrm{re}B^{1-1}\right) & \\
+4\mathrm{re}\,\rho^{10}_{00}\mathrm{re}B^{10}A_{00} & +4\left(\mathrm{re}\rho^{10}_{11}A_{11}\mathrm{re}B^{10}+\mathrm{re}\rho^{10}_{1-1}\mathrm{re}\left(B^{10}A_{1-1}\right)\right) & +4\mathrm{re}\,\rho^{10}_{10}\mathrm{re}\left(B^{10}A_{10}\right) \\
 & -4\left(\mathrm{re}\rho^{-10}_{11}A_{11}\mathrm{re}B^{10}+\mathrm{re}\rho^{-10}_{1-1}\mathrm{re}\left(A^{*}_{1-1}B^{10}\right)\right) & -4\mathrm{re}\,\rho^{10}_{-10}\mathrm{re}\left(B^{10*}A_{10}\right) \\
 & & +4\left(\mathrm{re}\rho^{10}_{01}\mathrm{re}\left(B^{10*}A_{10}\right)-\mathrm{re}\rho^{10}_{0-1}\mathrm{re}\left(A_{10}B^{10}\right)\right)
\end{array}
\qquad (16)
$$

Note that all combinations are of the form $\mathrm{re}\rho \times \mathrm{re}(AB)$. If re( ) is omitted, the quantity ( ) is already real due to the parity and hermiticity requirements.

The angular dependence of W is of the form:

$$W = B^{\lambda\lambda'}(\phi_e)\,\rho^{\lambda\lambda'}_{\sigma\sigma'}\,A_{\sigma\sigma'}(\theta,\phi)$$

Averages over appropriate combinations of θ, ϕ, ϕ_e isolate the components of ρ. The following expressions enable the explicit angular dependence of W to be shown.

Photon density matrix $B^{\lambda\lambda'}$

$$\begin{aligned}
B^{11} &= \Gamma_{11} = \widetilde{B}^{11}/\widetilde{B}_T = 1/2 \\
B_T &= B^{11} + B^{-1-1} = 1 \\
B^{00} &= \Gamma_{00} = \widetilde{B}^{00}/\widetilde{B}_T = \widetilde{\Gamma}_L/\widetilde{\Gamma}_T = \epsilon \\
B_{1-1} &= -\Gamma_{1-1}\, e^{2i\phi_e} = \widetilde{B}^{1-1}/\widetilde{B}_T \\
B_{10} &= -\Gamma_{10}\, e^{i\phi_e} = \widetilde{B}^{10}/\widetilde{B}_T
\end{aligned} \tag{17}$$

Rho decay matrix $A_{\sigma\sigma'} = \frac{4\pi}{3} Y^1_\sigma(\theta,\phi)\, Y^{1*}_{\sigma'}(\theta,\phi)$

$$\begin{aligned}
1/3\, A_{00} &= \cos^2\theta \\
1/3\, A_{11} &= 1/2 \sin^2\theta \\
1/3\, A_{1-1} &= -1/2 \sin^2\theta\, e^{2i\phi} \\
1/3\, A_{10} &= \frac{-1}{\sqrt{2}} \sin\theta\cos\theta\, e^{i\phi}
\end{aligned} \tag{18}$$

A rotation of the photon density matrix (or decay matrix) can be expressed:

$$\begin{aligned}
A_{\sigma\sigma'}(\phi) &= A_{\sigma\sigma'}(0)\, e^{i(\sigma-\sigma')\phi} \\
B^{\lambda\lambda'}(\phi_e) &= B^{\lambda\lambda'}(0)\, e^{i(\lambda-\lambda')\phi_e}
\end{aligned} \tag{19}$$

Example 1 — Average over ϕ and ϕ_e

$$W(\theta) = \iint \frac{d\phi_e}{2\pi}\,\frac{d\phi}{2\pi}\, e^{i(\lambda-\lambda')\phi_e}\, B^{\lambda\lambda'}(0)\, \rho^{\lambda\lambda'}_{\sigma\sigma'}\, A_{\sigma\sigma'}(0)\, e^{i(\sigma-\sigma')\phi}$$

$$= B^{\lambda\lambda}\, \rho^{\lambda\lambda}_{\sigma\sigma} A_{\sigma\sigma}$$

$$1/3W(\theta) = \cos^2\theta \left(\rho^{00}_{00}\,\Gamma_L + \Gamma_T\,\rho^{11}_{00}\right) + \sin^2\theta \left(\rho^{00}_{11}\,\Gamma_L + \frac{1}{2}\,\rho^{11}_{11}\,\Gamma_T + \frac{1}{2}\,\rho^{-1-1}_{11}\,\Gamma_T\right)$$

This exhibits the well-known fact that longitudinal rho's (lower index = 0) decay with a $\cos^2\theta$ distribution and transverse rho's decay with a $\sin^2\theta$ distribution. If ρ^{00}_{00} and ρ^{11}_{11} are the only large terms, the $\cos^2\theta$ and $\sin^2\theta$ terms directly exhibit the longitudinal and transverse photon cross sections.

Averaging $W(\theta)$ over θ gives a function independent of the angular variables.

$$W(t) = \Gamma_L\left(\rho^{00}_{00} + \rho^{00}_{11} + \rho^{00}_{-1-1}\right) + \Gamma_T\left(\rho^{11}_{00} + \rho^{11}_{11} + \rho^{11}_{-1-1}\right)$$

$$= 1 + \epsilon\left(d\sigma^L_\gamma/dt\right)/\left(d\sigma^T_\gamma/dt\right)$$

and

$$\frac{d\sigma_{ep}}{d\Omega_{k'}dk'_0 dt} = \tilde{\Gamma}_T\,\frac{d\sigma^T_\gamma}{dt}\,W(t) = \tilde{\Gamma}_T\left(\frac{d\sigma^T_\gamma}{dt} + \epsilon\,\frac{d\sigma^L_\gamma}{dt}\right)$$

as was previously shown.

It is helpful to note that an average over ϕ_e causes all terms with $\lambda\neq\lambda'$ to drop out, while averaging over ϕ causes all terms with $\sigma\neq\sigma'$ to vanish.

Some reasonable approximations can be made which reduce the number of terms present in W (see (16)). They are the following:

1. Amplitudes involving helicity flip of two are negligible; i.e., $T^1_{-1} \sim 0$. This is valid for small t if the target spin is $\leq 1/2$. This seems to hold for real photoproduction.[7]

2. Amplitudes for helicity flip one are small. Again, this seems to be true for photoproduction[7] when t is small. In this case, the products of amplitudes like ρ_{00}^{11} are ~ 0, and terms like ρ_{01}^{11} are assumed non-negligible.

3. The same as (2), except for dropping <u>all</u> products of amplitudes with helicity flip one; i.e., ρ_{01}^{11} and ρ_{00}^{11} are both dropped.

4. Limit of $q^2 \rightarrow 0$. Amplitudes with an upper index of zero should go to zero as they have internal factor of $\sqrt{|q^2|/q_4^2}$. The terms in $\rho_{\sigma\sigma'}^{\lambda\lambda'}$ have the properties:

$$\lim_{q^2 \rightarrow 0} \begin{Bmatrix} \rho_{11}^{01} \\ \rho_{11}^{00} \end{Bmatrix} \longrightarrow 0 \text{ as } \begin{Bmatrix} \sqrt{|q^2|} \\ |q^2| \end{Bmatrix} \text{ etc.}$$

<u>Example 2</u> — Real photon scattering ($q^2 \rightarrow 0$), see Appendix C for complete formula.

For $q^2 \rightarrow 0$, $\Gamma_{1-1} = \Gamma_{11}$. Using this and restrictions 1, 2, 3 two terms remain:

$$W = 2\left[\rho_{11}^{11}\left(B^{11}A_{11}\right) + \text{re}\left(B^{1-1}A_{1-1}\right)\rho_{1-1}^{1-1}\right]$$

$$W = 3\,\Gamma_{11}\sin^2\theta\left(\rho_{11}^{11} + \rho_{1-1}^{1-1}\cos 2\Psi\right)$$

where $\Psi = \phi + \phi_e$ = angle between the pion plane and the electron plane. Photoproduction of rho's from hydrogen indicates that $\rho_{1-1}^{1-1} \sim \rho_{11}^{11} \sim 1$ which means that rho decays parallel to the polarization of the photon, a property of diffraction mechanisms.[8] Note for a spin zero target $\rho_{1-1}^{1-1} \equiv \rho_{11}^{11}$.

The following angular averages were computed using assumption 1, $\left(T_{-1}^{1} = T_{1}^{-1} = 0\right)$. The missing terms can be easily calculated by inspecting Eq. (16). The expressions below can be further simplified by applying assumptions 2, 3, and 4. Data analysis from experiments can indicate where this is reasonable.

A Average over the electron plane ϕ_e

$$1/3\ W(\theta, \phi) = \cos^2\theta \left(\Gamma_{00}\,\rho^{00}_{00} + 2\Gamma_{11}\,\rho^{11}_{00}\right)$$

$$+ \sin^2\theta \left(\Gamma_{11}\,\rho^{11}_{11} + \Gamma_{00}\,\rho^{00}_{11} - \rho^{00}_{1-1}\,\Gamma_{00}\cos 2\phi\right)$$

$$- \sqrt{2}\sin 2\theta\cos\phi \left(\Gamma_{00}\,\mathrm{re}\,\rho^{00}_{10} + \Gamma_{11}\,\mathrm{re}\,\rho^{11}_{10}\right)$$

B Average over the pion plane ϕ

$$1/3\ W(\theta, \phi_e) = \cos^2\theta \left[\Gamma_{00}\rho^{00}_{00} + 2\,\Gamma_{11}\rho^{11}_{00} - 2\,\Gamma_{1-1}\rho^{1-1}_{00}\cos 2\phi_e - 4\,\Gamma_{10}\,\mathrm{re}\,\rho^{10}_{00}\cos\phi_e\right]$$

$$+ \sin^2\theta \left[\Gamma_{00}\rho^{00}_{11} + \Gamma_{11}\rho^{11}_{11} - 2\Gamma_{10}\cos\phi_e\,\mathrm{re}\,\rho^{10}_{11}\right]$$

C Average over $\phi - \phi_e$.

$$1/3\ W(\theta, \Psi) = \cos^2\theta \left[2\Gamma_{11}\rho^{11}_{00} + \Gamma_{00}\,\rho^{00}_{00}\right]$$

$$+ \sin^2\theta \left[\Gamma_{00}\,\rho^{00}_{11} + \Gamma_{11}\rho^{11}_{11} + \Gamma_{1-1}\rho^{1-1}_{1-1}\cos 2\Psi\right]$$

$$+ \sqrt{2}\,\Gamma_{10}\sin 2\theta\cos\Psi \left[\mathrm{re}\,\rho^{10}_{10} - \mathrm{re}\,\rho^{10}_{0-1}\right]$$

D Average over θ and $\phi - \phi_e$

$$1/3\ W(\Psi) = \Gamma_{00}\left(2\,\rho^{00}_{11} + \rho^{00}_{00}\right) + 2\Gamma_{11}\rho^{11}_{00}$$

$$+ 2\left(\Gamma_{11}\rho^{11}_{11} + \Gamma_{1-1}\rho^{1-1}_{1-1}\cos 2\Psi\right)$$

Other density matrix elements can be separated by averaging over different combinations of angles; for example, $2\phi_e \pm \phi$. These expressions are easily obtained from (16).

The analogy to photoproduction leads one to suspect that all helicity flip amplitudes are small for small t. If so, the elements ρ^{00}_{00}, ρ^{11}_{11}, ρ^{1-1}_{1-1}, and re ρ^{10}_{10} will dominate the decay distributions. ρ^{00}_{00} and ρ^{11}_{11} are then easily separated by the distribution of example 1, $1/3\, W(\theta) = 1/2\, \Gamma_T\, \rho^{11}_{11} \sin^2\theta + \Gamma_L\, \rho^{00}_{00} \cos^2\theta$, while ρ^{1-1}_{1-1} and ρ^{10}_{10} are determined from expressions D and C, respectively. The term in expression C, $1/3\, W(\theta,\Psi) = \text{terms} + \sqrt{2}\, \Gamma_{10} \sin 2\theta \cos\Psi$ re ρ^{10}_{10}, is especially important if T^0_0 is smaller than T^1_1. In this case, ρ^{00}_{00} would be small, and the interference term would be the only information available for longitudinal photons. Of course, T^0_0 could be large with a phase angle $\delta_{00} \simeq \pi/2$ with respect to T^1_1 resulting in:

$$\text{re } \rho^{10}_{10} \approx \left|T^1_1\right| \left|T^0_0\right| \cos\delta_{00} \approx 0.$$

Thus, in the worst case one obtains no information on longitudinal photons, while in the best case the magnitudes of the phase angle and amplitude are determined for T^0_0. Appendix B shows a graphical determination of the phase and amplitude of T^0_0 for a simple case.

V. VECTOR DOMINANCE

There are numerous variations on this idea. In a paper by J. J. Sakurai[9] the assumption that $\sum_A \left|\langle A \middle| J^{(\rho)} \cdot \rho^{T,L} \middle| p\rangle\right|^2 \delta(\sum p_i - \sum p_f)$ varied little with q^2 was used with the vector dominance relation

$$\langle A | J^{em}_\mu | p\rangle = \left(m^2_\rho / f_\rho\right) \left(m^2_\rho - q^2\right)^{-1} \langle A | J^{(\rho)}_\mu | p\rangle$$

to obtain the following relations for total cross sections for virtual transverse and longitudinal photons.

$$\sigma^{T}(q^2) = \frac{e^2}{f_\rho^2} \left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2 \sigma^{T}_{\rho p}\Big|_{\rm shell}$$

$$\sigma^{L}(q^2) = \frac{e^2}{f_\rho^2} \left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2 \left(-q^2/m_\rho^2\right) \xi(K)\sigma^{T}_{\rho p}\Big|_{\rm shell}$$

and $|_{\rm shell}$ means "evaluated on the mass shell,"

$J_\mu^{(\rho)}$ is the rho current source,

$J_\mu^{\rm em}$ is the photon current source,

$\xi(K) = \sigma^{L}_{\rho p}(K)/\sigma^{T}_{\rho p}(K)$,

f_ρ is the γ-ρ coupling constant,

A is any final state.

The striking property of this result is the very large longitudinal cross section for $|q^2| > m_\rho^2$.

One can use these relations and naively apply the optical theorem to get the q^2 dependence of the imaginary parts of the nonspin flip amplitudes for electroproduction of rhos. Using dispersion relations in photon energy one can show that the real parts of the amplitude will have the same q^2 dependence as the imaginary parts. Thus the amplitude has the q^2 dependence shown below.

$$\langle p|J^{\rm em}\cdot\epsilon^{T}|p\rho\rangle\Big|_{\rm no\ flip} = \frac{m_\rho^2/f_\rho}{m_\rho^2 - q^2}\,\langle p|J^{(\rho)}\cdot\rho^{T}|p\rho\rangle\Big|_{\rm shell}$$

$$\langle p|J^{\rm em}\cdot\epsilon^{L}|p\rho\rangle\Big|_{\rm no\ flip} = \sqrt{-q^2/q_0^2}\,\left(\frac{m_\rho^2/f_\rho}{m_\rho^2 - q^2}\right)\left(\frac{q_0}{m_\rho}\right)\langle p|J^{(\rho)}\cdot\rho^{L}|p\rho\rangle\Big|_{\rm shell}$$

where $\rho^L = \frac{1}{m_\rho}\left(E_\rho \hat{p}_\rho, |p|\right)$ is the longitudinal polarization vector and ρ^T = transverse polarization vector. The matrix elements on the left-hand side are those used earlier to form the production density matrix $\rho^{\lambda\lambda'}_{\sigma\sigma'}$. These two relations result in a prediction for the q^2 dependence of the differential cross sections for virtual photoproduction of rhos. In doing this, it has to be assumed that either spin flip amplitudes are very small, or that the spin flip amplitudes have the same q^2 dependence as the nonflip amplitudes.

The result for the differential cross sections for the virtual photoproduction of rho mesons is

$$\frac{d\sigma^T}{dt}(q^2) = \left.\frac{d\sigma_\gamma}{dt}\right|_{\text{shell}} \left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2$$

$$\frac{d\sigma^L}{dt}(q^2) = \left.\frac{d\sigma_\gamma}{dt}\right|_{\text{shell}} \left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2 \xi(K) \left(\frac{-q^2}{m_\rho^2}\right) .$$

The electroproduction cross section for the electroproduction of rho mesons becomes

$$\frac{d^3\sigma}{d\Omega_{k'} dk_0' dt} = \tilde{\Gamma}_T \left.\frac{d\sigma_\gamma}{dt}\right|_{\text{shell}} \left[\frac{1 - \epsilon\, \xi(K)\, q^2/m_\rho^2}{\left(1 - q^2/m_\rho^2\right)^2}\right]$$

$$\approx \tilde{\Gamma}_T \left.\frac{d\sigma_\gamma}{dt}\right|_{\text{shell}} \left[\frac{1}{1 - q^2/m_\rho^2}\right] \quad \text{for } \epsilon, \xi \approx 1 .$$

Thus if nonspin flip amplitudes determine the q^2 dependence of the reaction, the electroproduction of rhos will have the same q^2 dependence as the electroproduction total cross sections. Observation of a different q^2 dependence for rho production would mean that the original assumptions going into the vector dominance calculation were incorrect, or that spin flip amplitudes were large and had a different q^2 dependence from that of the nonflip amplitudes. In the latter case, because no prediction is made for flip amplitudes, vector dominance would have to be tested by removing the contribution to $\left(d\sigma^{T,L}/dt\right)(q^2)$ due to flip amplitudes.

In this paper it has been assumed that the effects of the width of the ρ and nonresonant 2-pion production are small. These effects are of the order of 10% in the photoproduction of ρ-mesons from hydrogen.[10]

VI. SUMMARY

The angular correlation of electrons and pions in the reaction $ep \rightarrow ep\rho^0$, $\rho^0 \rightarrow \pi^+\pi^-$ allows a separation of the longitudinal and transverse contributions to the cross section. Longitudinal-transverse separation is very difficult to obtain in the usual single-arm spectrometer experiments, requiring data to be taken over a wide range of incident beam energies and has only been done in the region of the isobars.[11] An angular correlation experiment, however, allows the separation to be effected at one incident beam energy. In addition, the specific reaction considered is crucial to vector dominance, regarding the q^2-dependence and magnitude of the longitudinal photon-nucleus amplitudes. The formulae developed for separation are:

$$\frac{d\sigma(ep \rightarrow ep\rho,\ \rho \rightarrow 2\pi)}{d\Omega_{k'}\, dk_0'\, dt\, d\left(\frac{\phi_e}{2\pi}\right) d\left(\frac{\Omega_\pi}{4\pi}\right)} = \tilde{\Gamma}_T \frac{d\sigma_\gamma^T}{dt} W$$

where

$$\tilde{\Gamma}_T = \frac{-\alpha}{2\pi^2} \frac{k'}{k} \frac{K}{q^2} \left(1 + \frac{2m^2}{q^2} + \frac{2k_0 k_0' + q^2/2}{|\vec{q}|^2}\right)$$

$$K = k_0 - k_0' + q^2/2M$$

$W(\theta, \phi, \phi_e, q^2, q_4, t)$ is the angular correlation function (see Eq. (16)) and $\frac{d\sigma^T}{dt}(q^2, q_4, t)$ is the differential "cross section" for transverse virtual photons.

The calculations have been made with hydrogen targets in mind, but are valid for heavy nuclei. Experiments measuring $\frac{d\sigma^L}{dt}(q^2, A, t)$ and $\frac{d\sigma^T}{dt}(q^2, A, t)$ are possible and important in determining whether longitudinal and transverse virtual photons interact similarly to transverse real photons. That is, do virtual photons scatter coherently with an A dependence characteristic of strongly interacting particles? For coherent processes, heavy targets can probably be treated as if they have spin zero, which simplifies the analysis.

The calculational technique used here is applicable to other reactions, such as $ep \to eN^*$, $N^* \to N\pi$.[12]

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APPENDIX A

It is very probable that rho production by virtual photons is a coherent process similar to rho production by real photons. The amplitude for production from a heavy nucleus then is the sum of the amplitudes for all the individual nucleons with modifications for final state absorption.[13] One suspects that the spin dependences in this sum should cancel to order 1/A due to the nucleons pairing off with opposite spin. This argument has been elaborated by Goldhaber and Goldhaber.[14] Thus, even if the spin dependent part of the amplitude for nucleon scattering were large, the contribution to the total cross section would be of order $\frac{1}{A^2}$ for coherent nucleus scattering. This negligible spin dependence would allow the target to be treated as a spin-zero object for which

$$\rho_{\sigma-\sigma'}^{\lambda-\lambda'} = (-1)^{\lambda'-\sigma'} \rho_{\sigma\sigma'}^{\lambda\lambda'}$$

Using this condition, the production matrix can be written as the product of amplitudes with no target spin dependence:

$$\rho_{\sigma\sigma'}^{\lambda\lambda'} = U_{\sigma}^{\lambda} U_{\sigma'}^{\lambda'*}$$

The phase of U_1^1 can be taken as zero. The five amplitudes for the reaction are:

$$U_1^1 = a_{11}$$

$$U_0^0 = a_{00}\, e^{i\,\delta_{00}}$$

$$U_{-1}^1 = a_{1-1}\, e^{i\,\delta_{1-1}}$$

$$U_0^1 = a_{10}\, e^{i\,\delta_{10}}$$

$$U_1^0 = a_{01}\, e^{i\,\delta_{01}}$$

where the magnitudes a_{mn} are real and the phases δ_{mn} are relative to the phase of U_1^1.

Using this parametrization, the production density matrix elements can be expressed $\rho_{\sigma\sigma'}^{\lambda\lambda'} = a_{\lambda\sigma} a_{\lambda'\sigma'} e^{i(\delta_{\lambda\sigma} - \delta_{\lambda'\sigma'})}$. Because re ρ is detected by the angular correlations, the phase terms appear as cos δ and sign ambiguities will occur. For example, the angular distribution is unchanged if all the phase angles are reversed. Nevertheless, fits to the data could determine the magnitude of the five amplitudes and the magnitudes of the four phases.

APPENDIX B

If all helicity flip amplitudes are zero, the angular distribution given by expression C is considerably simplified.

$$1/3\ W(\theta,\psi) = \Gamma_{00}\rho_{00}^{00}\cos^2\theta + \sin^2\theta\left[\Gamma_{11}\rho_{11} + \Gamma_{1-1}\rho_{1-1}^{1-1}\cos 2\psi\right]$$

$$+ \sqrt{2}\ \rho_{10}\ \mathrm{re}\left(\rho_{10}^{10}\right)\ \sin 2\theta\ \cos\psi$$

Assuming that $\rho_{1-1}^{1-1} = \rho_{11}^{11}$, which is apparently true for photoproduction,[8] and that $K^2 \gg q^2 \gg \mu^2$ so that $\epsilon \approx 1$, we obtain

$$1/3\ W(\theta,\psi) = \rho_L \cos^2\theta + \rho_T \sin^2\theta\cos^2\psi + \rho_I \sin 2\theta\cos\psi$$

In the above ρ_L, ρ_T, ρ_I are functions of K, q^2 and t, and

$$\rho_L = \rho_{00}^{00} = \left|T_0^0\right|^2$$

$$\rho_T = \rho_{11}^{11} = \left|T_1^1\right|^2$$

$$\rho_I = \mathrm{re}\left(T_1^{1*}T_0^0\right) = \left|T_1^1\right|\left|T_0^0\right|\cos\delta_0^0$$

δ_0^0 = phase angle between T_0^0 and T_1^1

The longitudinal-transverse interference term may be determined by forming the asymmetry

$$\alpha_{IF} = \frac{N_1 - N_2}{N_1 + N_2}$$

where

$$N_1 = \int_{30^o}^{60^o} \sin\theta\, d\theta \int_{-60^o}^{60^o} d\psi\ W(\theta,\psi)$$

and

$$N_2 = \int_{120^\circ}^{150^\circ} \sin\theta \, d\theta \int_{-60^\circ}^{60^\circ} d\psi \, W(\theta, \psi) .$$

Performing these integrations gives the equation

$$\alpha_{IF} \cong \frac{2.5\rho_I}{1.5\rho_L + \rho_T} = \frac{2.5 \times \cos \delta_0^0}{1 + 1.5\, x^2} ,$$

where

$$x = \left|T_0^0\right| / \left|T_1^1\right| .$$

A similar asymmetry may be formed that involves ρ_L and ρ_T.

$$\alpha_E = \frac{N_I - N_{II}}{N_I + N_{II}}$$

where

$$N_I = \int_{30^\circ}^{150^\circ} \sin\theta \, d\theta \int_{-30^\circ}^{30^\circ} d\psi \, W(\theta, \psi)$$

$$N_{II} = \int_{30^\circ}^{150^\circ} \sin\theta \, d\theta \int_{60^\circ}^{120^\circ} d\psi \, W(\theta, \psi)$$

This asymmetry compares the number of ρ^o mesons decaying parallel to the photon polarization to those decaying perpendicular to the polarization. Evaluation of the integrals gives

$$\alpha_E \cong \frac{\rho_T}{\rho_T + .67\rho_L} = \frac{1}{1 + .67\, x^2}$$

A graphical solution for x and $\left|\delta_0^0\right|$ is shown in Fig. 3. As can be seen, an experiment which measured α_E and α_{IF} to about 10 percent would be a strong test of vector dominance.

APPENDIX C

The limit of (16) can be taken to obtain the angular correlations between the photon polarization vector and the decay angles of the rho meson in its rest frame. As $q^2 \to 0$ all terms with upper indices of zero (longitudinal photons) go to zero.

$$W_\gamma = \lim_{q^2 \to 0} W = 2A_{00}\left(\rho_{00}^{11} B^{11} + \rho_{00}^{1-1} \text{ re } B^{1-1}\right)$$

$$+ 2\left(\rho_{11}^{11} B^{11} A_{11} + \rho_{1-1}^{1-1} \text{ re}\left(B^{1-1} A_{1-1}\right)\right)$$

$$+ 2\left(\rho_{-1-1}^{11} B^{11} A_{11} + \rho_{-11}^{1-1} \text{ re}\left(B^{1-1} A_{1-1}^*\right)\right)$$

$$+ 4\left(\text{re } \rho_{1-1}^{11} B^{11} \text{ re} A_{1-1} + \text{re } \rho_{11}^{1-1} A_{11} \text{ re} B^{1-1}\right)$$

$$+ 4\left(\text{re } \rho_{10}^{11} B^{11} \text{ re} A_{10} + \text{re } \rho_{10}^{1-1} \text{ re}\left(A_{10} B^{1-1}\right)\right)$$

$$- 4\left(\text{re } \rho_{-10}^{11} B^{11} \text{ re} A_{10} + \text{re } \rho_{-10}^{1-1} \text{ re}\left(A_{10}^* B^{1-1}\right)\right) .$$

The cross section then becomes

$$\frac{d\sigma_\gamma}{dt\, d\left(\frac{\phi_e}{2\pi}\right) d\left(\frac{\Omega_\pi}{4\pi}\right)} = \frac{d\sigma_\gamma}{dt} W_\gamma(\theta, \phi, \phi_e, q_4, t)$$

where ϕ_e is now the plane of polarization of the photons with respect to the production plane. The case for helicity flip amplitudes = 0 was discussed in example 2. By averaging W_γ over ϕ_e the distribution for unpolarized photons is obtained:

$$\frac{1}{2\pi}\int W_\gamma \, d\phi_e = 2A_{00}\,\rho^{11}_{00}\,B^{11} + 2\rho^{11}_{11}\,B^{11}A_{11} + 2\rho^{11}_{-1-1}\,B^{11}A_{11}$$

$$+ 4 \text{ re } \rho^{11}_{1-1}\,B^{11} \text{ re} A_{1-1} + 4 \text{ re } \rho^{11}_{10}\,B^{11} \text{ re } A_{10}$$

$$- 4 \text{ re } \rho^{11}_{-10}\,B^{11} \text{ re} A_{10}$$

with our normalization $B_{11} = \frac{1}{2}$ and $A_{\sigma\sigma'}$ is given by Eq. (18).

$$1/3\ W_\gamma(\theta,\theta) = \rho^{11}_{00}\cos^2\theta + \frac{1}{2}\left(\rho^{11}_{11} + \rho^{-1-1}_{11}\right)\ \sin^2\theta$$

$$- \text{ re } \rho^{11}_{1-1}\sin^2\theta \cos 2\phi$$

$$- \frac{1}{\sqrt{2}} \text{ re}\left(\rho^{11}_{10} + \rho^{-1-1}_{10}\right)\sin 2\theta \cos\phi$$

This is the same as the usual result[5] for the production matrix using unpolarized photons. The usual notation has the following relation to the above quantities.

$$\rho_{00} = \rho^{11}_{00}$$

$$\rho_{11} = \frac{1}{2}\left(\rho^{11}_{11} + \rho^{-1-1}_{11}\right)$$

$$\rho_{1-1} = \rho^{11}_{1-1}$$

$$\rho_{10} = \frac{1}{2}\left(\rho^{11}_{10} + \rho^{-1-1}_{10}\right)$$

and

$$\rho_{00} + 2\,\rho_{11} = \rho^{11}_{00} + \rho^{11}_{11} + \rho^{-1-1}_{11} = \rho_T = 1 \quad .$$

B. Dieterle, "Angular Correlations in the Electroproduction of ρ Mesons"

REFERENCES

1. H. Blechschmidt, J.P. Dowd, B. Elsner, K. Heinloth, P. Karow, J. Rathje, D. Schmidt, J.H. Smith, A. Kanaris, and A.G. Wynroe, Nuovo Cimento LIIIA, 1581 (1968).

2. M. Gourdin, Nuovo Cimento XXI, 1094 (1961);
M. Gourdin, Nuovo Cimento XXXVII, 208 (1965);
S. D. Drell and J. D. Walecka, Ann. Phys. 28, 18 (1964);
S. M. Berman, Phys. Rev. 135, B1249 (1964); and
L. Hand and R. Wilson, Report No. SLAC-25, Part 2, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1963).

3. R. P. Feynman, Theory of Fundamental Processes, (W. A. Benjamin, Inc., New York, 1962); Chapter 20.

4. M. Jacob and G. C. Wick, Ann. Phys. 7, 404 (1959);
Hartmut Pilkuhn, The Interactions of Hadrons, (John Wiley and Sons, New York, 1967); Chapter 9.

5. Hartmut Pilkuhn, The Interactions of Hadrons, (John Wiley and Sons, New York, 1967); Chapter 9;
K. Gottfried and J. D. Jackson, Nuovo Cimento XXXIII, 309 (1964).

6. This is an historical convention. There is no reason compelling one to adopt it other than the fact that it is used fairly consistently in data analysis.

7. E. Lohrmann, "Resonances (Bubble Chamber)", Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies (1967); pp. 212 and 213. This review contains excerpts from data presented to this conference by the Cambridge Bubble Chamber Group and the ABBHHM Collaboration. The figures shown on p. 212 and p. 213 show that ρ_{1-1}^{Hel} and reρ_{10}^{Hel} are small for photoproduction of ρ^0 mesons indicating that double and single helicity flip amplitudes are small.

References (cont'd.)

8. L. Criegee, M.H. Garrell, C. Gottfried, A. Krolzig, G. Loeffler, A Saulys, K.P. Schüler, U. Timm, W. Zimmermann, H. Werner, G.B. Collins, P.W. Dougan, E. Von Goeler, and R.A. Carrigan, Jr., Phys. Letters 28B, 282 (1968); F. Cooper, Phys. Rev. 170, 1602 (1968).

9. J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969).

10. F. Bulos, W. Busza, R. Giese, R.R. Larsen, D.W.G.S. Leith, B. Richter, V.Perez-Mendez, A. Stetz, S.H. Williams, M. Beniston, and J. Rettberg, Phys. Rev. Letters 22, 490 (1969).

11. H.L. Lynch, J.V. Allaby, and D.M. Ritson, Phys. Rev. 164, 1635 (1967); W. Albrecht, F.W. Brasse, H. Dorner, W. Flauger, K. Frank, J. Gayler, H. Hultschig, J. May, and E. Ganszauge, Phys. Letters 28B, 225 (1968); and W. Bartel, B. Dudelzak, H. Krehbiel, J. McElroy, U. Meyer-Berkhout, W. Schmidt, V. Walther, and G. Weber, Phys. Letters 27B, 660 (1968).

12. This process has been calculated using a different approach by E. Ganszauge, "Inelastische electron-proton streung," Interner Bericht, DESY F 21/3, (May 1968).

13. S. J. Brodsky and J. Pumplin, "Photon-nucleus total cross sections," Phys. Rev. 182, 1794 (1969). K. Gottfried and D. R. Yennie, "Vector mesons and nuclear optics," Phys. Rev. 182, 1595 (1969).

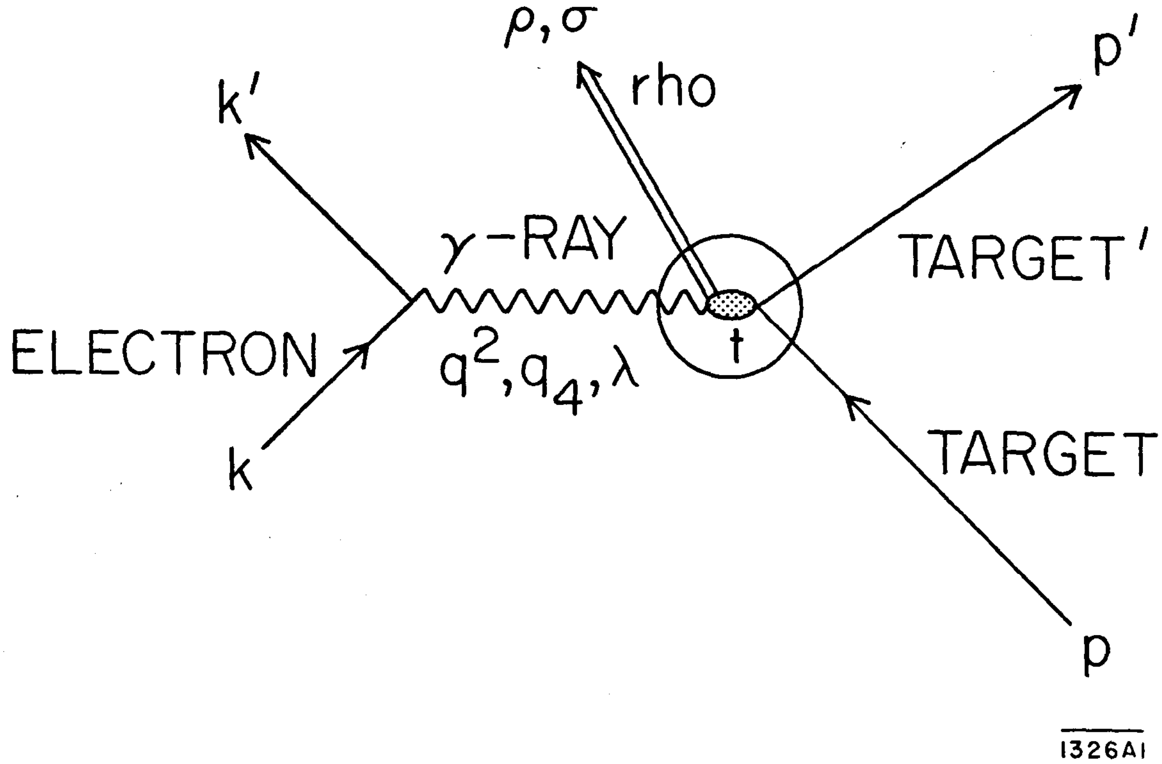
14. Alfred S. and Maurice Goldhaber, "Coherent high energy reactions with nuclei," in Preludes in Theoretical Physics, A. de-Shalit, H. Feshbach and L. Van Hove, Editors, (North Holland Publishing Co., Amsterdam, 1966).

FIGURE CAPTIONS

1. The electroproduction of rho mesons using a single photon exchange model. Incoming and outgoing particles are labeled with their four momenta. The photon state λ has "mass" $q^2 = (k - k')^2$, energy $q_4 = k_0 - k'_0$. σ is the helicity state of the rho meson, while the symbol $t = (p - p')^2$ denotes the four-momentum transfer squared to the target.

2. The coordinate systems used in the calculation of rho electroproduction followed by decay are depicted. The scattering process γ_{virt} + target $\rightarrow$ ρ^0 + target is evaluated in the C.M.S. using helicity amplitudes, thus $\rho_{\sigma\sigma'}^{\lambda\lambda'}$ is the corresponding production density matrix. The photon density matrix $B^{\lambda\lambda'}$ and ρ^0 decay matrix $A_{\sigma\sigma'}$ are transformed to the laboratory system and ρ^0 rest frame respectively and evaluated. $B^{\lambda\lambda'}$ and $A_{\sigma\sigma'}$ are defined in such a way that they are invariant under these transformations.

3. Graphical solution for x, δ_0^0 using experimental quantities α_E, α_{IF}. x and δ_0^0 are the longitudinal amplitude and phase with respect to the transverse amplitude. α_E and α_{IF} are asymmetries generated from the ρ^0 decay distribution and are defined in Appendix B. "Vector dominance" predicts $x = \left(|q^2|/m_\rho^2\right)^{1/2}$.



ρ, σ
rho
k'
p'
γ-RAY
TARGET$'$
ELECTRON
q^2, q_4, λ
t
k
TARGET
p
1326A1

Fig. 1

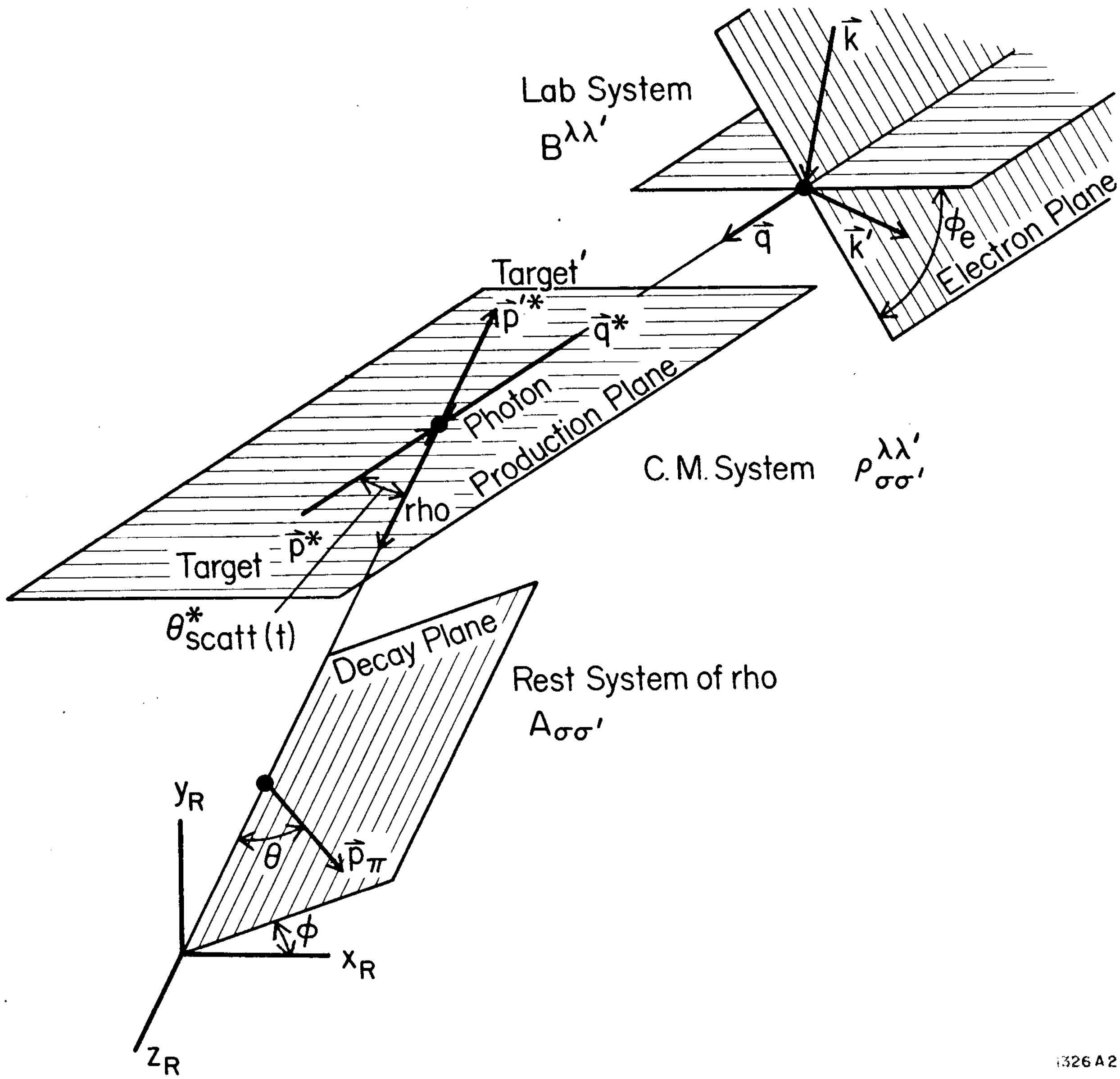

Lab System
$B^{\lambda\lambda'}$
$\vec{k}$
$\vec{q}$
$\vec{k}'$
ϕ_e
Electron Plane
Target'
$\vec{p}'^*$
$\vec{q}^*$
Photon
Production Plane
C. M. System $\rho^{\lambda\lambda'}_{\sigma\sigma'}$
rho
Target
$\vec{p}^*$
$\theta^*_{scatt}(t)$
Decay Plane
Rest System of rho
$A_{\sigma\sigma'}$
y_R
θ
$\vec{p}_\pi$
ϕ
x_R
z_R
1326A2

Fig. 2

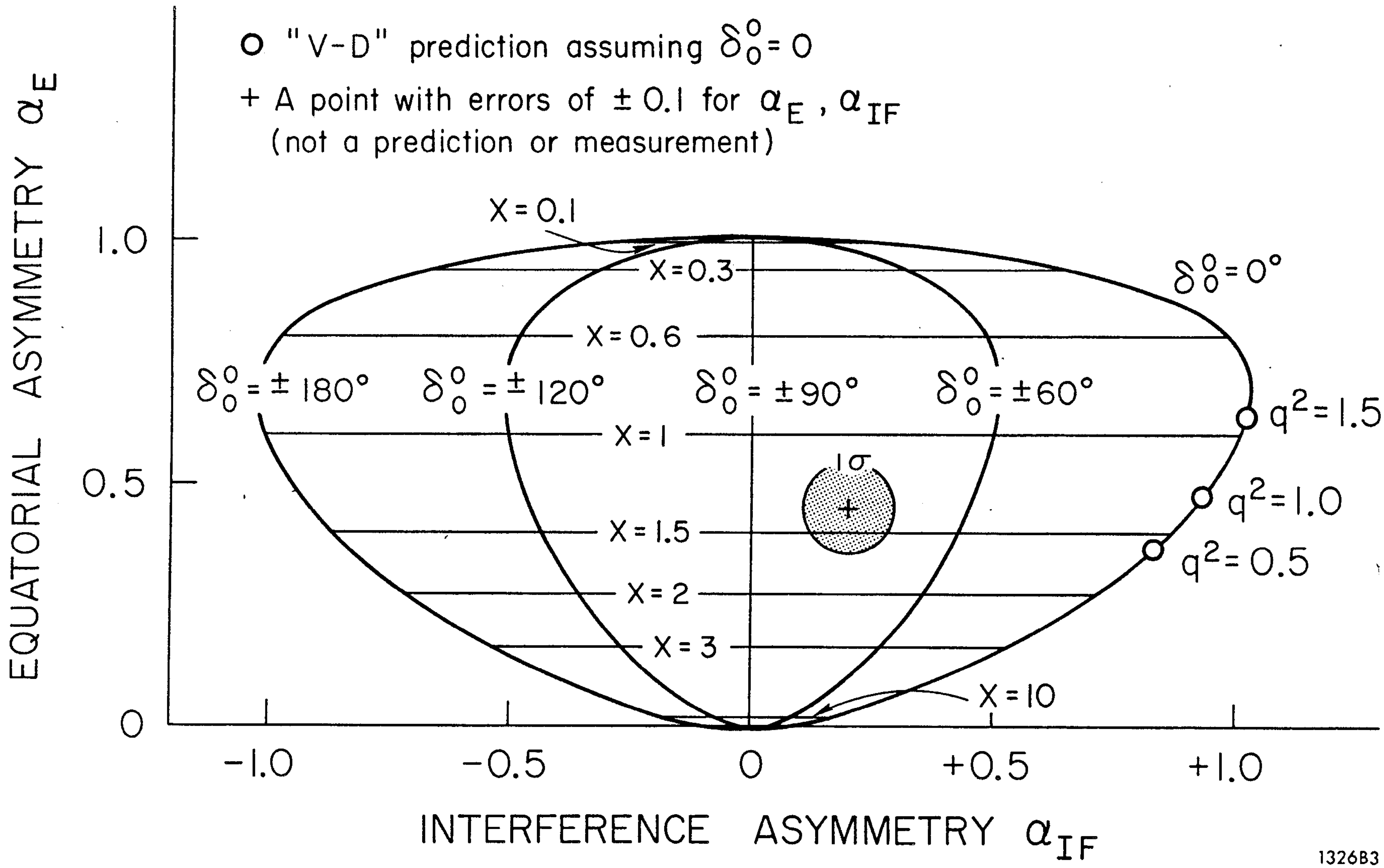

○ "V-D" prediction assuming $\delta_0^0 = 0$
+ A point with errors of ± 0.1 for α_E, α_{IF}
(not a prediction or measurement)
X = 0.1
X = 0.3
X = 0.6
X = 1
X = 1.5
X = 2
X = 3
X = 10
$\delta_0^0 = 0°$
$\delta_0^0 = \pm 180°$
$\delta_0^0 = \pm 120°$
$\delta_0^0 = \pm 90°$
$\delta_0^0 = \pm 60°$
$q^2 = 1.5$
$q^2 = 1.0$
$q^2 = 0.5$
1σ
EQUATORIAL ASYMMETRY α_E
INTERFERENCE ASYMMETRY α_{IF}
1.0
0.5
0
−1.0
−0.5
0
+0.5
+1.0
1326B3

Fig. 3