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AGREEMENT OF THE VECTOR DOMINANCE MODEL IN A COMPLETE TRANSVERSALITY SYSTEM WITH POLARIZED AND UNPOLARIZED PHOTOPRODUCTION DATA*

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ABSTRACT

Agreement with the Vector Dominance Model predictions are obtained in a system where the pion produced vector mesons are subjected to a complete transversality projection. In this system the admixture of polarizations from the element ρ_{10}^{v} are suppressed by a dynamical unitary transformation on the helicity frame spin density matrix. The tests are applied to pion photoproduction data by polarized and unpolarized photons; also an independent search is made for the γ-ρ coupling constant at $q^2 = 0$, yielding a value of $\gamma_{\rho}^{2}/4\pi = 0.40 \pm 0.03$.

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Recently, apparent serious discrepancies have been shown[1-3] to exist in the validity of the Vector Dominance Model (VDM) predictions from the reaction

$$\pi^- p \rightarrow \rho^o n \tag{1}$$

as compared with polarized photoproduction data from the reactions

$$\gamma p \rightarrow \pi^+ n \quad \text{and} \quad \gamma n \rightarrow \pi^- p \tag{2}$$

In these and other[4] comparisons, the transversality of the ρ^o meson from reaction (1) is imposed by evaluating the spin density matrix elements in the ρ^o's helicity reference frame. Whereby, the transverse components of the ρ^o polarization is projected by the factor of $\rho^H_{11}(t)$ multiplying $d\sigma/dt(\pi^- p \rightarrow \rho^o n)$.

In view of the spin rotation properties of massive particles under Lorentz transformations, theoretical considerations[5] have shown that the unique choice of the ρ^o helicity reference frame, *a priori*, is not well justified. That is for VDM applications a criterion of choice on the quantization axis, which lies in the production plane of reaction (1), does not exist free of additional assumptions.

It has been shown[6] that the combination of spin density matrix elements of $\rho_{11} + \rho_{1-1}$ describes a vector particle's transverse polarization state perpendicular to the production plane and, similarly, $\rho_{11} - \rho_{1-1}$ describes the state with transverse polarization parallel to the production plane. As such, the pion photoproduction polarization asymmetry $\Sigma(t) = (\sigma_\perp - \sigma_\parallel)/(\sigma_\perp + \sigma_\parallel)$ from reactions (2) can be combined to yield a VDM relationship free of ω^o-ρ^o interference term[7] by:

$$A(t) = \frac{\Sigma^+(t) + R(t)\,\Sigma^-(t)}{1 + R(t)} = \frac{\rho_{1-1}(t)}{\rho_{11}(t)} \tag{3}$$

where, R(t) is the π^- to π^+ unpolarized photoproduction cross-sectional ratio.

Białas and Zalewski have observed[9] that the ratio ρ_{1-1}/ρ_{11} from reaction (1) achieves a maximal value in the Donohue-Högaasen reference frame. In this system the decay intensity distribution of the vector particle, $I(\theta, \phi) = \langle y, Sy \rangle$, is reduced to a quadratic form; S being the hermitian spin density operator acting on y, the basis spin vector with spin eigenfunction components. The reduction to quadratic form has been obtained[10] by Donohue and Högaasen through a dynamical unitary transformation which, for example, rotates the helicity frame about the normal to the production plane by an angle ψ, given by $\tan 2\psi = -2\sqrt{2}\,\mathrm{Re}\,\rho_{10}/(\rho_{00}-\rho_{11}+\rho_{1-1})$, and which diagonalizes the spin density matrix to yield a vector particle intensity distribution in terms of the three real eigenvalues. One of these eigenvalues is the quantity: $\rho_{11}+\rho_{1-1}$ and, in this rotated system $\mathrm{Re}\,\rho'_{10}=0$. It should also be noted that the strict requirement of $\mathrm{Re}\,\rho'_{10}=0$ is incidental to the diagonalization of the spin density matrix.

In reference (9) the data of reaction (1) has not been handled directly to yield fitted density matrix elements in the dynamically rotated system; and thus to account for the important error correlations. Also, the large s-wave interference terms have not been abstracted from the ρ^o region, in a similarly direct fashion.

A posteriori to the Białas and Zalewski successful VDM tests, in this letter we present our arguments for a physical justification of a complete transversality system; develop a general ρ^o decay intensity distribution with the s-wave interference terms in this system; and, also, apply the data of a large compilation[11] of reaction (1) near 4.0 GeV to test the Vector Dominance Model at $q^2 = 0$ with polarized[12,13] and unpolarized[14-16] photoproduction data of reactions (2) at 3.4 and 5.0 GeV, q^2 is the photon mass.

At $q^2 = 0$ a γ-to-ρ^o transition can occur only on ρ^o spin density states of ρ^v_{ij}, which describe purely transverse polarizations. The massless photon's transverse polarizations allow for the presence of photon spin densities of ρ_{11}, ρ_{-1-1} and only the admixture of states ρ_{1-1}. However, a massive vector particle, such as the ρ^o produced in reaction (1), has transverse and longitudinal polarizations; here, not only the spin densities of ρ_{11}, ρ_{-1-1} with the admixture of ρ_{1-1} are allowed to be present, but in addition, the state of ρ_{00} and the admixture of transverse-longitudinal state of ρ_{10} participate. In order to apply VDM at $q^2 = 0$, we must project out of the ρ^o vector-meson not more than the entire photon polarization properties. Thus the transversality projection on the ρ^o vector-meson must be described in such a system which does not allow for the presence of a quantum-mechanical admixture of states from transverse to longitudinal ρ^o polarizations; that is, a system where the spin density element of $\rho_{10} = 0$. This requirement needs to be imposed only at $q^2 = 0$.

Our argument is eminently supported by the measurement of the spin density element ρ_{10} in a reaction where a direct γ-to-ρ^o transition occurs at $q^2 = 0$; the reaction is: $\gamma p \to \rho^o p$. A detailed measurement[17] of this reaction, both as a function of incident photon energy and in the helicity or Jackson systems, yields a value for the ρ^o's density matrix element of $\rho_{10} \approx 0$.

Hence, it becomes clear that the Donohue-Högaäsen system for VDM applications is fortuitous. Specifically for VDM applications, the requirement is a general unitary transformation which suppresses the element ρ_{10} in the vector-meson's spin density matrix, but which need not necessarily diagonalize this matrix.

Since the ρ^o in reaction (1) is produced with considerable s-wave interference terms, we start with a 4×4 spin density matrix S, which describes a vector-meson along with a competing resonant or non-resonant scalar component. This real self-adjoint matrix is:

$$S = \begin{pmatrix} \rho_{00}^{s} & \rho_{10}^{int} & \rho_{00}^{int} & -\rho_{10}^{int} \\ \rho_{10}^{int} & \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{00}^{int} & \rho_{10} & \rho_{00} & -\rho_{10} \\ -\rho_{10}^{int} & \rho_{1-1} & -\rho_{10} & \rho_{11} \end{pmatrix} \tag{4}$$

The dipion decay intensity distribution in the ρ^o region is given by $I(\theta, \phi) = \langle y, Sy \rangle$, where the basis vector $y = \left\{ Y_0^0, Y_1^1, Y_1^0, Y_1^{-1} \right\}$. The spherical harmonics $Y_\ell^m(\theta, \phi)$ are given as a function of the polar and azimuthal decay angles θ and ϕ, obtained in the dipion rest frame with the helicity quantization axis. The decay intensity distribution as obtained from the above scalar product reduces to the familiar form (cf. reference (4), Eq. (6)). After a unitary transformation U, this distribution is expressed by $I(\theta, \phi, \psi) = \langle \eta, S'\eta \rangle$, where $S' = USU^{-1}$ and $\eta = Uy$. The explicit form[18] of this transformation can be summarized in terms of the orthonormal $d^j_{\lambda\mu}(\psi)$ functions, from the rotation group representation. Thus, the vector part of the density matrix elements are transformed by $\rho'_{\lambda\mu} = \sum_{ij} d^1_{\lambda i}(\psi)\rho_{ij} d^1_{j\mu}(-\psi)$. The desired requirement, where the transformed $\rho'_{10} = 0$, is satisfied by the same condition on ψ as in the Donohue-Högaasen system.

It can be shown, that one of the four real eigenvalues of S', the transformed spin density matrix, is again the quantity $\rho'_{11} + \rho'_{1-1}$. The invariance property of this quantity is explicitly tested below. Hence, the new ρ^o decay intensity distribution is given in terms of the helicity angles θ and ϕ, and the dynamical rotation angle ψ, by:

$$\begin{aligned} I(\theta,\phi,\psi) = (4\pi)^{-1}\Big\{1 &+ (\rho_{11}-\rho_{00})\, 1/2 \Big[(1-3\cos^2\theta)(3\cos^2\psi-1) \\ &+ 3\sin 2\psi \sin 2\theta \cos\phi - 3\sin^2\psi \sin^2\theta \cos 2\phi\Big] - \rho_{1-1}\, 3/2 \Big[(1+\cos^2\psi)\sin^2\theta\cos 2\phi \\ &+ \sin 2\psi \sin 2\theta \cos\phi + \sin^2\psi(3\cos^2\theta - 1)\Big] - 2\sqrt{6}\,\rho^{int}_{10}\Big[\cos\psi\sin\theta\cos\phi \\ &+ \sin\psi\cos\theta\Big] + 2\sqrt{3}\,\rho^{int}_{00}\Big[\cos\psi\cos\theta - \sin\psi\sin\theta\cos\phi\Big]\Big\} \end{aligned} \tag{5}$$

where the trace normalization condition of $\rho_{00} + 2\rho_{11} + \rho^s_{00} = 1$ has been used. We note that at $\psi = 0$, this distribution reduces to the familiar form but without the ρ_{10} term (cf. reference (4), Eq. (6)).

A detailed fitting procedure has been followed on the data of reaction (1), to solve for the five unknown parameters in Eq. (5). In terms of ρ^o production, the data has been divided in intervals of $\cos\theta_{c.m.}$, such that in the parameters determination the data would participate in equal parts of statistical significance. The true ρ^o production cross section as a function of $\cos\theta_{c.m.}$ is known from a previous study,[4] where reflections of isobar resonances are handled properly in the ρ^o resonance fit along with phase space. We have used the program MINFUN to minimize the negative logarithm of the likelihood function from Eq. (5), and thus, to solve for the dynamical rotation angle ψ and the new spin density matrix elements. Several passes were made through the program's parameter search and convergence modes. The obtained solutions for the five parameters were perturbed and recycled to test their validity. The analytic gradients of the likelihood function was supplied to the program, to achieve a more accurate parameter convergence. For a final validity check, the integrated distributions $I(\theta, \psi)$ and $I(\phi, \psi)$ were computed and compared with the data.

Figure 1(a) exhibits the behavior[19] of the dynamical rotation angle ψ, as obtained from our fitting procedure. The goodness of our solutions is manifested in Fig. 1(b), where ϵ, the ratio of the eigenvalue[20] $\rho_{11} + \rho_{1-1}$ in the transversality over the eigenvalue in the helicity system, is given. As it is expected, this ratio averages around unity. The helicity system spin density matrix elements were obtained from a previous study.[4]

Figure 2(a) exposes the deficiency of the conventional helicity frame, specifically in VDM comparisons of reaction (1). In VDM, the ρ^o spin density matrix element ratio ρ_{1-1}/ρ_{11} is a direct measure of $A(\pi^+, \pi^-)$, the π^+-π^--averaged polarization asymmetry in photoproduction. Contrary to photoproduction experiments[12,13], large negative values for $A(\pi^+, \pi^-)$ are predicted in the helicity system. Figure 2(b) displays the ratio ρ_{1-1}/ρ_{11} in the proposed transversality system, which results to a remarkable agreement of VDM with currently available[12,13,21] knowledge on $A(\pi^+, \pi^-)$.

The π^+-π^--averaged photoproduction cross section, from reactions (2), is predicted by a VDM expression. With the assumptions of time-reversal invariance and isospin conservation, this expression is given by:

$$\frac{\alpha}{4} \left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1} \left(\frac{p_{\pi^-}}{p_\rho}\right)^2_{\mathrm{c.m.}} \frac{E(s_\gamma)}{E(s_\rho)} \; P_{\rho^o}(t) \frac{d\sigma}{dt} (\pi^- p \to \rho^o n) \tag{6}$$

where, $\gamma_\rho^2/4\pi$ is the γ-to-ρ^o coupling constant at $q^2 = 0$, $E(s)$ is the cross-sectional energy dependence[22] of reaction (1), and $P_{\rho^o}(t)$ is the ρ^o transverse polarization projection. For unpolarized (perpendicularly polarized) photoproduction cross sections,[23] the projection is $\rho_{11}(t), \left(\rho_{11}(t) + \rho_{1-1}(t)\right)$, evaluated in the complete transversality system. Our VDM points carry errors propagated from uncertainties in the reaction (1) cross section[4] and in the determinations of $\rho_{1-1}(t)$ and $\rho_{11}(t)$; moreover, an error of 0.03, in the fitted value of $\gamma_\rho^2/4\pi$ is included.

Figure 3(a) shows the momentum transfer distribution of perpendicularly polarized photoproduction cross sections at 3.4 GeV, in comparison with the VDM prediction from expression (6). The goodness of this comparison with the abstracted[23] $\sigma_{\perp}^{\pm}$ points is apparent. To enhance the validity of VDM comparisons further, we suggest that absolute cross sections of reactions (2) be measured with linearly polarized photons near 4.0 GeV. Equally good VDM comparisons are obtained with unpolarized photoproduction data at 3.4 and 5.0 GeV, as shown in Fig. 3(b) and 3(c), respectively.

An independent evaluation of the γ-ρ^{o} coupling constant is obtained by treating $\gamma_{\rho}^{2}/4\pi$ in expression (6), as an unknown parameter. In the above three cases, separate fits are made between the smoothed photoproduction cross sections and the VDM points. The average value from these fits, yields a γ-ρ^{o} coupling constant at $q^{2} = 0$, to be: $\gamma_{\rho}^{2}/4\pi = 0.40 \pm 0.03$. Our results contradict the negative conclusion of a recent VDM comparison[24] of the type similar to Fig. 3(a).

ACKNOWLEDGEMENT

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REFERENCES

1. C. Geweniger et al., Phys. Rev. Letters 28B, 155 (1968).
2. R. Diebold and J. A. Poirier, Phys. Rev. Letters 22, 255 (1969).
3. L. J. Gutay et al., Phys. Rev. Letters 22, 424 (1969).
4. I. Derado and Z. G. T. Guiragossián, Phys. Rev. Letters 21, 1556 (1968).
5. H. Fraas and D. Schildknecht, Nucl. Phys. B6, 395 (1968).
6. M. Krammer and D. Schildknecht, Nucl. Phys. B7, 583 (1968).
7. In this and the following VDM relationships the contributions from ω^o and ϕ^o terms are neglected. We have shown elsewhere,[8] that in a VDM relationship at $q^2 = 0$ and photon energies 2-18 GeV, the ω^o contribution in amplitude is $\sim$15% and that of the ϕ^o is $\sim$5%. Hence, in these cross-sectional relations free of ω^o-ρ^o interference, neglect of the ω^o term corresponds to a 2% correction.
8. Z. G. T. Guiragossián and A. Levy, "The γ-ρ^o Coupling Constant, Compton Scattering, and Total Hadronic γ-p Cross Sections," Report No. SLAC-PUB 535, Stanford Linear Accelerator Center, Stanford University (1968) (to be published).
9. A. Białas and K. Zalewski, Phys. Letters 28B, 436 (1969).
10. J. T. Donohue and H. Högaasen, Phys. Letters 25B, 554 (1967).
11. A sample of 7984 events of the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ was compiled as part of a Stanford Linear Accelerator-Purdue-Notre Dame collaboration (cf. reference 4); see, D. H. Miller, L. J. Gutay, P. B. Johnson, V. P. Kenney and Z.G.T. Guiragossián, Phys. Rev. Letters 21, 1489 (1968) and P. B. Johnson et al., Phys. Rev. 176, 1651 (1968).
12. C. Geweniger et al., Phys. Letters 29B, 41 (1969).
13. Z. Bar-Yam, J. P. Dowd, J. de Pagter and W. Kern, (to be published); and Z. Bar-Yam (private communication).
14. Z. Bar Yam, et al., Phys. Rev. Letters 19, 40 (1967).
15. A. M. Boyarski et al., Phys. Rev. Letters 20, 300 (1968).

16. P. Heide et al., Phys. Rev. Letters 21, 248 (1968).

17. Aachen-Berlin-Bonn-Hambury-Heidelberg-München Collaboration, Phys. Rev. 175, 1669 (1968).

18. J. T. Donohue, Brookhaven National Laboratory (private communication).

19. The dynamical angle ψ, which suppresses ρ_{10}, has a behavior which may be particularly connected to the relative strength and phase of two opposite-parity exchanges in reaction (1), i.e. π and A_2. One of us (Z. G. T. G.) would like to thank Prof. R. Blankenbecler for a valuable discussion on this problem. Also, cf. reference 22, below.

20. In Eq. (5) one of the fitted parameters is $(\rho_{11}-\rho_{00})$, the decoupling of which can be achieved only by the trace condition and an independent knowledge of ρ^{s}_{00}. A theoretical estimation of $\rho^{s}_{00} = 0.072$ is used according to the work of L. Durand, III and Y. T. Chiu, Phys. Rev. Letters 14, 329 (1965) and private communication to one of us (Z. G. T. G.).

21. As indicated, we have used a theoretical estimate on $\Sigma^{-}(\gamma n \rightarrow \pi^{-} p)$ from a study fitting well measurements of $\Sigma^{+}(\gamma p \rightarrow \pi^{+} n)$ in the range $0 < |t| < 0.4 (\mathrm{GeV}/c)^2$. J. Frøyland and D. Gordon, "Cut Conspiracy and Pion Evasion in π^{+} and π^{-} Photoproduction," MIT Report CTP-38 (1968) (to be published). We thank Dr. D. Gordon for several helpful discussions and a private communication.

22. In the range of 2 to 16 GeV/c for reaction (1), the cross-sectional energy dependence is given by a fit to be: $\sigma(s) = 340\ s^{-3} + s^{-0.6}$. This form is suggestive to π and A_2 exchanges taking place. G. Bellini et al., Nuovo Cimento 53A, 798 (1968).

23. $\sigma^{\pm}_{1}$ is abstracted from the polarization asymmetry $\Sigma^{\pm}$, with the unpolarized photoproduction cross section $\sigma^{\pm}_{0}$, by $\sigma^{\pm}_{1} = \sigma^{\pm}_{0}(1 + \Sigma^{\pm})$. $\sigma^{-}_{0} = R\sigma^{+}_{0}$, where R is the π^{-}/π^{+} unpolarized photoproduction ratio on Deuterium. In resonable cases, we have extrapolated R and σ_0 in t.

24. R. Diebold and J. A. Poirier, Phys. Rev. Letters 22, 906 (1969). The authors here do not include a time reversal factor of $(p_{\pi^-}/p_\rho)^2_{c.m.}$ (cf. Reference 6 above), their VDM prediction uses a value of $\gamma_\rho^2/4\pi = 0.52$ instead of our fitted value of 0.40.

FIGURE CAPTIONS

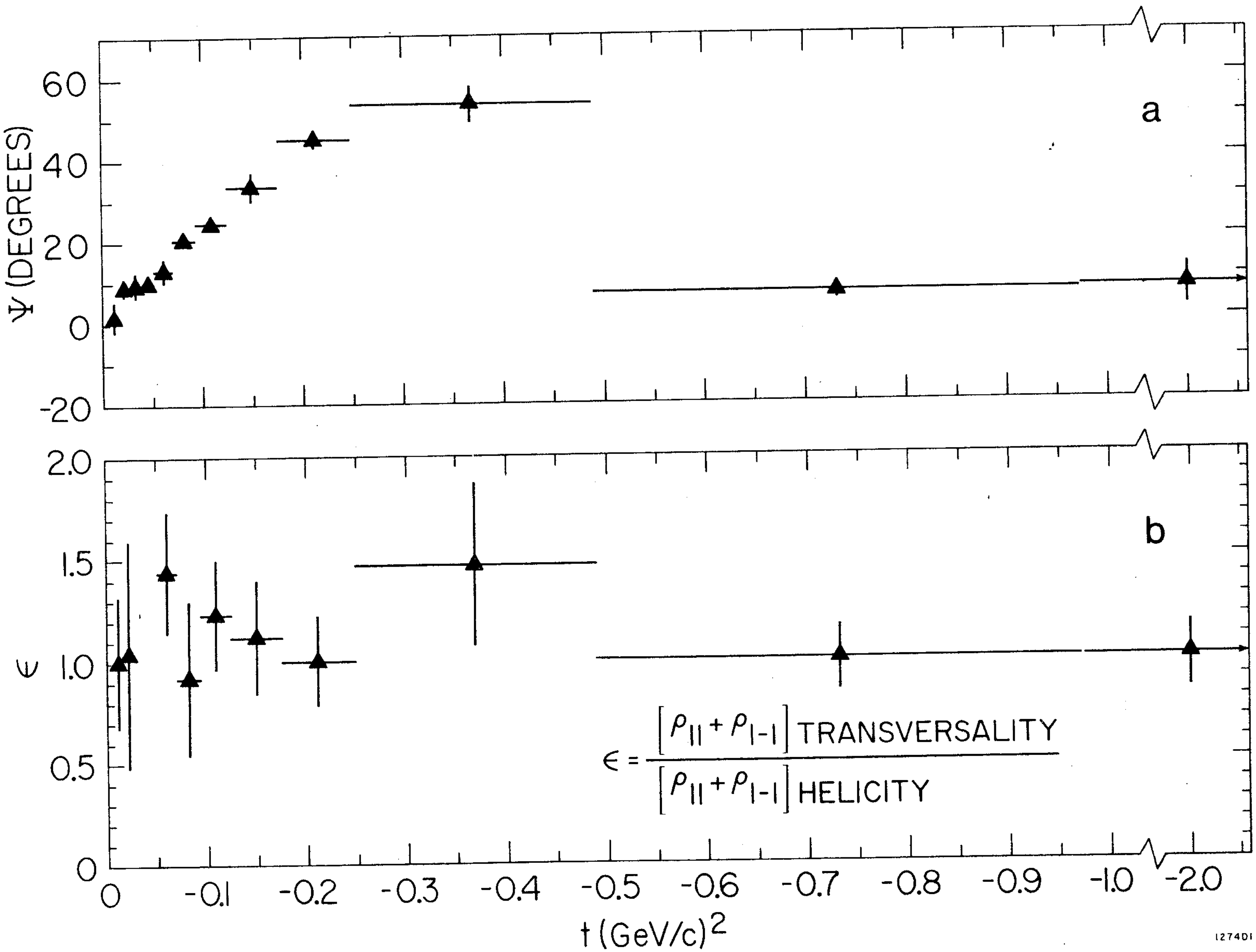
1. (a) Behavior of the transversality condition's dynamical rotation angle ψ, from a unitary transformation which suppresses the ρ^o-meson spin density matrix element ρ_{10}, in $\pi^- p \rightarrow \rho^o n$.

   (b) Comparison of the eigenvalue $\rho_{11} + \rho_{1-1}$ in the transversality and helicity systems. t is the momentum transfer of pion-produced ρ^o's.

2. (a) ρ^o spin density matrix element ratio ρ_{1-1}/ρ_{11} evaluation in the conventional helicity system.

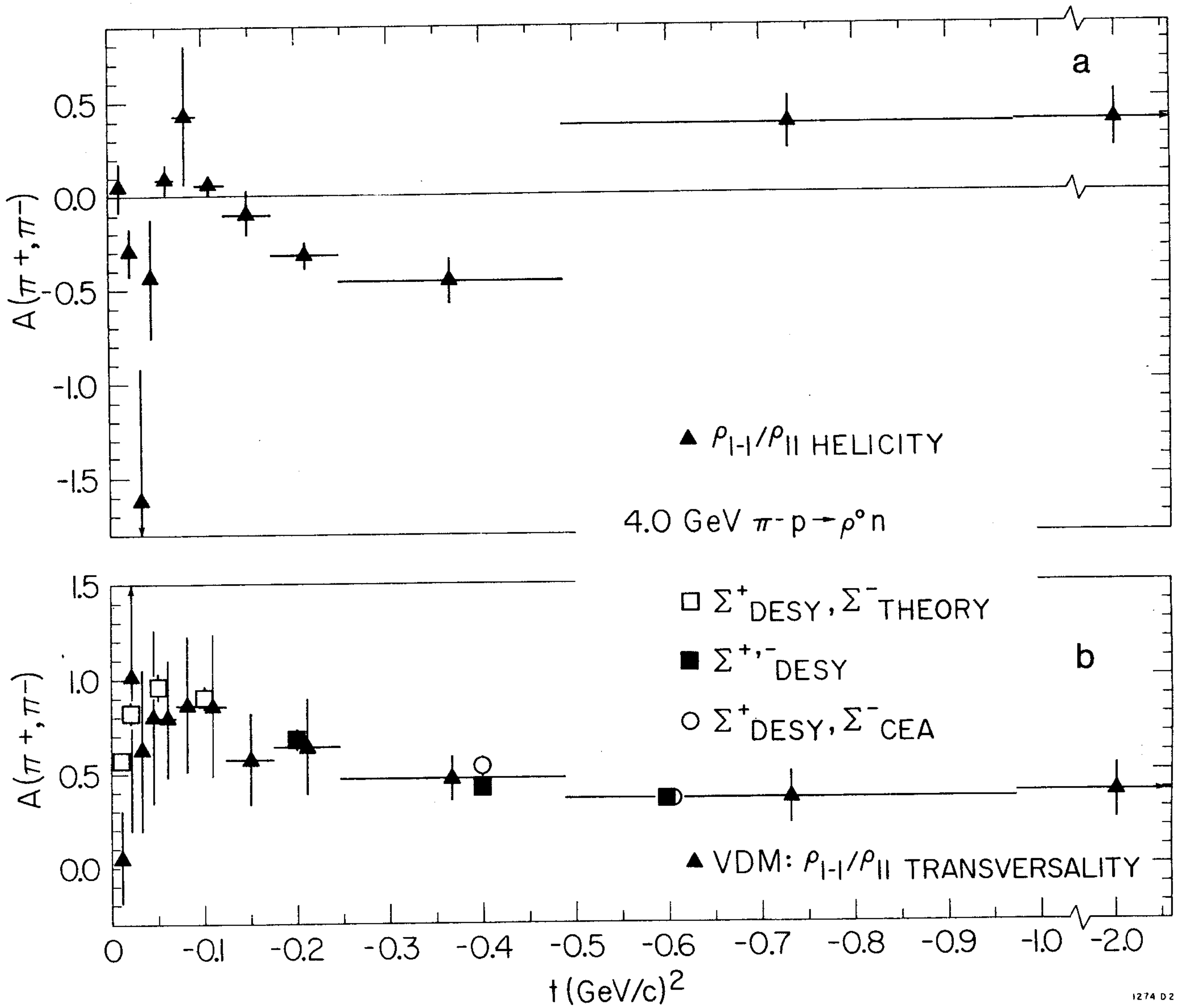
   (b) VDM prediction of $A(\pi^+, \pi^-)$, the π^+-π^- - averaged photoproduction polarization asymmetry. ρ_{1-1}/ρ_{11} of ρ^o's, from 4.0 GeV $\pi^- p \rightarrow \rho^o n$ data, evaluated in the proposed transversality system, for VDM comparison with current experiments (CEA, DESY) and theoretical calculation (Frøyland and Gordon).

3. Complete transversality system comparison of VDM predictions, using the average of the best fit coupling constant, with π^+-π^- - averaged photoproduction data; (a) at 3.4 GeV, by photons linearly polarized perpendicular to the production plane, (b) at 3.4 GeV and (c) at 5 GeV, by unpolarized photons.



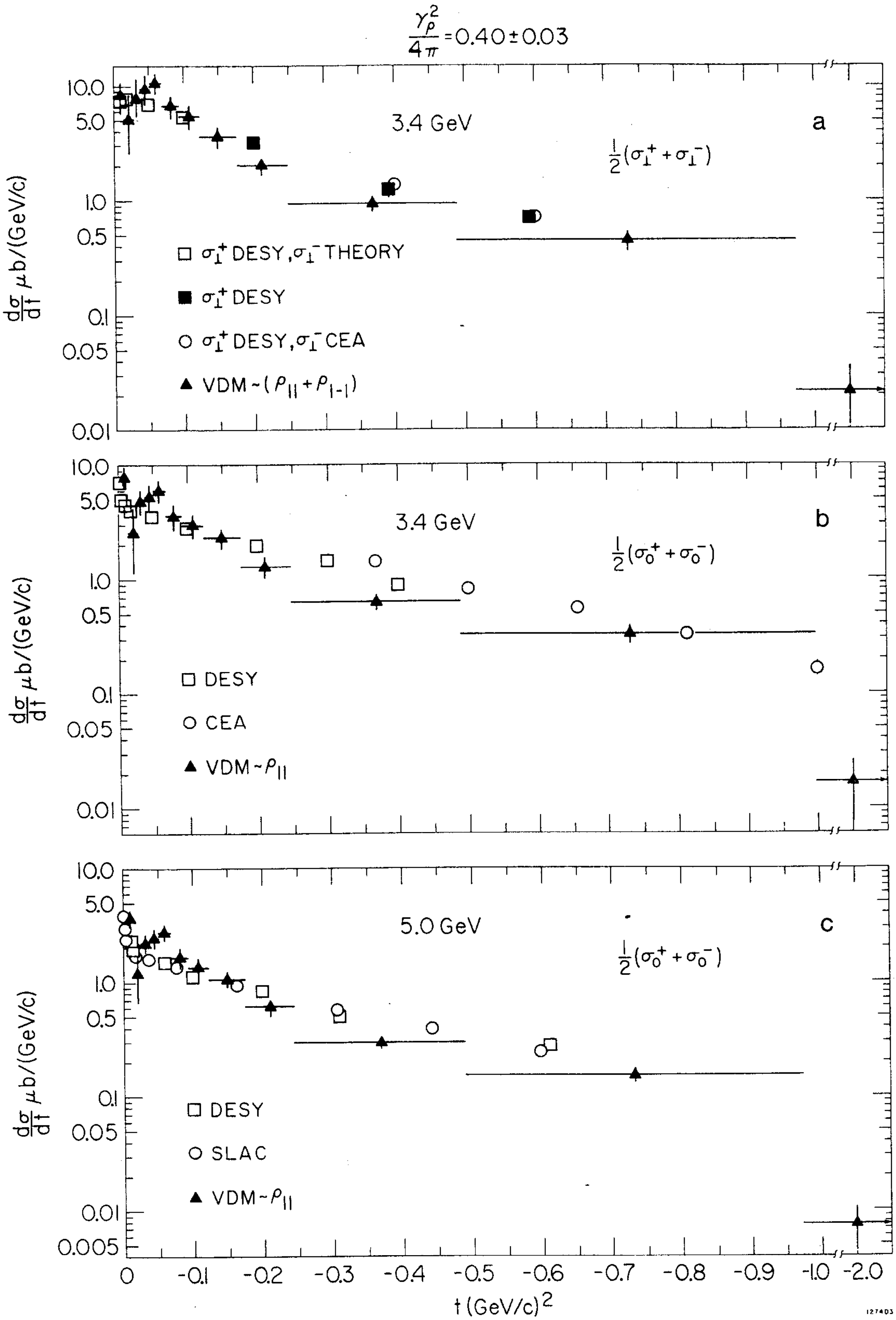
60
40
20
0
-20
Ψ (DEGREES)
a
2.0
1.5
1.0
0.5
0
ε
b
$\epsilon = \frac{[\rho_{11}+\rho_{1-1}]\ \text{TRANSVERSALITY}}{[\rho_{11}+\rho_{1-1}]\ \text{HELICITY}}$
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
-2.0
t (GeV/c)^2
1274D1

Fig. 1



a
b
A(π+, π−)
▲ ρ1-1/ρ11 HELICITY
4.0 GeV π− p → ρ° n
□ Σ+DESY, Σ−THEORY
■ Σ+,−DESY
○ Σ+DESY, Σ−CEA
▲ VDM: ρ1-1/ρ11 TRANSVERSALITY
t (GeV/c)2
1274 D2

Fig. 2



$\frac{\gamma_\rho^2}{4\pi} = 0.40 \pm 0.03$
3.4 GeV
a
$\frac{1}{2}(\sigma_\perp^+ + \sigma_\perp^-)$
$\sigma_\perp^+$ DESY, $\sigma_\perp^-$ THEORY
$\sigma_\perp^+$ DESY
$\sigma_\perp^+$ DESY, $\sigma_\perp^-$ CEA
VDM ~ $(\rho_{11} + \rho_{1-1})$
3.4 GeV
b
$\frac{1}{2}(\sigma_0^+ + \sigma_0^-)$
DESY
CEA
VDM ~ ρ_{11}
5.0 GeV
c
$\frac{1}{2}(\sigma_0^+ + \sigma_0^-)$
DESY
SLAC
VDM ~ ρ_{11}
$\frac{d\sigma}{dt}\ \mu b/(GeV/c)$
t (GeV/c)^2
127403

Fig. 3