

THE SIGN OF THE  $\pi^0 \rightarrow \gamma\gamma$  DECAY AMPLITUDE <sup>†</sup>

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ABSTRACT

The experimentally observed constructive interference between the amplitude for the Primakoff effect and the other amplitudes involved in  $\pi^0$  photoproduction is interpreted as showing that the  $\pi^0 \rightarrow \gamma\gamma$  decay amplitude and  $g_{\pi NN}$  have opposite signs. The experimental  $\pi^0 \rightarrow \gamma\gamma$  amplitude thus agrees in sign and approximately in magnitude with the result of computing the triangle graph in perturbation theory with a single elementary proton going around the fermion loop.

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In certain special situations one can determine the sign of a strong interaction scattering amplitude by observing its interference with a known, real amplitude due to the electromagnetic interactions. Such is the case in establishing experimentally the sign (and magnitude for that matter) of the real part of the near forward  $\pi^\pm p$  and  $pp$  elastic scattering amplitudes at high energy<sup>1</sup>. In these particular processes one can observe the interference of the real coulomb (one photon exchange) amplitude with the real part of the strong interaction amplitudes.

One of the few other places in high energy physics where one can observe such an interference is in  $\pi^0$  photoproduction. Here one can observe the interference of the one photon exchange amplitude (Primakoff effect<sup>2</sup>) with the remainder of the near forward photoproduction amplitude due to direct channel resonances or meson exchange. A measurement of the sign in this interference will then determine the sign of the  $\pi^0 \rightarrow \gamma\gamma$  decay amplitude involved in the Primakoff effect relative to the rest of the photoproduction amplitude, whose sign can in turn be related back to the sign of  $g_{\pi NN}$  in the Born terms by the use of dispersion relations, or more generally, by the use of finite energy sum rules.

The sign of the  $\pi^0 \rightarrow \gamma\gamma$  decay amplitude is currently of some interest due to the recent work of Adler<sup>3</sup> who has shown that the partially conserved axial-vector current (PCAC) equations for the neutral members of the axial-vector current octet must be modified by the addition of an additional term with a specific form. This modification, due to the presence of closed loop triangle diagrams in spinor electrodynamics, changes the PCAC prediction of a vanishing  $\pi^0 \rightarrow \gamma\gamma$  decay amplitude as the pion four-momentum

vanishes, to a prediction of a non-zero amplitude which is proportional to a weighted average of the squares of the charges of the elementary fermions involved in the closed loop triangle graphs. The sign and magnitude of the  $\pi^0 \rightarrow \gamma\gamma$  amplitude is then a possible way of choosing between different models of elementary particles, as has been noted by Okubo<sup>4</sup>. All of the determinations of the sign of the  $\pi^0 \rightarrow \gamma\gamma$  amplitude which Okubo discusses are, however, rather indirect or depend themselves on some model or additional assumption which is not completely free of doubt. The experimental observation of constructive interference by Braunschweig et al.<sup>5</sup> in  $\pi^0$  photoproduction is, on the other hand, free of theoretical assumptions and hence provides a direct and relatively clean determination of the sign.

The translation of the experimental observation of constructive interference<sup>5</sup> into a statement about the relative sign of the  $\pi^0$  decay amplitude and  $g_{\pi NN}$  is basically just a simple, straightforward, but nerve-wracking exercise in defining amplitudes and watching signs carefully. We start by defining our photoproduction S-matrix element as<sup>6</sup>

$$S_{\gamma + N \rightarrow \pi + N} = (2\pi)^4 i \delta(p_2 + q - p_1 - k) \sqrt{\frac{M_N^2}{E_1 E_2 2w_1 2w_2}} \bar{u}(p_2) T u(p),$$

$$\begin{aligned} T = & A_1(s, t) i \gamma_5 \left(\frac{1}{2}\right) (\gamma \cdot \epsilon \gamma \cdot k - \gamma \cdot k \gamma \cdot \epsilon) \\ & + A_2(s, t) 2i \gamma_5 [P \cdot \epsilon q \cdot k - P \cdot k q \cdot \epsilon] \\ & + A_3(s, t) \gamma_5 [\gamma \cdot \epsilon q \cdot k - \gamma \cdot k q \cdot \epsilon] \\ & + A_4(s, t) 2 \gamma_5 [\gamma \cdot \epsilon P \cdot k - \gamma \cdot k P \cdot \epsilon - i M_N \left(\frac{1}{2}\right) (\gamma \cdot \epsilon \gamma \cdot k - \gamma \cdot k \gamma \cdot \epsilon)] \end{aligned}$$

where  $k$  and  $p_1$  are the four-momenta of the initial photon and nucleon.  $\epsilon_\mu$  is the photon polarization vector, and  $q$  and  $p_2$  are the four-momenta of the final pion and nucleon, respectively. We define  $P = (p_1 + p_2)/2$  and  $s = -(k + p_1)^2$ ,  $t = -(k - q)^2$ ,  $u = -(k - p_2)^2$ , or equivalently, define the variables  $\nu = -P \cdot k / M_N$ ,  $\nu_1 = -q \cdot k / 2M_N$ . To take case of the isospin we write each invariant amplitude  $A_i$  (to be taken between nucleon isospinors) as

$$A_i = A_i^{(+)} \frac{1}{2} \{ \tau_\alpha, \tau_3 \} + A_i^{(-)} \frac{1}{2} [ \tau_\alpha, \tau_3 ] + A_i^{(0)} \tau_\alpha,$$

where  $\alpha$  is the isospin index of the final pion. For  $\gamma + p \rightarrow \pi^0 + p$  we have  $A_i(\gamma + p \rightarrow \pi^0 + p) = A_i^{(+)} + A_i^{(0)}$ . The explicit Born terms for the amplitude  $A_1$  read

$$\begin{aligned} A_1^{(+)\text{B}} &= A_1^{(0)\text{B}} = -\frac{eg}{2} \left[ \frac{1}{M_N^2 - s} + \frac{1}{M_N^2 - u} \right] \\ &= -\frac{eg}{4M_N} \left[ \frac{1}{\nu_B - \nu} + \frac{1}{\nu_B + \nu} \right] \\ A_1^{(-)\text{B}} &= \frac{eg}{2} \left[ \frac{1}{M_N^2 - s} - \frac{1}{M_N^2 - u} \right] \\ &= \frac{eg}{4M_N} \left[ \frac{1}{\nu_B - \nu} - \frac{1}{\nu_B + \nu} \right] \end{aligned}$$

where  $\nu_B = \nu_1 = -q \cdot k / 2M_N$ .

Next, we define the  $\gamma\gamma \rightarrow \pi^0$  S-matrix element as

$$S_{\gamma+\gamma \rightarrow \pi^0} = i(2\pi)^4 \delta(q - k_1 - k_2) \frac{1}{\sqrt{8q_0 k_{10} k_{20}}} (-i) \epsilon_{\mu\nu\lambda\sigma} \epsilon_\mu^{(1)} \epsilon_\nu^{(2)} k_{1\lambda} k_{2\sigma} F(q^2)$$

where  $k_1, k_2$  and  $q$  are the four-momenta of the initial photons and final  $\pi^0$  respectively, and  $\epsilon_\mu^{(1)}, \epsilon_\nu^{(2)}$  are the polarization vectors of the photons. If we now compute  $F(0)$  from the triangle graph with a single proton (charge coupling only) in the elementary fermion loop, we obtain using the renormalized charge, pion-nucleon coupling constant, and nucleon mass,

$$F(0) = -\frac{e^2 g_{\pi NN}}{4\pi^2 M_N},$$

in agreement with the old calculation of Steinberger<sup>7</sup>.

We are now ready to calculate the one-photon exchange (Primakoff effect) amplitudes for  $\gamma + p \rightarrow \pi + p$ . We find

$$A_1^P = A_2^P = A_3^P = 0$$

$$A_4^{(+P)} = A_4^{(0P)} = (-\frac{1}{2} eF)/(-t),$$

where  $-t$  is positive in the physical region for photoproduction. (In perturbation theory this means we would have

$$A_4^{(+P)} = A_4^{(0P)} = \frac{e^3 g}{8\pi^2 M_N} \frac{1}{(-t)},$$

i. e.,  $A_4^{(+)}$  and  $A_4^{(0)}$  have the same sign as  $e g_{\pi NN}$ .)

Now we are ready to compare amplitudes and determine the sign of the amplitude for  $\pi^0 \rightarrow \gamma\gamma$ . The amplitudes  $A_1, A_2, A_3, A_4$  can be determined at high energies at least as to sign by using finite energy sum rules to relate them to the resonance parameters (or more generally, a phase shift analysis) at low energies. This has been done, for example, by Di Vecchia et al.<sup>8</sup>, who find that the amplitude  $A_4^{(+)}$  (which, coming from "isovector" photons is much bigger than  $A_4^{(0)}$ , coming from "isoscalar" photons) has an imaginary, and hence<sup>8</sup> real part, which has the same sign as  $eg_{\pi NN}$ . Now since the sign of  $A_4^{(+)P}$  is that of  $-eF$  and experimentally one observes constructive interference, we see that  $-eF$  has the same sign as  $eg_{\pi NN}$ , i. e.,  $F$  and  $g_{\pi NN}$  have opposite signs. This agrees with the results of Okubo<sup>4</sup> as to the sign of the amplitude. It also agrees with the sign given by a single "elementary" proton going around the closed loop in the triangle graph, a model which also gives approximately the correct magnitude for the decay rate of  $\pi^0 \rightarrow \gamma\gamma$ !

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FOOTNOTES

1. See, for example, K. J. Foley et al., Phys. Rev. Letters 19, 193 (1967);  
ibid, 19, 857 (1967).
2. H. Primakoff, Phys. Rev. 81, 899 (1951).
3. S. L. Adler, Institute for Advanced Study Preprint (1968). The author  
thanks Professor Adler for discussions on his work on modifications of  
PCAC.
4. S. Okubo, University of Rochester Preprint (1968).
5. M. Braunschweig et al., Phys. Letters 26B, 405 (1968).
6. We work in the metric where  $p^2 = -M^2$ ,  $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu}$ ,  $\gamma_5^2 = 1$ ,  
 $\epsilon_{1234} = +1$ ,  $\bar{u}(p)u(p) = 1$ ,  $\epsilon_\mu^* \epsilon_\mu = +1$ ,  $(p)_4 = i(p)_0 = iE$ , etc. In this paper  
 $M_N$  is the nucleon mass,  $+e$  is the proton's charge,  $e^2/4\pi = 1/137$ , and  
 $g_{\pi NN} = g$  is the renormalized pion-nucleon coupling constant,  $g^2/4\pi \approx 14.8$ .
7. J. Steinberger, Phys. Rev. 76, 1180 (1949).
8. P. Di Vecchia et al., Nuovo Cimento 55A, 809 (1968). These authors are  
specifically concerned with calculating the parameters of  $\omega$  Regge pole  
exchange, which they assume dominates the  $A_i^{(+)}$  amplitudes at high energy.  
For our purposes it does not matter what are the specific Regge poles or  
cuts which are being exchanged. We do, however, need to use the fact  
that we have dominantly odd signature trajectories exchanged with  
 $0 \leq \alpha_{\text{eff}}(0) \leq 1$ , as appears to be the case experimentally, so that the real  
and imaginary parts of the amplitude have the same sign at high energies.