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COMPARISON OF PHASE SHIFT ANALYSIS SOLUTIONS TO π^-p SCATTERING DATA*

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ABSTRACT

A comparison between the π^-p scattering data and the predictions of the various phase shift analyses is presented. In particular, it is pointed out that the dispersion relation fit of the CERN group fails to represent the data.

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In the past few years, several experiments making precise determinations of the $\pi^{\pm}p$ elastic differential cross section[1-5] and proton polarization,[6-11] over wide energy regions have made possible extensive phase shift analyses.[12-18] The results of these analyses have led to substantial progress in the understanding of pion-nucleon scattering. One of the features of these studies was the discovery, in different partial waves, of several resonances which could not be readily identified as peaks in total cross sections.[13-15] A notable example of this is the so-called "third pion-nucleon resonance" at 1688 MeV, which was shown to include resonant states in S_{31}, D_{15}, F_{15} and possibly other partial waves. More recently, by using partial wave dispersion relations to provide a smooth energy dependence to the data, the CERN group succeeded in achieving smoothness in the resulting partial wave phase shifts, which led to the "announcement" of the existence of nine new resonances of low elasticity.[19]

In this letter we present a comparison between the elastic $\pi^{-}p$ scattering data and the predictions of the phase shift analyses of various groups. We wish to point out a discrepancy between the experimental data and the CERN dispersion relation solution. We show below that this particular smooth solution is not a faithful representation of the experimental measurements of $\pi^{-}p$ scattering. The effect on the determination of the pion nucleon resonance parameters is also briefly discussed.

The $\pi^{-}p$ elastic scattering data previously available[1-3] are shown in Fig. 1 together with our own new data.[20] The strong structures around 1520 MeV and 1688 MeV resulting from the formation of nucleon resonances are clearly visible. The two curves of the predicted cross sections of the phase shift analysis solutions are calculated with the absorption parameters and phase shifts (η's and

δ's) given by the various groups[21,22] using the expression

$$\sigma_{el} = 4\pi \lambda\!\!\!^{-2} \sum_{\ell} \left(J + \frac{1}{2}\right) \left|T_{\ell}^{\pm}\right|^2 ,$$

where

$$T_{\ell}^{\pm} = \frac{\eta_{\ell}^{\pm} e^{2i\delta_{\ell}^{\pm}} - 1}{2i} , \quad \text{and } J = \ell \pm \frac{1}{2} .$$

The solid curve in Fig. 1 is the $\pi^- p$ elastic cross section predicted by the CERN phase shift solution using experimental data as input together with constraints from partial wave dispersion relations (referred to as CERN-EXP. hereafter), whereas the dashed curve is that predicted by the CERN dispersion relation fit (referred to as CERN-TH. hereafter, also referred to as CERN I[23] in literature).[24] It is clearly evident that though the CERN-EXP. solution describes the data well, there are marked discrepancies between the CERN-TH. predictions and the experimental data. The CERN-TH. solution is unable to reproduce the sharp peaks around 1520 and 1688 MeV.

To investigate further the nature of these discrepancies, we compared the predicted $\pi^- p$ differential cross sections with those measured in our experiment. Here the differential cross sections are calculated from the usual expression[25]

$$\frac{d\sigma}{d\Omega} = \left|f(\theta)\right|^2 + \left|g(\theta)\right|^2 ,$$

where

$$f(\theta) = \frac{1}{k} \sum_{\ell} \left[(\ell + 1)\, T_{\ell}^{+} + \ell\, T_{\ell}^{-}\right] P_{\ell} (\cos \theta) ,$$

$$g(\theta) = \frac{i}{k} \sin \theta \sum_{\ell} \left(T_{\ell}^{+} - T_{\ell}^{-}\right) \frac{dP_{\ell} (\cos \theta)}{d \cos \theta} .$$

We show in Figs. 2(a) - 2(g) our measured differential cross sections at six of our energies.[20] These six energies are indicated by the arrows in Fig. 1. The data agree very well with previous measurements.[1-3] The solid and dashed curves again correspond to the predictions of CERN-EXP. and CERN-TH. respectively. The discrepancies in both the forward and backward regions between the CERN-TH. predictions and the data are quite obvious.

The total $\pi^- p$ scattering cross sections predicted by these phase shift solutions may be obtained most conveniently from the forward scattering amplitude by using the optical theorem

$$\sigma_{total} = \frac{4\pi}{k} \text{ Im } f(0^o)$$

In Fig. 3, we have compared these with the recent precision measurements of $\pi^- p$ total cross section by Carter et al.[26] Again, the CERN-TH. solution fails to agree with the data in exactly the same fashion as illustrated in Fig. 1.

We have also made the same comparisons for the phase shift solutions of the Saclay,[13] Berkeley,[21] and Glasgow groups[22] in Figs. 4-6. While the solution from the first two groups do not posses the smoothness of the CERN-TH. solution, they do have the virtue that their predicted cross sections do show reasonable agreement with the data. The Glasgow solution also employs an energy dependent fit and shows qualitative agreement with $\pi^- p$ elastic scattering data.[24]

In smoothing the partial wave phase shifts, by imposing theoretical considerations, one hopes to eliminate the wobbles and structures which were generated purely due to inadequate or inconsistent input data. In principle, one of the most important merits of this smoothing is that one can not only determine the resonance parameters more accurately, but also identify the very inelastic resonances with more certainty. However as also pointed out by others,[13,27] it is often extremely difficult to determine the significance of the fine structures observed in the partial wave phase shifts and to determine which of these structures needs to be smoothed out. Therefore, it is very important that the final smooth solution faithfully describes the experimentally well-measured quantities such as the total and differential cross sections. In the case of CERN-EXP., the procedure has been successful in that the phase shift parameters are considerably smoother than those obtained by other energy independent analyses. On the other hand the CERN-TH. solution, from which the published resonance parameters were obtained, fails to agree with the rapidly varying $\pi^- p$ measured cross section, as illustrated above. This disagreement is reflected in the nucleon resonance parameters determined from this solution. Since the predicted resonance peaks in the cross section appear to be lower and wider than observed in the data, the elasticity is underestimated and the resonance width overestimated. This effect can be seen in Fig. 7, where the absorption parameter, η, the phase shift δ, and the elastic partial wave cross sections are shown for three resonating waves — D_{13}, D_{15}, F_{15} — as examples. We see that the apparently good agreement of the two solutions for η and δ is extremely deceptive since the cross sections are very sensitive functions of both parameters. The underestimate of the strength of the coupling to the elastic channel, and the overestimate of the width is apparent in each case for the CERN-TH. solution.[28] It is easily seen that these lead to the discrepancy between the predicted elastic cross section and the data as illustrated in Fig. 1.

In summary, we find that the smooth phase shift solution of CERN, obtained from the partial wave dispersion relations — CERN-TH. or CERN I, is not a good representation of the measured $\pi^- p$ elastic scattering data.[29] Thus, while we agree with the need for obtaining smooth partial wave phase shifts, we emphasize that one must be careful in carrying this out, so as not to produce misleading results.

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21. The phase shift solutions of CERN-EXP., CERN-TH., Saclay, and Berkeley were obtained at the Conference on πN Scattering at Irvine (1967) through the courtesy of C. Johnson and H. Steiner.

22. A. T. Davies and R. G. Moorhouse, paper presented at the 14th International Conference on High Energy Physics at Vienna (1968). The Glasgow solutions were communicated to us by R. G. Moorhouse.

23. CERN-TH. is referred to as CERN I since the resonance parameters quoted by Ref. 12 above and the Particle Data Group — Rev. Mod. Phys. (January 1969), come from this solution. For clarification of this point, see footnote on p. 87 of Lovelace's Heidelberg report of Ref. 19.

24. The CERN-EXP. solution is obtained by an iterative fitting procedure in which information from partial wave dispersion relations in addition to the experimental data is used in the overall phase shift analysis. The theoretical

"data points" are weighted with large errors. CERN-TH. is the dispersion relation calculation using the parameters of their final fit. The fact that in the final iteration the two solutions are different indicates the limitations of their method.

25. The Coulomb scattering contribution amounts to less than 0.5% at the lowest energy considered here.

26. A. A. Carter, K. F. Riley, and R. J. Tapper; D. V. Bugg, R. S. Gilmore, K. M. Knight, D. C. Salter, G. H. Stafford, J. D. Davies, J. D. Dowell, P. M. Hattersley, R. J. Homer, and A. W. O'Dell, Phys. Rev. 168, 1457 (1968).

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28. For example, in the case of F_{15}, we estimate for CERN-TH., $\Gamma = 190$ MeV, $x = 0.56$, and for CERN-EXP., $\Gamma = 110$ MeV, $x = 0.65$ based on the elastic partial wave cross sections.

29. Since completing this work it has been brought to our attention that the N.B.S. group (J. Coyne, S. Meshkov, W. Parke, R. Ponzini and H. Williams) had independently observed this discrepancy.

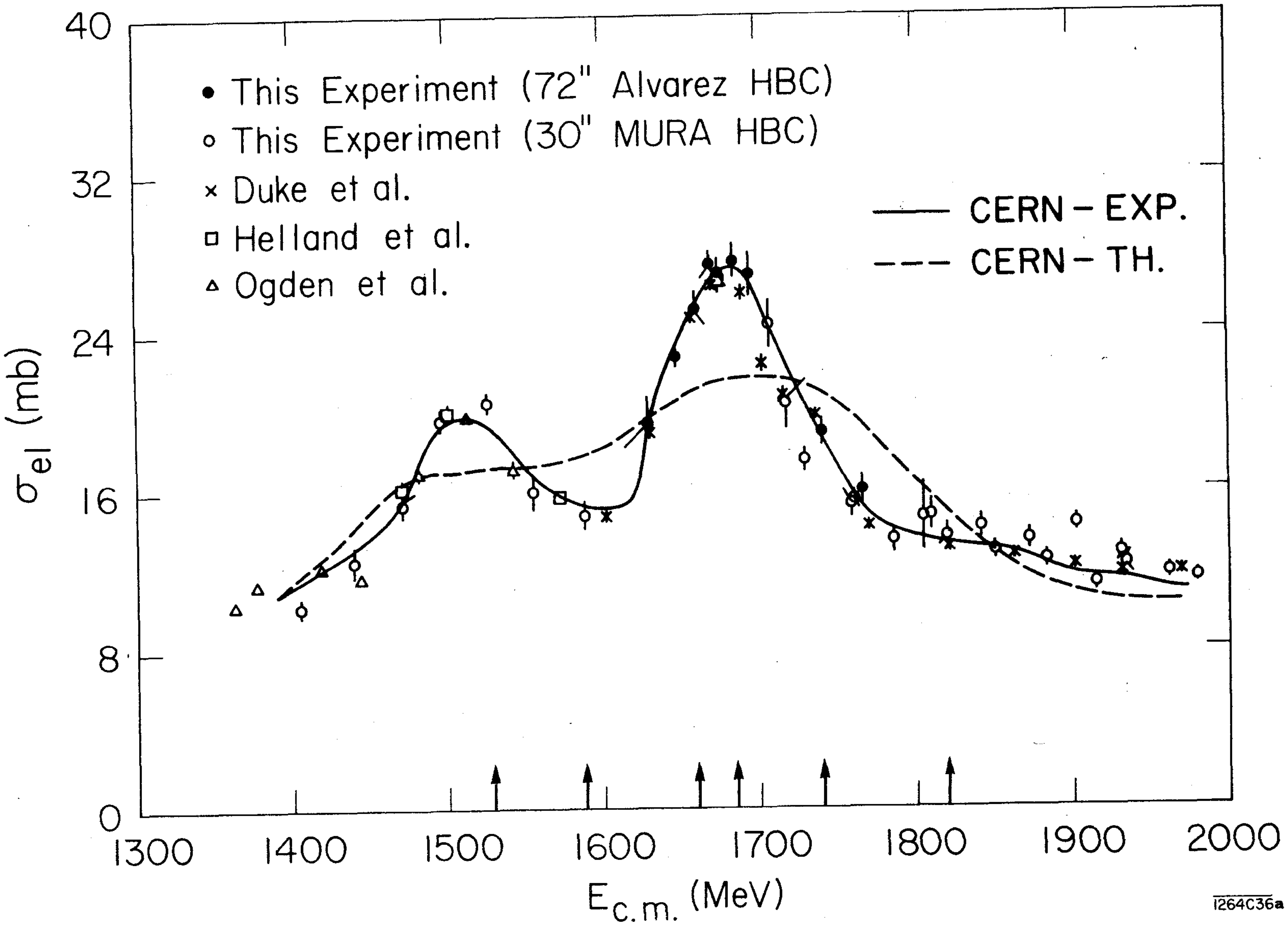
FIGURE CAPTIONS

1. $\pi^- p$ elastic scattering cross section measurements of Duke et al., Helland et al., Ogden et al., and this experiment. Solid and dashed curves correspond to the predicted $\pi^- p$ elastic cross section of CERN-EXP. and CERN-TH. phase shifts respectively. The arrows indicate the six energies chosen for differential cross section comparison as shown in Fig. 2.
2. $\pi^- p$ differential cross sections measured in this experiment at six sample energies. Those predicted by CERN-EXP. and CERN-TH. are shown as solid and dashed curves for comparison.
3. Total $\pi^- p$ cross section measurements by Carter et al. compared with those predicted by CERN-EXP. and CERN-TH.
4. Predicted $\pi^- p$ elastic cross sections of the Saclay, Berkeley and Glasgow solutions compared with the same data as shown in Fig. 1.
5. Predicted $\pi^- p$ differential cross sections of the Saclay, Berkeley and Glasgow solutions compared with the same data as shown in Fig. 2.
6. Predicted $\pi^- p$ total cross sections of the Saclay, Berkeley and Glasgow solutions compared with the same data as shown in Fig. 3.
7. (a) - (c) Partial wave absorption parameter, η, for D_{13}, D_{15}, and F_{15}.

   (d) - (f) Partial wave phase shift, δ, for D_{13}, D_{15}, and F_{15}.

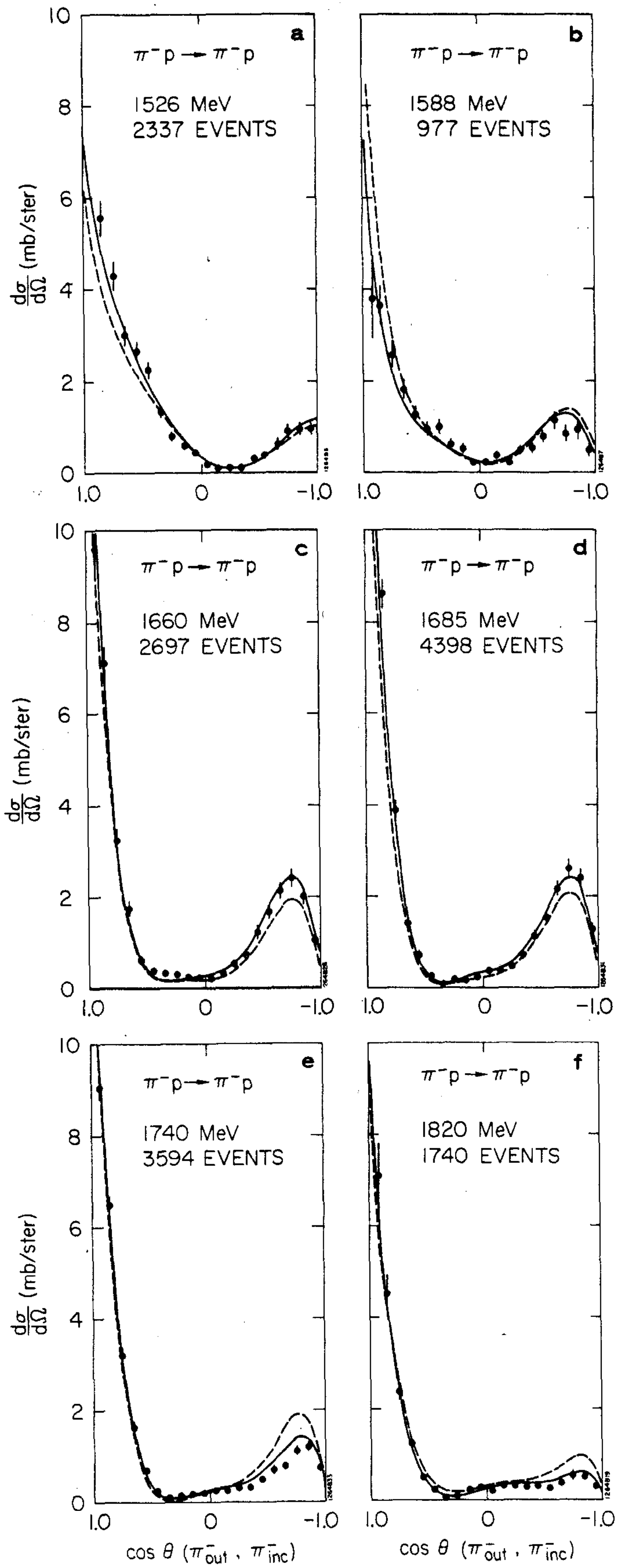
   (g) - (i) Partial wave $\pi^- p$ elastic cross section for D_{13}, D_{15}, and F_{15}.

   Those of CERN-EXP. are shown as dots connected by solid lines, and those of CERN-TH. are shown as dashed curves. The solid line represents both when they agree.



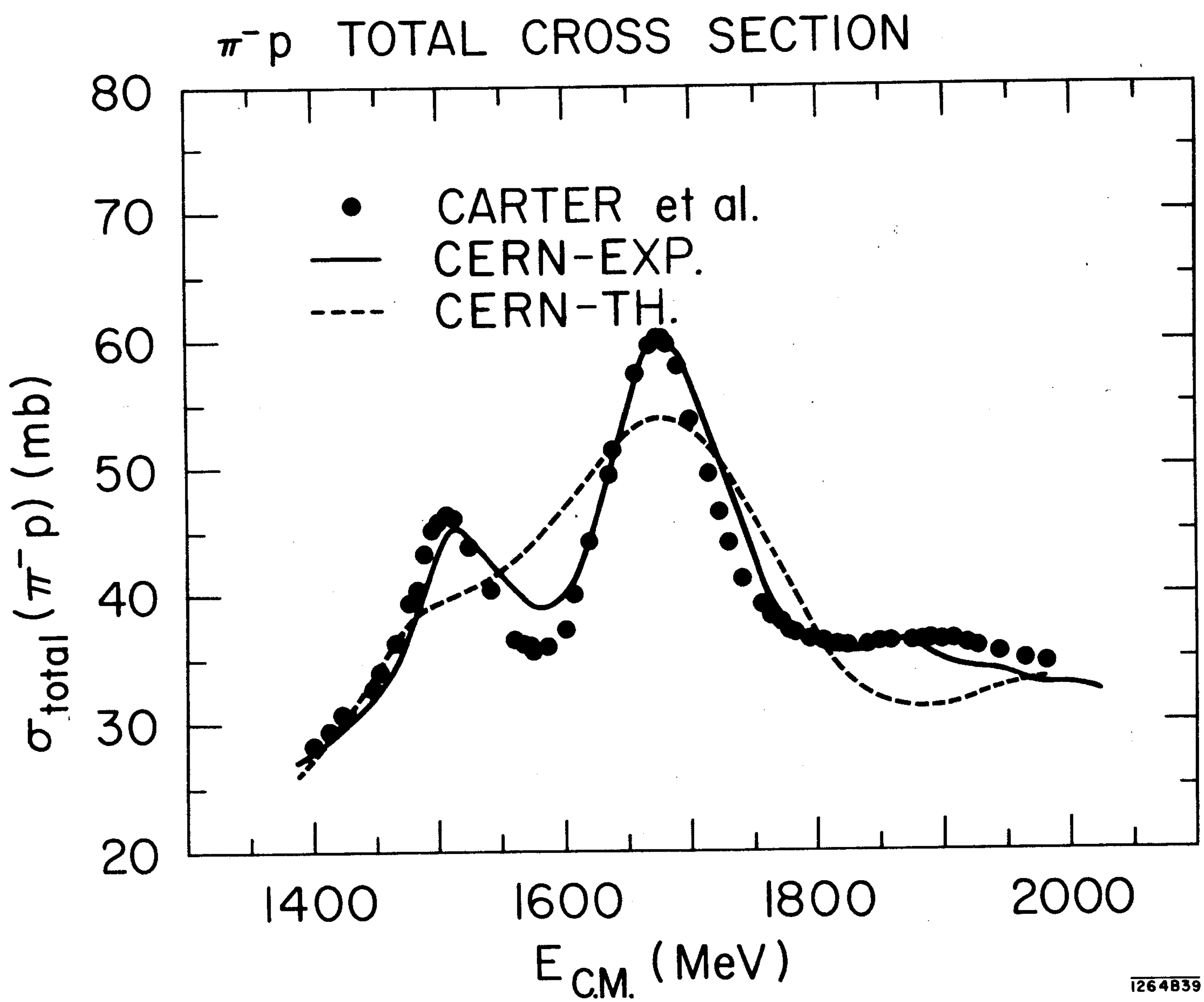
This Experiment (72" Alvarez HBC)
This Experiment (30" MURA HBC)
Duke et al.
Helland et al.
Ogden et al.
CERN – EXP.
CERN – TH.
σel (mb)
40
32
24
16
8
0
1300
1400
1500
1600
1700
1800
1900
2000
Ec.m. (MeV)
1264C36a

Fig. 1



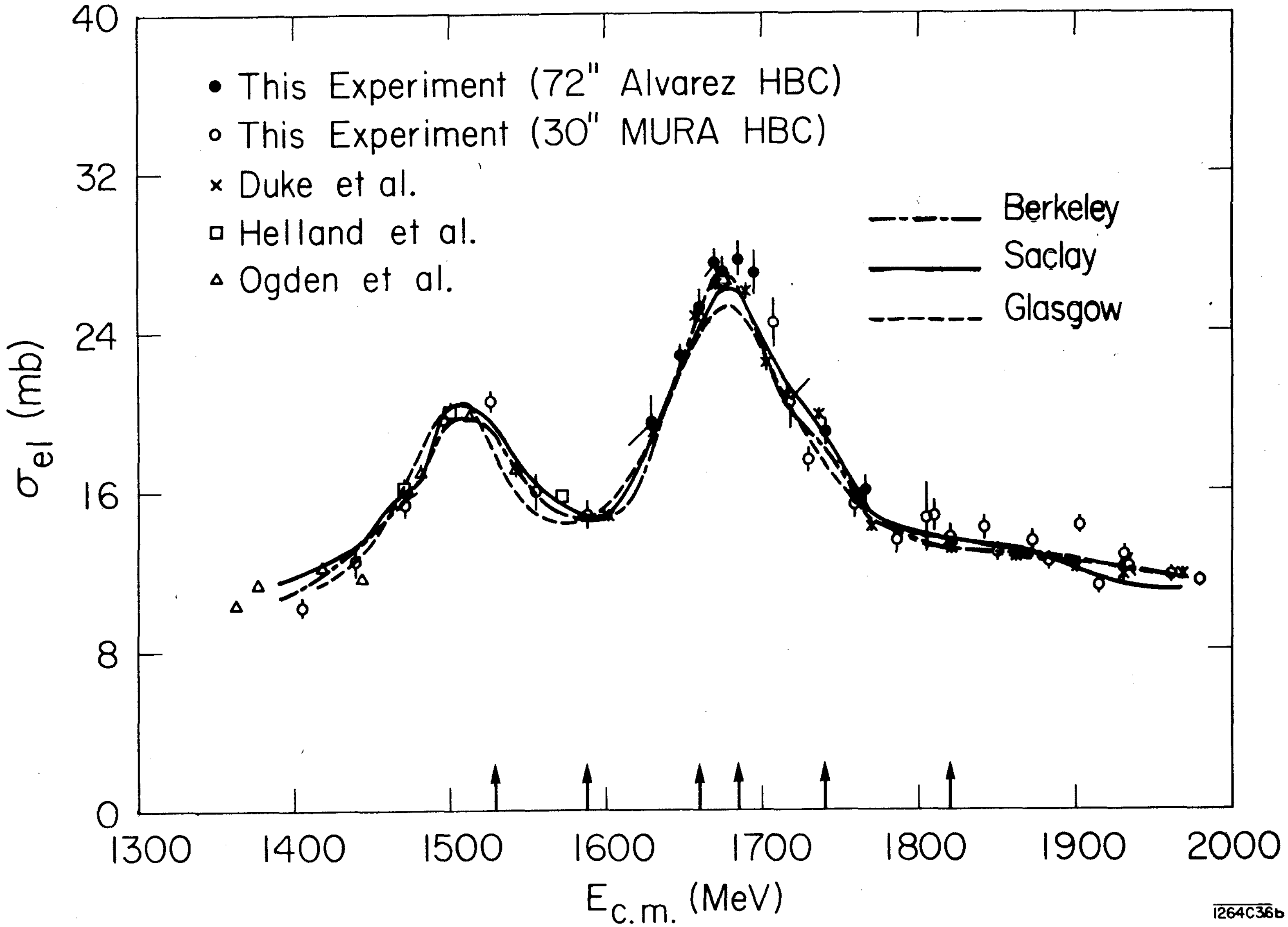
a
π⁻p → π⁻p
1526 MeV
2337 EVENTS
b
π⁻p → π⁻p
1588 MeV
977 EVENTS
c
π⁻p → π⁻p
1660 MeV
2697 EVENTS
d
π⁻p → π⁻p
1685 MeV
4398 EVENTS
e
π⁻p → π⁻p
1740 MeV
3594 EVENTS
f
π⁻p → π⁻p
1820 MeV
1740 EVENTS
dσ/dΩ (mb/ster)
cos θ (π⁻out , π⁻inc)

Fig. 2



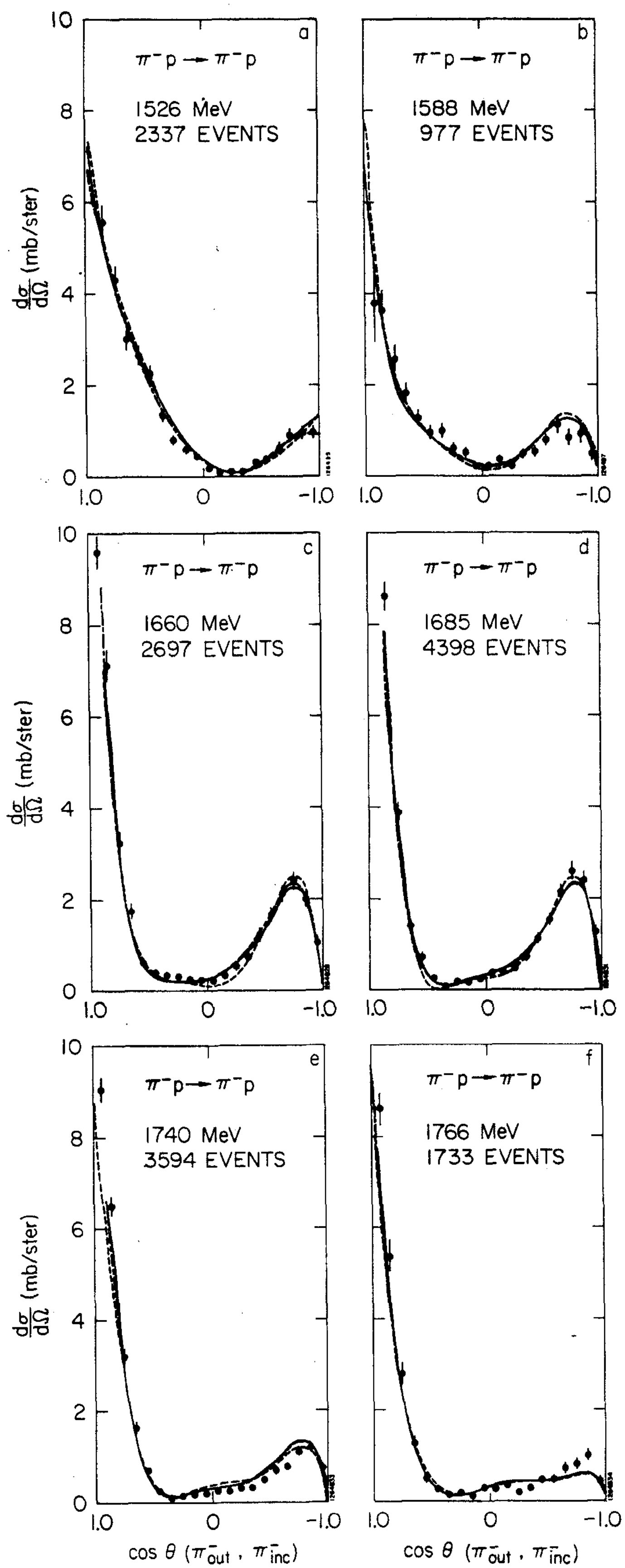
π⁻p TOTAL CROSS SECTION
80
70
60
50
40
30
20
CARTER et al.
CERN-EXP.
CERN-TH.
σ total (π⁻p) (mb)
1400
1600
1800
2000
E C.M. (MeV)
1264B39

Fig. 3



This Experiment (72" Alvarez HBC)
This Experiment (30" MURA HBC)
Duke et al.
Helland et al.
Ogden et al.
Berkeley
Saclay
Glasgow
σ_{el} (mb)
$E_{c.m.}$ (MeV)
0
8
16
24
32
40
1300
1400
1500
1600
1700
1800
1900
2000
1264C36b

Fig. 4



π⁻p → π⁻p
1526 MeV
2337 EVENTS
1588 MeV
977 EVENTS
1660 MeV
2697 EVENTS
1685 MeV
4398 EVENTS
1740 MeV
3594 EVENTS
1766 MeV
1733 EVENTS
a
b
c
d
e
f
dσ/dΩ (mb/ster)
cos θ (π⁻out , π⁻inc)

Fig. 5

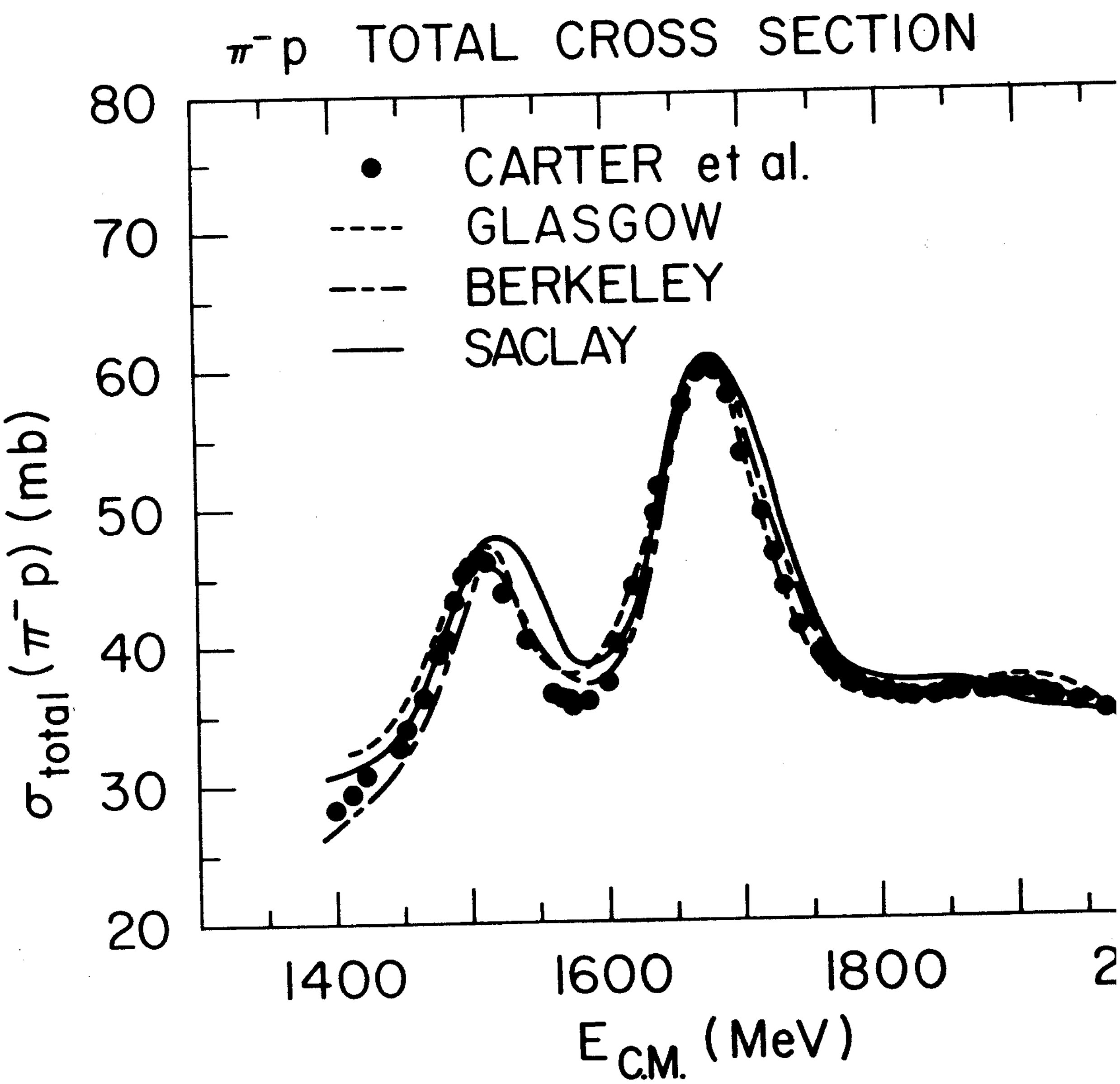

$\pi^- p$ TOTAL CROSS SECTION
CARTER et al.
GLASGOW
BERKELEY
SACLAY
80
70
60
50
40
30
20
$\sigma_{total}(\pi^- p)$ (mb)
1400
1600
1800
2
$E_{C.M.}$ (MeV)

Fig. 6

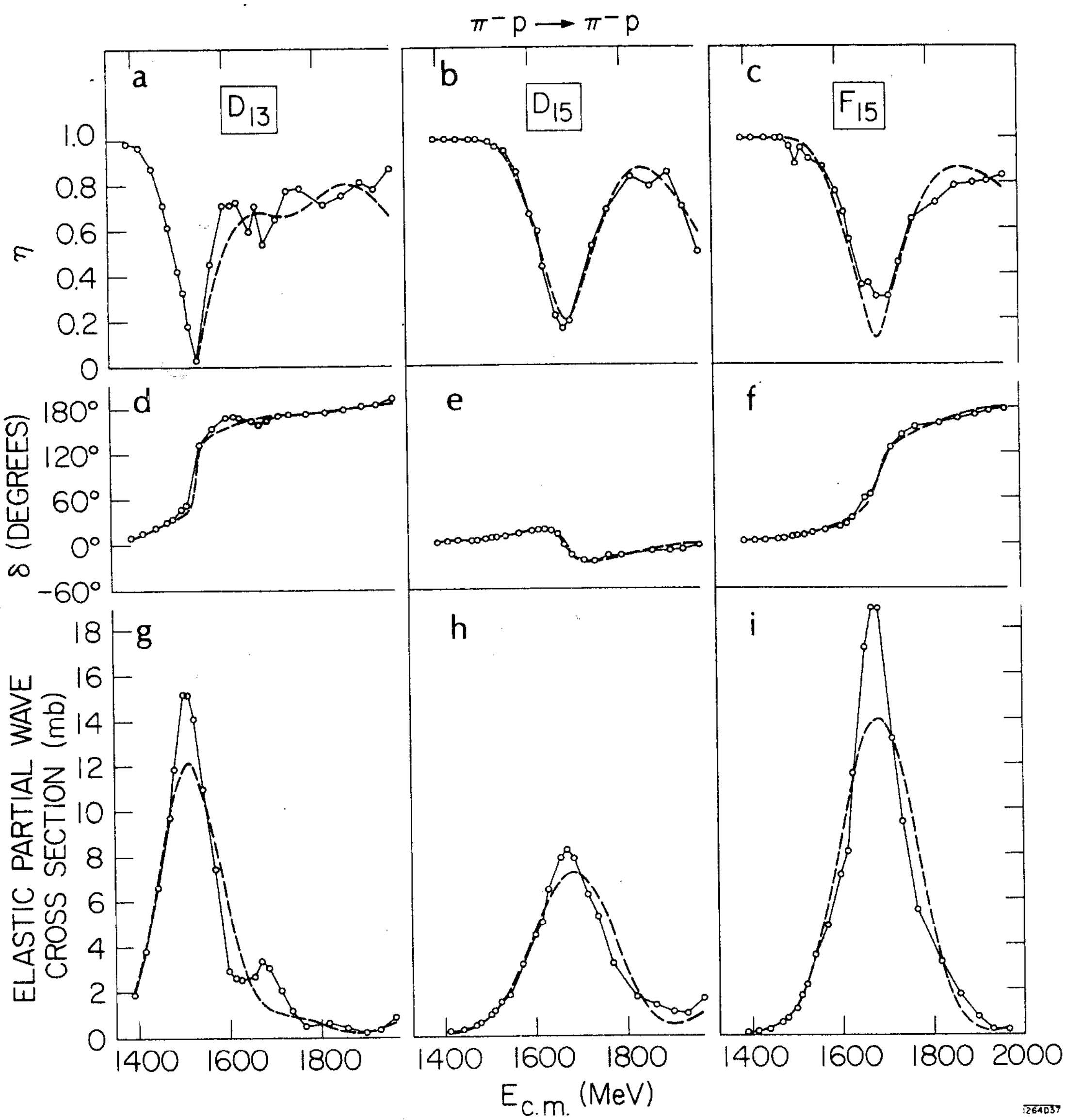

π⁻p → π⁻p
a
D13
b
D15
c
F15
η
1.0
0.8
0.6
0.4
0.2
0
d
e
f
δ (DEGREES)
180°
120°
60°
0°
-60°
g
h
i
ELASTIC PARTIAL WAVE CROSS SECTION (mb)
18
16
14
12
10
8
6
4
2
0
1400
1600
1800
2000
E c.m. (MeV)

Fig. 7