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TESTS OF EXCHANGE DEGENERACY BY PROCESSES RELATED BY CROSSING†

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ABSTRACT

High energy scattering processes which are related by $s \longleftrightarrow u$ crossing are shown to have the same differential cross section at a given energy if they are governed by the exchange of Regge trajectories which are exchange degenerate. Various experimental tests of this are discussed.

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The purpose of this note is to point out that the previously proposed idea of exchange degeneracy[1] of the vector and tensor meson Regge trajectories leads in a simple way to a number of interesting predictions of the equality of differential cross sections at small t for processes related by $s \leftrightarrow u$ crossing. Some of these predictions are already experimentally testable with currently available data, while many others will soon be testable using data from experiments in progress.

The basic idea is the following: Suppose we have a process of the type pseudoscalar meson + baryon $\rightarrow$ pseudoscalar meson + baryon, which at high energies and small momentum transfers is dominated by exchange of the vector and tensor meson Regge trajectories. For example, the process $K^+p \rightarrow K^o\Delta^{++}$ seems to be dominated by ρ and A_2 meson exchange at high energy. What is the relation of the amplitudes for the $s \leftrightarrow u$ crossed process (in this case $\bar{K}^o p \rightarrow K^-\Delta^{++}$) to those of the original one? The answer[2] is that the part of the amplitudes due to exchange of even signature trajectories remains unchanged, while that part due to odd signature trajectories just changes sign. This may easily (and nonrigorously) be seen by noticing that under the substitution $s \leftrightarrow u$, $\cos\theta_t \rightarrow -\cos\theta_t$, so that the even (odd) signature trajectories, being continuations of the even (odd) J partial waves, do not (do) change sign under $s \leftrightarrow u$ crossing. Such a sign change under $s \leftrightarrow u$ crossing of the part of the amplitudes due to exchange of odd signature trajectories, of course, leads to very different net amplitudes for the $s \leftrightarrow u$ crossed and uncrossed processes, depending on the relative phases of the Regge amplitudes. However, we know the phase of a Regge pole amplitude is given by the signature factor. For the odd signature trajectories, such as the vector meson trajectories, this is just $1 - e^{-i\pi\alpha_V(t)}$, while for even signature trajectories, such as the tensor meson trajectories, it is $1 + e^{-i\pi\alpha_T(t)}$. Exchange degeneracy in what we might call its "weak" form, just says that the vector and tensor mesons lie on degenerate trajectories, e.g. $\alpha_\rho(t) = \alpha_{A_2}(t)$, $\alpha_{K*}(t) = \alpha_{K**}(t)$, etc. Stated in terms of the phases of the Regge exchange amplitudes, this means that the odd and even signature trajectories have the phases $1 - e^{-i\pi\alpha(t)} = 2e^{+i\pi/2}\, e^{i\pi\alpha(t)/2} \sin(\pi\alpha(t)/2)$ and $1 + e^{-i\pi\alpha(t)} = 2e^{i\pi\alpha(t)/2} \cos(\pi\alpha(t)/2)$ respectively. In other words, they are just exactly 90^o out of phase with one another.

Thus, although the amplitudes for the $s \leftrightarrow u$ crossed and uncrossed amplitudes may be very different, the interchange $s \leftrightarrow u$ does not change the expression for the differential cross section if $\alpha_V(t) = \alpha_T(t)$, because the amplitudes due to Reggeized vector and tensor meson exchange differ by 90^o in phase and hence contribute incoherently to the differential cross section. We thus expect the same $d\sigma/dt$ at a given (high) energy for processes related by $s \leftrightarrow u$ crossing which are governed by exchange degenerate Regge trajectories.

So far we have discussed consequences of just the "weak" form of exchange degeneracy, $\alpha_V(t) = \alpha_T(t)$, i.e., the vector and tensor mesons lie on the same Regge trajectory. A "strong" form of exchange degeneracy would say that not only are the trajectories of the vector and tensor mesons the same, but so are their residues, i.e., $\beta_V(t) = \beta_T(t)$. This is in fact the way in which Regge theory can "explain" or "accommodate" certain amplitudes, e.g., those for $K^+n \rightarrow K^op$, being real at high energies, for $\alpha_V(t) = \alpha_T(t) = \alpha(t)$ and $\beta_V(t) = \beta_T(t) = \beta(t)$ lead to a net amplitude

$\beta_V(t)\left[1-e^{-i\pi\alpha_V(t)}\right] + \beta_T(t)\left[1+e^{-i\pi\alpha_T(t)}\right] = 2\beta(t)$, which is purely real. Note that the process related by s ⟷ u crossing has in this case a phase given by $-\beta_V(t)\left[1-e^{-i\pi\alpha_V(t)}\right] + \beta_T(t)\left[1+e^{-i\pi\alpha_T(t)}\right] = 2\beta(t)\,e^{-i\pi\alpha(t)}$. In either case, if every helicity amplitude has residues for even and odd signature trajectories related by the "strong" form of exchange degeneracy, we in particular predict no polarization of the final baryon, because all amplitudes then have the same phase (purely real, or a phase $e^{-i\pi\alpha(t)}$). This is of course similar to the case where only one Regge trajectory is exchanged and one expects no polarization because all amplitudes have the same phase.

There are a fairly large number of relations among particle differential cross sections which can be derived from the above considerations. First of all, a number of SU(3) relations among pseudoscalar meson-baryon differential cross sections simplify and can be tested with presently available data, as has been noted by Mathews[3]. In particular, the relations

$$\frac{1}{2}\frac{d\sigma}{dt}(\pi^- p \to \pi^o n) + \frac{3}{2}\frac{d\sigma}{dt}(\pi^- p \to \eta n) = \frac{1}{2}\frac{d\sigma}{dt}(K^- p \to \bar{K}^o n) + \frac{1}{2}\frac{d\sigma}{dt}(K^o p \to K^+ n)$$

and

$$\frac{1}{2}\frac{d\sigma}{dt}(\pi^+ p \to \pi^o \Delta^{++}) + \frac{3}{2}\frac{d\sigma}{dt}(\pi^+ p \to \eta\Delta^{++}) = \frac{1}{2}\frac{d\sigma}{dt}(K^+ p \to K^o \Delta^{++}) + \frac{1}{2}\frac{d\sigma}{dt}(\bar{K}^o p \to K^- \Delta^{++})$$

simplify to

$$\frac{1}{2}\frac{d\sigma}{dt}(\pi^- p \to \pi^o n) + \frac{3}{2}\frac{d\sigma}{dt}(\pi^- p \to \eta n) = \frac{d\sigma}{dt}(K^- p \to \bar{K}^o n)$$

and

$$\frac{1}{2}\frac{d\sigma}{dt}(\pi^+ p \to \pi^o \Delta^{++}) + \frac{3}{2}\frac{d\sigma}{dt}(\pi^+ p \to \eta\Delta^{++}) = \frac{d\sigma}{dt}(K^+ p \to K^o \Delta^{++}) ,$$

if these reactions are governed by exchange degenerate even signature (A_2) and odd signature (ρ) trajectories, as they seem to be[4].

Secondly, there are a few cases where relations on differential cross sections which follow from exchange degeneracy can be presently tested[5]. These involve the K*(890) and K**(1420) Regge trajectories, which appear to be exchange degenerate. In particular, there are data for $\frac{d\sigma}{dt}(K^-p \to \pi^-\Sigma^+)$ and $\frac{d\sigma}{dt}(\pi^-p \to K^o\Sigma^o)$ at 4.1 GeV/c and 4.0 GeV/c laboratory momenta respectively[6,7]. Assuming isospin 1/2 exchange, we have $\frac{d\sigma}{dt}(\pi^-p \to K^o\Sigma^o) = \frac{1}{2}\frac{d\sigma}{dt}(\pi^+p \to K^+\Sigma^+)$, so that $\alpha_{K*}(t) = \alpha_{K**}(t)$ tells us that $\frac{d\sigma}{dt}(K^-p \to \pi^-\Sigma^+) = \frac{d\sigma}{dt}(\pi^+p \to K^+\Sigma^+) = 2\frac{d\sigma}{dt}(\pi^-p \to K^o\Sigma^o)$. The agreement of this with experiment shown in Fig. 1 is rather good, although the statistical errors are fairly large. There are also data on $\frac{d\sigma}{dt}(K^-p \to \pi^-\Sigma^+)$ and $\frac{d\sigma}{dt}(\pi^+p \to K^+\Sigma^+)$ at 5.5 GeV/c and 5.4 GeV/c respectively[6,8], and these are shown in Fig. 2. Here the predicted equality of differential cross sections is not so well obeyed. In general, the fairly large experimental errors prevent us from saying anything conclusive using the presently available data.

Thirdly, the predicted equality of the differential cross sections for $K^-p \to \bar{K}^on$ and $K^op \to K^+n$, $K^+p \to K^o\Delta^{++}$ and $\bar{K}^op \to K^-\Delta^{++}$, $\pi^-p \to K^o\Lambda$ and $\bar{K}^op \to \pi^+\Lambda$, as well as $\pi^-p \to K^oY_1^{*o}$ and $\bar{K}^op \to \pi^+Y_1^{*o}$, etc. will soon be testable. Data are presently available for the first of each of the above pairs of processes, and data for the second of each of the pairs is now being analyzed from the SLAC K_2^op bubble chamber experiment[9]. Also from this experiment, it should be possible to look at the polarization of the final Λ in $\bar{K}^op \to \pi^+ + \Lambda$ and thus test the "strong" form of exchange degeneracy at small values of t. At present, we only have the measurements of large Λ

polarization in $\pi^- p \rightarrow K^o \Lambda$ at relatively low energies[7], a recent experiment[10] reporting a large Λ polarization in $K^- n \rightarrow \pi^- \Lambda$ at 4.5 GeV/c with poor statistics, and the large Σ polarization[8] in $\pi^+ p \rightarrow K^+ \Sigma^+$ at 5.4 GeV/c, again with poor statistics, to use in judging how well the "strong" form of exchange degeneracy holds. If the large values of the polarization persist, it will be particularly interesting to see if they change sign when one goes to the $s \leftrightarrow u$ crossed processes, as would be the case if the polarization arises due to the interference between amplitudes from Regge trajectories of opposite signature, rather than arising from the interference of two trajectories with the same signature and different values of $\alpha(t)$.

References

1. R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

2. D. H. Sharp and W. G. Wagner, Phys. Rev. 128, 2899 (1962).

3. R. D. Mathews, University of California Lawrence Radiation Laboratory preprint, UCRL-18611 (unpublished).

4. The analysis of Mathews indicates that $K^- p \to \bar{K}^0 n$ and $K^+ p \to K^0 \Delta^{++}$ have important contributions to $d\sigma/dt$ from both ρ and A_2 exchange at small t. Thus our prediction of equal cross sections for the $s \leftrightarrow u$ crossed reactions, $K^0 p \to K^+ n$ and $\bar{K}^0 p \to K^- \Delta^{++}$, is not completely trivial, as would be the case if only one trajectory were exchanged. Using SU(3) for the Regge residues, one can also then show that there are important contributions to $\pi^- p \to K^0 \Lambda$, $K^- p \to \pi^- \Sigma^+$, etc. from both K*(890) and K**(1420) exchange, so that the predictions for these processes are also not trivial because only one Regge trajectory is exchanged.

5. The relations between differential cross sections tested here are contained among the equations of A. Ahmadzadeh, Phys. Letters 22, 669 (1966), where they are derived using the "strong" form of exchange degeneracy plus SU(3) symmetry for the Regge residue functions. Only the "weak" form of exchange degeneracy, $\alpha_{K^*}(t) = \alpha_{K^{**}}(t)$, is necessary in the derivation of course.

6. J. S. Loos et al., Phys. Rev. 173, 1330 (1968).

7. O. I. Dahl et al., Phys. Rev. 163, 1430 (1967).

8. W. A. Cooper et al., Phys. Rev. Letters 20, 472 (1968).

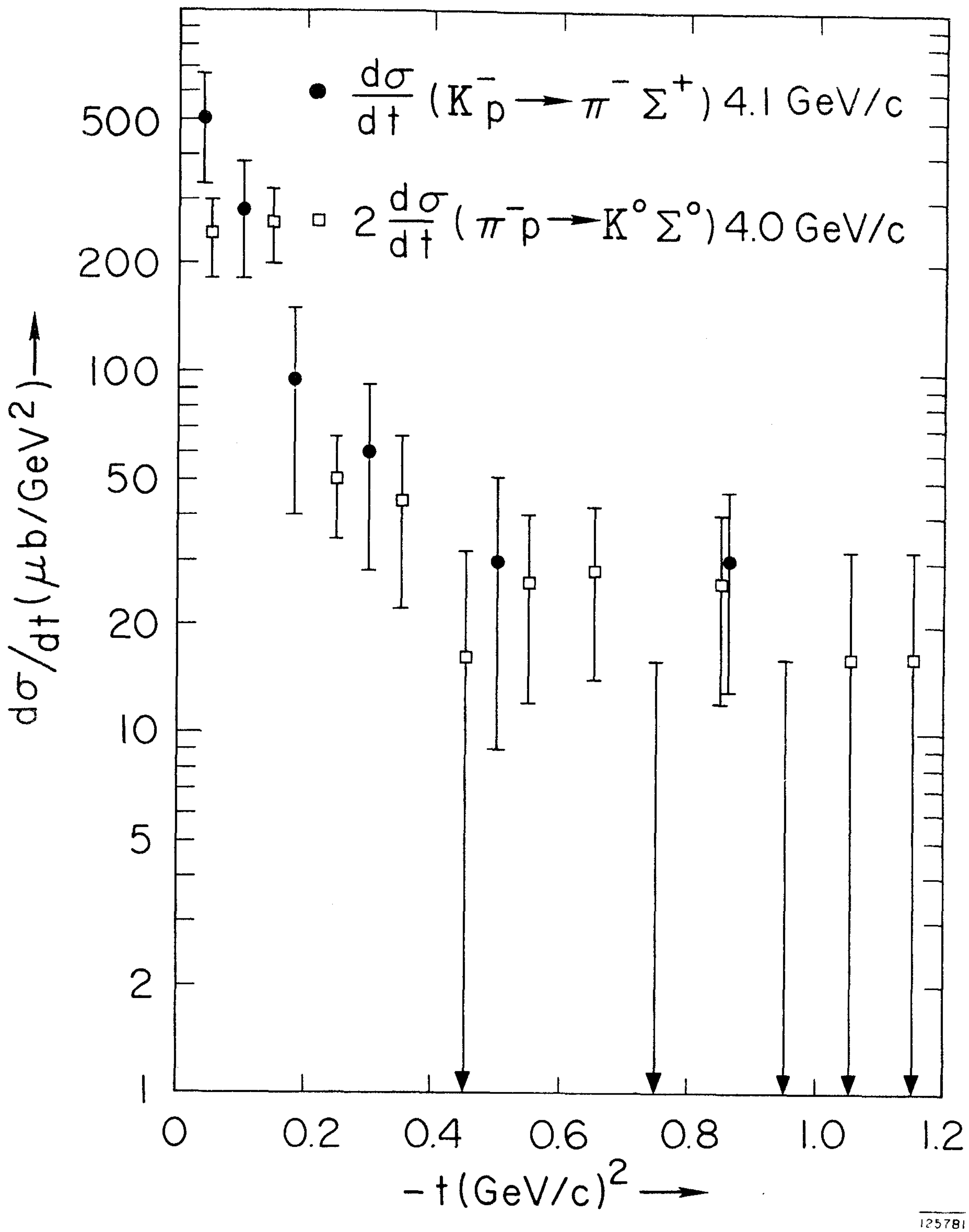
9. SLAC Group B, private communication.

10. W. L. Yen et al., Purdue University preprint C00-1428-106 (unpublished).

Figure Captions

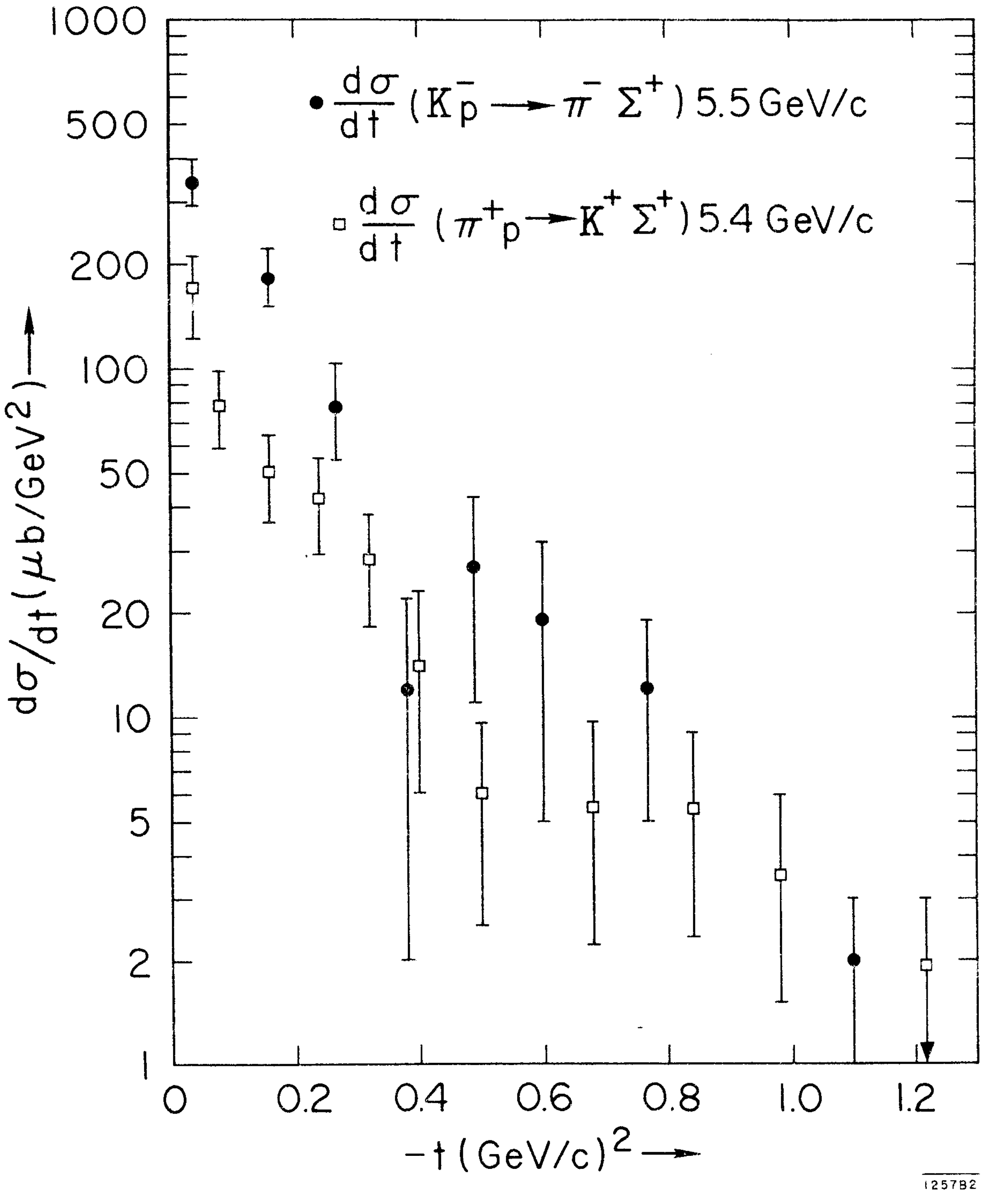
Figure 1 The differential cross section[6] for $K^-p \to \pi^-\Sigma^+$ at 4.1 GeV/c and two times the differential cross section[7] for $\pi^-p \to K^0\Sigma^0$ at 4.0 GeV/c.

Figure 2 The differential cross sections for $K^-p \to \pi^-\Sigma^+$ and $\pi^+p \to K^+\Sigma^+$ at laboratory momenta of 5.5 and 5.4 GeV/c respectively[6,8].



$\frac{d\sigma}{dt}(K^-p \rightarrow \pi^-\Sigma^+)$ 4.1 GeV/c
$2\frac{d\sigma}{dt}(\pi^-p \rightarrow K^\circ\Sigma^\circ)$ 4.0 GeV/c
500
200
100
50
20
10
5
2
1
$d\sigma/dt\,(\mu b/GeV^2)$
0
0.2
0.4
0.6
0.8
1.0
1.2
$-t\,(GeV/c)^2$
125781

Fig. 1



1000
500
200
100
50
20
10
5
2
1
$d\sigma/dt$ (μb/GeV^2) →
● $\frac{d\sigma}{dt}$ ($K^-p \to \pi^- \Sigma^+$) 5.5 GeV/c
□ $\frac{d\sigma}{dt}$ ($\pi^+p \to K^+ \Sigma^+$) 5.4 GeV/c
0
0.2
0.4
0.6
0.8
1.0
1.2
$-t$ (GeV/c)^2 →
1257B2

Fig. 2