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FIRST ORDER RENORMALIZATION OF THE WEAK MAGNETIC FORM-FACTOR DUE TO SYMMETRY BREAKING*

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ABSTRACT

A sum rule similar to that of Fubini, Furlan and Rossetti is used to show that the weak magnetic form-factor in strangeness changing semi-leptonic hyperon decays is renormalized to first order in the symmetry breaking Hamiltonian. This is in contrast to the Ademello-Gatto theorem which states that there is no such renormalization in the case of the strangeness changing charge form-factor.

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There is increasing experimental interest in the measurement of the weak magnetic form-factor in semi-leptonic hyperon decays.[1] This paper will argue that we should not expect very close agreement of these form-factors with their SU(3) symmetry values.

We start by considering the matrix element between the baryon octet states $|B(\vec{p})\rangle$ and $|B'(\vec{p})\rangle$ of the i'th component of the SU(3) vector current;

$$\langle B'(p')|J^{i}_{\mu}(X)|B(\vec{p})\rangle = e^{iq\cdot X} i\bar{\mu}(\vec{p}) \left[G_V^{B'B}(q^2)\gamma\mu + q\mu\sigma\mu\nu G_M^{B'B}(q^2) \right] \mu(\vec{p}) \qquad (1)$$

$$q \equiv p' - p$$

The Ademello-Gatto theorem[2] states that the charge form factor $G_V^{B'B}(0)$ is renormalized to no lower order in λ than λ^2 where the total Hamiltonian is given by $H = H_0 + \lambda H'$ with λ small,[3] and where H_0 conserves SU(3) invariances. Our problem is to see if a similar theorem holds for the weak magnetic form factor $G_M^{B'B}(0)$.

We derive our result from the commutation relation:

$$\left[Q^{4+15}_{t=0} ; J_0^{4-15}(\vec{X})_{t=0} \right] = J_0^3(\vec{X}) + \sqrt{3}\, J_0^8(\vec{X}) \equiv J_0^{3+3\sqrt{8}}(\vec{X}) \qquad (2)$$

Where Q^u is defined as $\int d^3\vec{X} J_0^j(\vec{X})$. We insert this commutator between the baryon states $\langle B'(p)|$ and $|B(\vec{p})\rangle$ and insert a complete set of intermediate states in the commutator yielding;[4]

$$\langle B'|(p')|J_0^{3+3\sqrt{8}}(\vec{X})|B(\vec{p})\rangle = \sum_{\alpha} \int \frac{d^3k}{(2\pi)^3} \langle B'(\vec{p}')|Q^{4+15}|\alpha(\vec{k})\rangle\langle\alpha(\vec{k})|J_0^{4-15}(\vec{x})|B(\vec{p})\rangle$$

$$-\sum_{\alpha} \int \frac{d^3k}{(2\pi)\beta} \langle B'(\vec{p}')|J_0^{4-15}(\vec{X})|\alpha(\vec{k})\rangle\langle\alpha(\vec{k})|Q^{4+15}|B(\vec{p})\rangle$$

(3)

On separating out the octet contribution we obtain:

$$\langle B'(p')|J_0^{3+3\sqrt{8}}(\vec{X})|B(\vec{p})\rangle = \sum_{B'} \int \frac{d^3k}{(2\pi)^3} \langle B'(\vec{p}\,')|Q^{4+15}|B'(\vec{k})\rangle\langle B'(\vec{k})J_0^{4-15}(\vec{X})|B(\vec{p})\rangle$$

$$-\sum_{B''} \int \frac{d^3k}{(2\pi)^3} \langle B'(\vec{p}\,')|J_0^{4-15}(\vec{X})|B''(\vec{k})\rangle\langle B''(\vec{k})|Q^{4+15}|B(\vec{p})\rangle$$

$$+\sum_{\alpha'} \int \frac{d^3k}{(2\pi)^3} \langle B'(\vec{p}\,')|Q^{4+15}|\alpha'(\vec{k})\rangle\langle \alpha'(\vec{k})|J_0^{4-15}(\vec{X})|B(\vec{p})\rangle$$

$$-\sum_{\alpha'} \int \frac{d^3k}{(2\pi)^3} \langle B'(\vec{p}\,')|J_0^{4-15}(\vec{X})|\alpha'(\vec{k})\rangle\langle \alpha'(k)|Q^{4+15}|B(\vec{p})\rangle \tag{4}$$

where $|B'(\vec{k})\rangle$ is an octet state and the $|\alpha(\vec{k})\rangle$ are the higher mass states.

Now let $\vec{p}\,'-\vec{p} = \vec{q} = q_b\hat{b}$ where $\vec{p} > \vec{q} = 0$ and differentiate both sides of Eq. (4) with respect to q_b and then take the limit as $q_b \to 0$, $p_0 \to \infty$. Carrying this out we obtain using Eq. (1):

$$G_M^{B'B}(0)^{3+3\sqrt{8}} = \sum_{G''} G_V^{B'B''4+15}(0)\, G_M^{B''B}(0)^{4-15} - G_M^{B'B''}(0)^{4-15} G_V^{B''}(0)^{B4+15}(0) + \alpha^{B'B} \tag{5}$$

Where the correction term is defined by (we assume

$$G_M^{B'B}(q^2) \underset{q^2\to 0}{\sim} \frac{A}{B\quad q^2} \text{ or } \frac{d}{dq_b}\; G_M^{B'B}(q^2) \sim \frac{q_b}{\left(q^2+B^2\right)} \to 0 \text{ as } q_b \to 0):$$

$$\alpha^{B'B} = \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha'} \lim_{p_0\to\infty} \lim_{q_b\to 0} \frac{d}{dq_b}\Big(\langle B'(\vec{p}\,'')|Q^{4+15}|\alpha'(\vec{k})\rangle\langle \alpha'(\vec{k})|J_0^{4-15}(0)|B(\vec{p})\rangle$$

$$-\langle B'(\vec{p})|J_0^{4-15}(0)|\alpha'(k)\rangle\langle \alpha'(\vec{k})|Q^{4+15}|B(\vec{p})\rangle\Big) = \alpha^{B'B(-)} - \alpha^{B'B(+)}. \tag{6}$$

(Note the interchange of the limit $p \to \infty$ and the sum over states.)

We shall take specific cases of Eq. (5) and show that we obtain a set of equations which may be solved for the weak magnetic form factors in terms of certain baryon anomalous magnetic moments and the correction terms $\alpha^{B'B}$. By the Ademello-Gatto theorem the charge form factors in Eq. (5) may be replaced by their SU(3) symmetry values with the introduction of an error of order λ^2 or smaller. Such corrections will be ignored as we are attempting to establish that there exist corrections to the weak magnetic form factors of order λ. This leads to the following set of equations:

$$2u_p + u_N/2M_N\alpha = -\sqrt{\tfrac{3}{2}}\, G_M^{p\rho} - \sqrt{\tfrac{1}{2}}\, G_M^{\Sigma^o p} + \alpha^{pp} \tag{7a}$$

$$3u_{nn}/2M_N\alpha = \sqrt{\tfrac{1}{2}}\, G_M^{\Lambda p} + \sqrt{\tfrac{3}{2}}\, G_M^{\Xi^-\Lambda} + \alpha^{\Lambda\Lambda} \tag{7b}$$

$$3u_{\Sigma^o\Sigma^o}/2M_N\alpha = \sqrt{\tfrac{3}{2}}\, G_M^{\Sigma^o p} + \sqrt{\tfrac{1}{2}}\, G_M^{\Xi^-\Sigma^o} + \alpha^{\Sigma^o\Sigma^o} \tag{7c}$$

$$2u_{\Xi^-} + u_{\Xi^o}/2M_N\alpha = -\sqrt{\tfrac{3}{2}}\, G_M^{\Xi^-\Lambda^o} - \sqrt{\tfrac{1}{2}}\, G_M^{\Xi^-\Sigma^o} + \alpha^{\Xi^-\Xi^-} \tag{7d}$$

$$u_{\Sigma^o\Lambda^o}/2M_N\alpha = \sqrt{\tfrac{3}{2}}\, G_M^{\Sigma^o p} + \sqrt{\tfrac{1}{2}}\, G_M^{\Xi^-\Lambda} + \alpha^{\Sigma^o\Lambda^o} \tag{7e}$$

where we have used isospin invariance to convert $G_M^{B'B\; 3+3\sqrt{8}}$ to $G_M^{B'B\; 3+\frac{1}{\sqrt{3}}8} = \frac{u^{B'B}}{2M_N\alpha}$, and where M_N is the nucleon mass. Equations (7a) through (7d) are degenerate but may be solved with the help of Eq. (7e) which involves the mixed moment $u_{\Sigma^o\Lambda^o}$.[5]

We will now attempt to estimate the order in λ of the correction terms $\alpha^{B'B}$. Consider the first term in Eq. (6)

$$\alpha^{B'B(-)} = \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha'} \lim_{p_b\to\infty} \lim_{q_b\to 0} \frac{d}{dq_b} \langle B'(\vec{p})|Q^{4+15}|\alpha'(\vec{k})\rangle\langle\alpha(\vec{k})|J_0^{4-15}(0)|B(\vec{p})\rangle \tag{8}$$

Now we use the following substitutions:

$$\langle B'(\vec{p})|\, Q^{4+15}|\alpha'(\vec{k})\rangle = \int d^3y \,\langle B'(\vec{p})|J_0^{4+15}(\vec{y})|\alpha'(\vec{k})\rangle\, \bar{y} - \int d^3y\, \langle B'(\vec{p})\frac{|q_0 J_0^{4+15}(y)|}{q_0}\alpha'(k)\rangle$$

$$= \int d^3y \frac{\langle B'(p)|D^{4+15}(y)|\alpha'(k)\rangle}{q_0} \quad (\vec{q}\to 0)$$

$$\langle\alpha(\vec{k})|J_0^{4-15}(0)|B(\vec{p})\rangle = \frac{\langle\alpha(\vec{k})|q_0 J_0^{4-15}(0)|B(\vec{p})\rangle}{q_0} = + \frac{\langle\alpha|q_b J_b^{4-15}|B(\vec{p})\rangle}{q_0} + \frac{\langle\alpha|D^{4-15}(0)|B(\vec{p})\rangle}{q_0}$$

with $D(x) \triangleq \partial_\mu J_\mu$ (10)

and note that the matrix element $\langle\alpha|D^{4-15}|B\rangle$ is manifestly of order λ. If we ignore the octet mass splitting between B and B' we are left with:

$$\alpha^{B'B(-)} = \sum_{\alpha'} \lim_{p_0\to\infty} \langle B'(p)|D^{4+15}(0)|\frac{\alpha'(\vec{k})\rangle\langle\alpha'(\vec{k})}{q_0^2}|J_b^{4-15}(0)|B(p)\rangle\,\delta^{(3)}(\vec{p}-\vec{k})$$

We now introduce the invariant state $|B(\vec{p})) = \sqrt{p_0}\,|B(\vec{p})\rangle$ and the invariant variable $V = p\cdot q = p_0 q_0$, $(\vec{q}\cdot\vec{p}=0)$ then suppressing the sum over the internal quantum numbers of the intermediate states we have

$$\alpha^{B'B(-)} = \epsilon_b \int \lim_{p_0\to\infty} (B'(\vec{p})|D^{4+15}(0)|\alpha(\vec{k})\rangle\,\langle\alpha(\vec{k})|J_b^{4-15}(0)|B(p))\,\delta^{(4)}(p-k+q)\frac{dV}{V^2} \qquad (12)$$

with

$$k_0 = p_0 + q_0 \quad . \quad \frac{dk_0}{q_0} = \frac{dV}{V} \; (\vec{q}=0)$$

where $\epsilon_b = 1$ with ϵ a polarization vector whose components are zero save for the b'th (this vector is invariant in any frame where $\vec{p}\cdot\vec{q} = 0$).

Next we consider the invariant $R^{(-)}$ given by:

$$R^{(-)} = \lim_{k \to 0} \epsilon_b \int e^{ik\cdot x}\, \theta(x_0) \left(B'(\vec{p}) \left[\left(\partial_u J^{u4+15}(x), J_b^{4-15}(0)\right)\right] \middle| B(p)\right) d^4x \qquad (13)$$

if we define:

$$B^{B'B}(v) = \epsilon_b \left(B'(\vec{p}) \middle| D^{4+15}(0) \middle| \alpha'(\vec{k})\right\rangle \left\langle \alpha'(\vec{k}) \middle| J_b^{4-15}(0) \middle| B(p)\right) \delta^{(4)}(p-k+q)$$

$$(\vec{q} = 0) \qquad (14)$$

insertion of a complete set of intermediate states between the operators in $R^{(-)}$ shows that its absorbtive part is given by:

$$\text{abs } R^{(-)}(v) = \text{Im } A^{(-)}(v) \qquad (15)$$

Therefore assuming that the field of the strange scalar meson the κ is given by:

$$D^{4-15}(x) = \frac{f_{\kappa^-}}{\sqrt{2}}\, m_K^2\, \phi^{\kappa^-}(x) \qquad (16)$$

we obtain

$$R^{(-)} = \frac{f_{\kappa^-}}{\pi} \int_{v'}^{\infty} \text{Im } A^{(-)}(v)\, dv \qquad (17)$$

$$\alpha^{B'B} = \frac{f_{\kappa}}{\pi} \int_{v'}^{\infty} \frac{\text{Im } A^{(-)}(v)\, dv}{v} \qquad (18)$$

From the form of $R^{(-)}$, Im $A^{(-)}(v)$ is the imaginary part of the invariant amplitude for the process: $\kappa^- + B \to B' + K^*$ for virtual κ and K^*; at zero momentum transfer- V' is the minimum value of V for contributions from intermediate states other than the octet.

Finally we have obtained with a similar analysis of $\alpha^{B'B(+)}$,

$$\alpha^{B'B} = \frac{f_{\kappa^-}}{\pi} \int_{v'}^{\infty} \frac{\mathrm{Im}\, A^{\kappa^- + B \to B' + K^{*-}}(v)\, dv}{v} - \int_{v''}^{\infty} \frac{\mathrm{Im}\, A^{\kappa^+ + B \to B' + K^{*+}}(v)\, dv}{v}$$

$$(v' \neq v'') \qquad (19)$$

Now the κ^- vertex in Eq. (18) is suppressed by a factor of λ as this vertex is proportional to a matrix element of the divergence of the strange vector current. However, there is no such suppression of the K^{*-} vertex which, in fact we expect to be of the same order of magnitude as a photoproduction vertex, divided by the fine structure constant α. Thus unless there is a fortuitous cancellation between $\alpha^{B'B(-)}$ and $\alpha^{B'B(+)}$, we expect the correction term $\alpha^{B'B}$ to be of first order in λ. It should be noted that this result is not very surprising when we note that the electromagnetic moments of the baryons themselves arise out of the amplitude for the photoproduction of pions and are, in fact, given by integrals of the photoproduction amplitude as of the same functional form as our $\alpha^{B'B}$ and vanish if the axial vector current is conserved.[4]

We should also remark that we have assumed that we may interchange the limit $p_0 \to \infty$ and the sum over intermediate states in $\alpha^{B'B}$. Adler and Dashen[7] indicate that this interchange of limits in the case of the commutator $\left[Q^i, J_b^j(0)\right]$ may be valid depending on the details of the model one chooses for the currents. By necessity we make the assumption that the interchange is valid.

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   N. Brene, B. Helleson and M. Roos, Phys. Letters 11, 344 (1961);

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2. M. Ademello and R. Gatto, Phys. Rev. Letters 13, 264 (1965);

   S. Fubini and G. Furlan, Physics 1, 229 (1965);

   Ademello and Gatto claim their result holds for $G_M^{B'B\,4+i5}$ also, however, their argument is based on the algebraic structure of the vector form factor, and involves equating two such form factors at q = 0 but in this case since $G_M^{B'B}$ appears as $G_M^{B'B}\, q_v \sigma_{uv}$ the $q \rightarrow 0$ limit gives 0 = 0 identically.

3. If we let H' be dimensionless of order 1, then small is defined by

   $\lambda \ll E^{rest}_{\text{excited states}} - E^{rest}_{\text{octet states}}$.

4. We essentially follow the derivation of the Fubini-Furlan-Rossetti sum rule.

   S. Fubini, G. Furlan and C. Rossetti, Nuovo Cimento 40, 1171 (1965);

   We have suppressed a summation over the spins of the external baryons.

5. The sum of Eqs. (7a) - (7d) yields an estimate for the correction terms in terms of the electromagnetic anomalous magnetic moments:

$$\frac{1}{2M_N\alpha}\left(2U_{pp} + U_{nn} + 3U_{\Lambda\Lambda} + 3U_{\Sigma^o\Sigma^o} + 2U_{\Xi^-\Xi^-} + U_{\Xi^o\Xi^o}\right) = \left(\alpha^{pp} + \alpha^{\Lambda\Lambda} + \alpha^{\Sigma^o\Sigma^o} + \alpha^{\Xi^-\Xi^-}\right)$$

6. To be more explicit, if we consider the currents in a specific representation, the quark model say

$$J_\mu^{4+15}(x) = \bar{q}(x)\, \lambda^{4+15}\, \gamma_\mu\, q(x)$$

$$J_\mu^{Em}(x) = \bar{q}(x)\, \lambda^{3+1/\sqrt{3}\ 8}\, \gamma_\mu\, q(x)$$

where q(x) is the quark field and define in analogy with Eq. (13) a quantity

$$R^{(-)pp}_{Em} = \lim_{k \to 0} \epsilon_b \int d^4x\, e^{ik\cdot x}\, \theta(x_0) \left(p(\vec{p}')\middle|\left(\partial_\mu A^3_\mu(x),\ J^3_b(0)\right|p(\vec{p})\right)$$

we notice that $R^{(-)pp}_{\epsilon_m}$ differs from $R^{(-)}$ essentially by the replacement of $\partial_\mu J^{4+15}_\mu$ by $\partial_\mu A^3_\mu$. Now $\partial_\mu A^3_\mu$ is proportional to chiral SU(3) × SU(3) breaking which is large, of order 1, whereas, $\partial_\mu J^{4+15}_\mu$ is of order λ. Thus we expect $\alpha^{B'B(-)}$ to order λ with respect to $\alpha^{p'p(-)}_{Em}$, where $\alpha^{p'(-)}_{Em}$ is obtained from $R^{(-)pp}_{Em}$ in the same way that $\alpha^{B'B(-)}$ is obtained from $R^{(-)}$. But from Ref. 4 we have that $\alpha^{pp(-)}_{Em} = \frac{\mu_p - \mu_n}{2M_N\alpha}$ which is of the same order of magnitude as the baryon octet terms in Eqs. (7). Thus we expect $\alpha^{B'B(-)}$ to be a correction term of order λ.

7. S. Adler and R. Dashen, Current Algebras and Applications to Particle Physics, (W. A. Benjamin and Co., New York, 1968); p. 259-262.