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THEORETICAL IDEAS ON HIGH-ENERGY INELASTIC ELECTRON-PROTON SCATTERING*†

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INTRODUCTION

The data on inelastic electron-proton scattering reported by the MIT-SLAC electron-scattering group[1] represents a study of the proton under conditions of extreme violence. The incident electron energy is 7 - 18 BeV, the transverse momentum of the secondary electron is 1 - 2 BeV and the mass of the produced system of hadrons is 3 - 5 BeV. Under these circumstances we may expect to learn much about proton structure at small distances.

In order to analyze the data, we may write the cross section for the process, with only the final electron detected, as[2,3]

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^2\frac{\theta}{2}} W_2(Q^2, \nu)\cos^2\frac{\theta}{2}\left[1 + 2\tan^2\frac{\theta}{2}\left(1 + \frac{\nu^2}{Q^2}\right)\frac{\sigma_t}{\sigma_t + \sigma_\ell}\right]$$

$$\approx \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} W_2(Q^2,\nu)\left[1 + \frac{\nu^2}{2EE'}\,\frac{\sigma_t}{\sigma_t+\sigma_\ell}\right] \quad \frac{\theta}{2} \ll 1$$

W_2 (and W_1) are the structure-functions of Drell and Walecka, which are related kinematically to the cross sections σ_t and σ_ℓ for absorption of transverse and longitudinal virtual photons on the proton.[3] The laboratory energy of the virtual photon is $\nu = E - E'$, and $Q^2 = 4EE'\sin^2\frac{\theta}{2}$ is the square of the virtual-photon four momentum. The Feynman diagram for the process is illustrated in Fig. 1.

The salient feature of the data appears to be that the structure function νW_2 (for $q^2 > 0.5\ \text{BeV}^2$) can be represented as a function of the single variable ν/Q^2, as shown in Fig. 2. The other structure function, taken above to be $\sigma_t/(\sigma_t+\sigma_\ell)$ has not yet been separated from the data, and for the data reported at Vienna is a small contribution. With the wider-angle data now being analyzed, this separation will soon be made.

Another way of stating the result shown in Fig. 2 is that it appears that the cross section for the process depends only upon the natural invariants characterizing

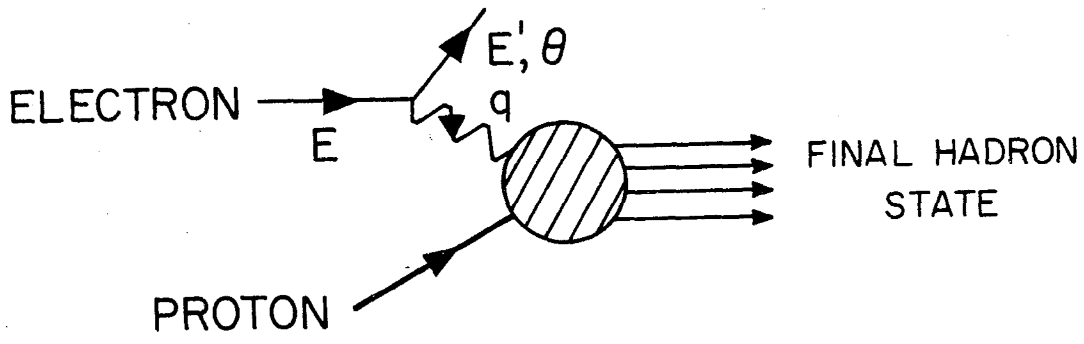

E', θ
ELECTRON
q
E
FINAL HADRON
STATE
PROTON

Fig. 1

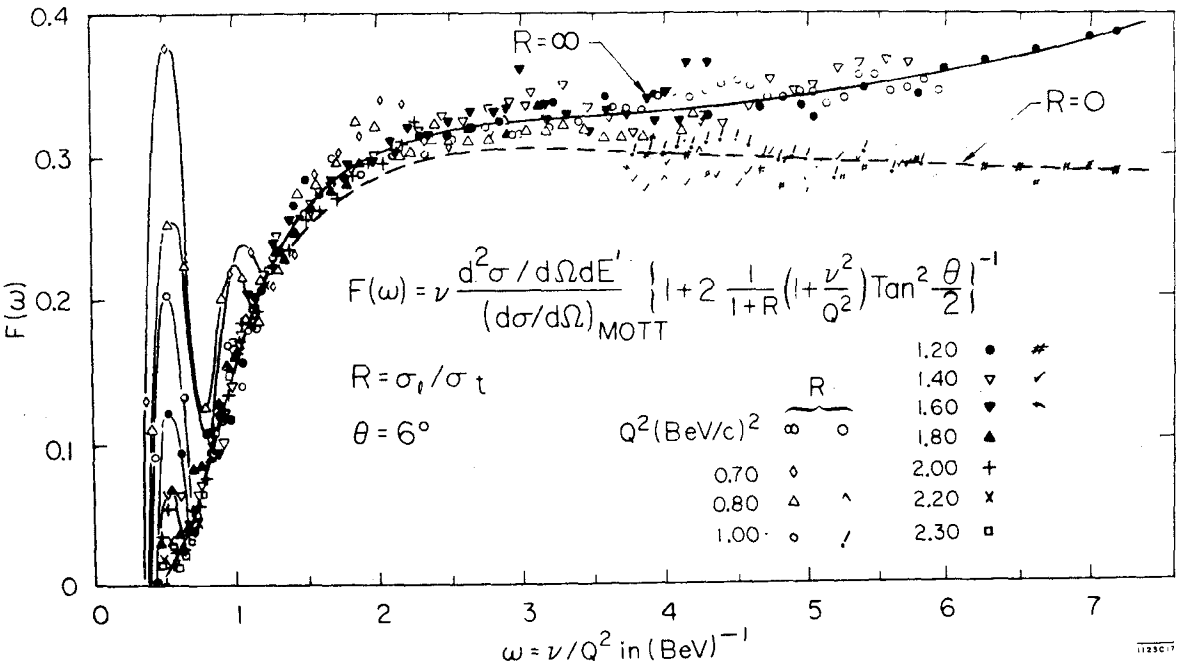

R=∞
R=0
F(ω) = ν (d²σ/dΩdE')/(dσ/dΩ)MOTT {1+2 1/(1+R) (1+ν²/Q²) Tan² θ/2}⁻¹
R = σl/σt
θ = 6°
Q²(BeV/c)²
R
∞ 0
0.70
0.80
1.00
1.20
1.40
1.60
1.80
2.00
2.20
2.30
F(ω)
ω = ν/Q² in (BeV)⁻¹
0 1 2 3 4 5 6 7
0 0.1 0.2 0.3 0.4

Fig. 2

the kinematics: s, t, and u in standard Mandelstam language. The scaling property $\nu W_2 = F(\nu/Q^2)$ is then just a consequence of dimensional analysis. In this case there is no natural size or length characterizing the scattering. This is in contrast to pure strong-interaction phenomena, where overwhelmingly the secondary particles emerge with small transverse momenta $\lesssim 400$ MeV, and where transverse-momentum distributions are precipitously exponential. In contrast, here the electron distribution falls off as a power of the transverse momentum $\sim p_\perp^{-4}$.

A second feature of the data is its apparently diffractive character. Recalling that $\nu W_2 \cong Q^2(\sigma_t + \sigma_\ell)/4\pi^2\alpha$, we see that at high virtual-photon energy ν the cross section approaches a constant, just as if the photon were a hadron. There is no bump in νW_2, as had been speculated on the basis of quasi-elastic scattering from pointlike constituents within the proton.[4]

There are various theoretical models which try to explain or at least describe these features.[5,6,7,8,9,10] None work really well, or are totally satisfying. However, before going into these models, we will first discuss other possible experiments which may exhibit scale-invariance or the pointlike behavior at high transverse momentum. Then we will discuss three theoretical descriptions of the data; these are:

1) incoherent scattering from pointlike constituents within the proton — the "parton" model, or "Thomson nucleon"

2) vector dominance, or "Rutherford electron"

3) current commutators.

Finally, we will discuss the implications of these models for future electron (and muon) scattering experiments such as electron scattering from deuterium, separation of σ_t from σ_ℓ, the A-dependence of electron scattering from nuclei, and the nature of the final hadron states in these processes.

IMPLICATIONS FOR OTHER PROCESSES

The scale-invariance, or the pointlike character, of the data is in some way a consequence of the fact that in this experiment we use a probe, the electron, which is approximately pointlike, and which interacts weakly with hadronic matter. It may well be that this is a general characteristic of hadron processes initiated by a weakly coupled pointlike probe. In addition to e, these include μ, ν, and γ-ray. Clearly the process

$$\mu + p \longrightarrow \mu + \text{anything}$$

should behave like electron scattering; this is a test of quantum electrodynamics and μ - e universality. In

$$\nu + p \longrightarrow \mu + \text{anything}$$

the total cross section should rise linearly with laboratory energy if scale invariance holds, and if the Fermi-coupling remains local (no intermediate boson). Dimensional analysis gives

$$\sigma_{\nu p}^{tot}(s) \propto G^2 s = G^2(2ME_{lab}) .$$

The somewhat less obvious process

$$\gamma + p \longrightarrow \gamma + \text{anything}$$

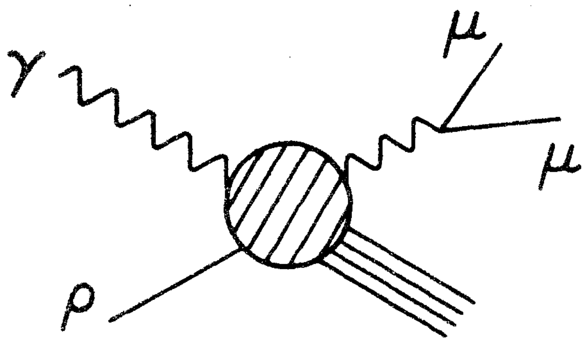
likewise could be pointlike, and exhibit a transverse momentum distribution falling off as a power, similar to that for the electron, $\sim p_{\perp}^{-4}$. If the proton does contain pointlike constituents which carry the charge, then it should be possible to see the constituents by looking. That is, we shine high-frequency light on the proton and observing the light scattered incoherently, at high transverse momentum, by the pointlike constituents.

The γ-ray background for this experiment from photoproduced π^0's is formidable. However it falls off exponentially with increasing transverse momentum, and the signal from this Compton process, if indeed it behaves as described above, must eventually dominate the background at sufficiently high transverse momentum. While at present energies the predicted signal appears at best to be of the same order of magnitude as the noise,[6] the situation improves markedly as the energy increases up to the next generation of accelerators in the 100 BeV region. I will come back later as to how this estimate was made.

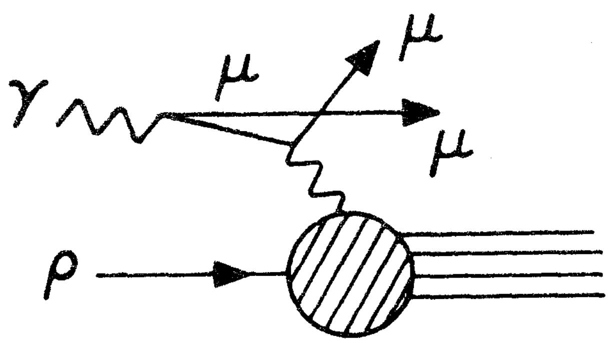
A variant of this process with perhaps better signature is μ-pair photoproduction

$$\gamma + p \longrightarrow \mu^+ + \mu^- + \text{anything}$$

where again high transverse momentum muons are observed. The Compton diagrams in Fig. 3



essentially measure the previous inelastic γ-ray process, divided by $\sim \frac{1}{137}$ in rate. The Bethe-Heitler diagrams of Fig. 4 measure inelastic μ-p scattering



by using the virtual μ-pair spectrum contained in the γ-rays. This flux of virtual muons is $\sim \frac{1}{137}$ the γ-ray flux, which at an electron machine is considerably larger than the muon flux attainable in a conventional beam. Therefore in this way, μ-p inelastic scattering may be studied at higher transverse momentum than in conventional experiments.

Finally the energy dependence of the total cross section for storage-ring processes

$$e^+ + e^- \longrightarrow \text{hadrons}$$

may conceivably be scale-invariant: $\sigma^{tot} \sim \alpha^2 E_{cm}^{-2}$. In any case this process has a direct bearing on the same kinds of questions as in the electron scattering.

MODELS FOR ELECTRON SCATTERING

Going back to the electron scattering process itself, we consider three different theoretical descriptions. The first is an ancient idea. The proton is visualized as granular; composed of pointlike constituents (Feynman calls then partons) from which the electron Coulomb scatters incoherently. The new feature, not present in all the historical examples, is that this process is here extreme - relativistic. Feynman[7] has exploited this feature to give a concrete description of what the function νW_2 measures. In the center-of-mass frame of electron and proton, at high energy, the proton is Lorentz-contracted into a thin pancake, from which the electron scatters instantaneously provided the energy uncertainty ΔE is big enough. Futhermore, the internal motion of the stuff inside the proton is slowed down by time-dilatation. So in this way the <u>instantaneous</u> charge distribution of the proton is seen by the electron, and the scattering is an incoherent sum of scatterings from its pointlike constituent partons.[4,7]

In the more concrete terms that Feynman developed, the notion of "internal motion" is replaced by "intermediate virtual state," which is taken as a state of N non-interacting pointlike constituents, (perhaps an eigenstate of H_0, whatever that is). The lifetime of such a given virtual state increases in proportion to the proton energy, again because of time-dilatation, as the energy tends to ∞. So <u>during</u> the act of scattering, Feynman envisages the scattering taking place from this beam of temporarily non-interacting partons. As $\underset{\sim}{P} \to \infty$ for the proton, the longitudinal momentum of the parton also goes to ∞, and in particular the four-momentum of the parton (which is supposed to be light) becomes a given fraction (the longitudinal fraction) of the proton's four-momentum

$$p_\mu^{(\mathrm{Parton})} = x\, P_\mu^{(\mathrm{Proton})} \qquad x < 1$$

Given all this, the cross section can be worked out, and

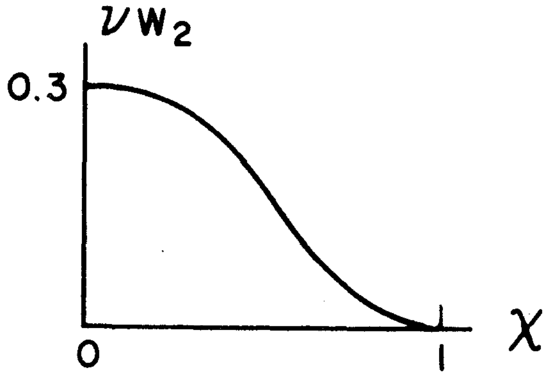
$$\nu W_2 = \sum_N x P_N(x) \left\langle \sum_i Q_i^2 \right\rangle_N \qquad x = \frac{Q^2}{2M\nu}$$

where N stands for a configuration of N protons in the proton. $P_N(x)$ is the distribution of longitudinal fraction and Q_i is the charge of the ith parton. So νW_2 measures the longitudinal momentum distribution of the constituents in a rapidly moving proton, as well as their mean-square charges. The scaling invariance comes out of the calculation — it must, because no natural length was put into the model.

Attempts to fit the data have been made by Drell, Levy, and Yan[5] in a cut-off charge-symmetric theory of bare nucleons and pions with γ_5 coupling. Emmanuel Paschos and I[6] have tried a quark model. I will not describe these in detail, but just indicate the successes and problems encountered by Paschos and me:

1) Within a factor of 2 the general magnitude of νW_2 comes out well (as well, of course as the scale-invariance).

2) To do better is difficult; the predictions are generally too big. The reason is that there is a sum rule stating $\int_0^1 dx \left[\nu W_2\right] = \langle Q^2 \rangle$ = mean-square charge per parton. The function νW_2, plotted vs x, is shown in Fig. 5



and the area under it is ≈ .16. Most models put more charge into the constituents than that.

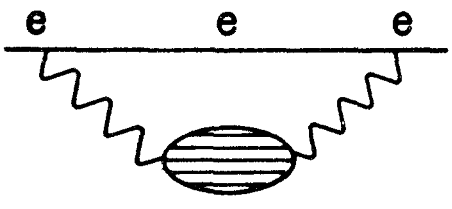
3) The shape of the curve is not naturally explained, in particular the feature $\nu W_2 \longrightarrow 0.3$ as $x \longrightarrow 0$ ($\nu \longrightarrow \infty$). But there are enough unknown parameters in the model that a fit can be made. Feynman says he can get the behavior as $x \longrightarrow 0$ in terms of features of nucleon-nucleon scattering, but I haven't seen the details.

Paschos and I propose[6] that the model be tested by looking at the partons — with light, as discussed earlier. The same Feynman arguments can be used, and we find, in comparison with electron scattering under identical conditions (same E_i, E_f, θ ; hadron final state not observed), the ratio

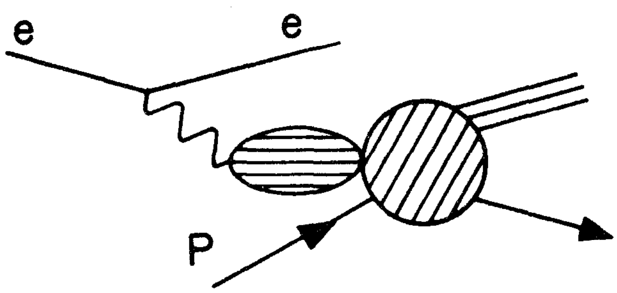
$$\frac{d\sigma_\gamma}{d\sigma_e} = \frac{(E_i - E_f)^2}{E_i E_f}$$

independent of parton spin and distribution of longitudinal momentum — provided partons carry unit charge. If the partons are point-quarks, there is an extra factor $\langle Q^4 \rangle / \langle Q^2 \rangle \sim 0.4$ in our model. The experiment appears feasible at SLAC energies.

The second explanation is along the lines of vector-dominance. This model, instead of Thomson proton, may be called the Rutherford model of the electron. By this I mean that an electron has a pointlike core plus a very weak polarization cloud of virtual hadrons surrounding it, as in Fig. 6.



The proton collides with the cloud (Fig. 7), diffractively if the energy is large.



If only the ρ is put in, and one says

$$\sigma_t = \sigma_\gamma \left(\frac{m_\rho^2}{q^2 + m_\rho^2} \right) \qquad\qquad \sigma_\ell \ll \sigma_t$$

one underestimates the cross section by a factor ~ 2 at $q^2 \sim 1-2$ BeV^2. However, the <u>shape</u> of νW_2 — its diffractive character — is nicely understood on the basis of the model, or the related Regge-pole statement that at high q^2, only the Pomeranchuk trajectory contributes. This has been emphasized by Harari.[13] One way to repair the numerical disagreement is to add in contributions from other vector states more massive than ρ. In this way, Ritson[14] at Stanford has found a credible fit. Brodsky and Pumplin[11] argue that vector-dominance plus the scale-invariance of the electron scattering data determine the dependence upon mass of the direct photon-vector meson couplings. In this way the energy dependence (as $E \longrightarrow \infty$) of the colliding beam process $e^+ + e^- \longrightarrow$ hadrons should be E_{cm}^{-4}. But crude estimates of the magnitude of cross section required appear to give an answer unreasonably large.

Another way to make repairs is to lay the blame on σ_ℓ. Berman and Schmidt[12] some time ago found a reasonably good fit with $\sigma_\ell \sim 3\sigma_t$ for $q^2 \sim 1-2$ BeV^2, and using ρ-dominance only. The new data seem to throw doubt on this explanation, but the calculations haven't been pushed further.

We now turn to the third model, which is formal and relates νW_2, as well as the other form factor W_1, to commutators of electromagnetic current densities. Here these have been important contributions by Cornwall and Norton[8] and by Callan and Gross.[9] A simple way of summarizing the results of Cornwall and Norton is the statement[10]

$$\nu W_2 \xrightarrow[q^2 \to \infty]{} \lim_{P_z \to \infty} \frac{\omega}{2\pi} \int d^3x \int d\tau \, e^{-i\omega\tau} \langle P_z \left| \left[J_x\left(\underset{\sim}{x}, \frac{\tau}{P_0}\right), J_x(0) \right] \right| P_z \rangle - (x \longrightarrow z)$$

where $\omega = \dfrac{Q^2}{M\nu} = 2x$

If this limit exists and in finite, the scale-invariance of the data follows. By taking moments

$$2\int_0^2 d\omega \, \omega^{n-1} \left[\nu W_2\right] = \int d^3x \langle P_z \left| \frac{-i^n}{P_0^n} \left[\frac{\partial^n J_x(\underset{\sim}{x}, 0)}{\partial t^n}, J_x(0) \right] \right| P_z \rangle_\infty - (x \longrightarrow z)$$

one extracts equal-time commutators of the current with its nth time-derivative, n odd. Feynman has argued that the even moments can be expressed in terms of equal-time <u>anti-commutators</u>. The argument, translated from Feynman's private language into English, is that once the commutator is known asymptotically, the Wightman axioms determine the Wightman functions asymptotically and thus the anticommutator as well.

In practice, the most interesting moments thus far are for n = 0 and 1. For n = 0, the integral appears to diverge logarithmically. The inequality derived[15] from Adler's neutrino sum-rule

$$\int_0^{\tilde{\nu}(q^2)} d\nu \left[W_{2p} + W_{2n}\right] > \frac{1}{2}$$

is trivially satisfied if $\tilde{\nu}(q^2)$, the energy required to saturate the Adler sum-rule

$$\int_0^{\tilde{\nu}(q^2)} d\nu \left[\beta(\nu, q^2)^{\bar{\nu}p} - \beta(\nu, q^2)^{\nu p}\right] \cong 1$$

[For neutrino processes, β is analogous to W_2] is chosen large enough. Writing $\tilde{\nu} = x_{max} q^2$, a value of $x_{max} \sim 2$-3 gives the integral $\int d\nu\, W_{2p} \sim .25$.

However, if Pomeranchuk-exchange <u>is</u> the mechanism operating in the electron scattering it would appear that the Adler sum-rule is violated. Measurement of W_{2n} will clarify this question, which is at present uncertain. What can be said is that the trend of the data as function of q^2 is favorable, but that the magnitude of W_2 is small by a factor 2-3 for easy saturation of this sum-rule inequality.

The moment for n = 1

$$\int_0^2 d\omega \left[\nu W_2\right] = Q^2 \int \frac{d\nu}{\nu^2} \left[\nu W_2\right]_{q^2 \to \infty}$$

has been studied by Callan and Gross. It is related to $\langle P \left| \left[\frac{\partial J_i}{\partial t}, J_j\right] \right| P\rangle_\infty$ and appears to be finite and nonvanishing (≈ 0.3). Callan and Gross[9] have argued that by studying the ratio of the moments for W_2 and for the other form factor W_1, the quark model (in Lagrangian form: the "gluon" model) and field-algebra are distinguishable. In particular they conclude that

$$\sigma_t / \sigma_\ell \longrightarrow \begin{cases} 0 & \text{Field algebra} \\ \infty & \text{Quark model} \end{cases}$$

as $q^2 \longrightarrow \infty$. This is consistent with the expectations based upon the $n = 0$ divergent sum-rules[4] for W_1, or on the parton-picture. This Callan-Gross argument, however, stands on a more solid footing, being based on moments of the data which are finite although they depend upon Lagrangian models which may not exist.[17, 18]

In conclusion, the status of these three theoretical descriptions is summarized in Table I, and the predictions of the model for future electron scattering experiments is recorded in Table II.

Table I

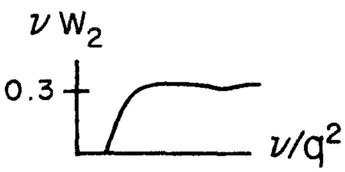
Features of the Data	Partons	Vector-Dominance	Current-Commutators
Scale-invariance	Yes	No apparent reason (This may have been remedied by Sakurai[18])	Yes, in sense that $p \longrightarrow \infty$ commutator exists
Shape 	Not easily explained	Yes	No statement
Magnitude: $\nu W_2 \lesssim 0.3$	Data a factor ~ 2 too small	Data too big (This may have been remedied by Sakurai[18])	Gross[19] predicts $n = 1$ moment from rest-frame commutator within a factor 2.

Table II

Predictions	Partons	Vector-Dominance	Current-Commutators
σ_t/σ_ℓ	∞ spin $\frac{1}{2}$ 0 spin 0 ? spin 1	$\ll 1$ [Berman-Schmidt] [M_ρ^2/Q^2 (Sakurai[18])]	∞ {Quarks or spin $\frac{1}{2}$ 0 Field algebra
σ_n/σ_p	0.8 Quarks 1 as $\nu/Q^2 \to \infty$ Drell et al. Model dependent in general	≈ 1	$\neq 1$ in general [The commutators are isospin - dependent]
A-dependence	$\sim$ A	$A^{2/3}$ (actually $\sim A^{0.8}$) for $\nu \gg 6$ BeV	??
Nature of final hadron states	Perhaps more longitudinal momentum in proton	Similar to γp or πp distribution	??

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2. W_1 and W_2 were defined by S. Drell and J. Walecka, Ann. Phys. 28, 18 (1964). One writes ($q^2 = -Q^2$)

$$(2\pi)^3 \frac{E}{M_p} \sum_n \langle P|j_\mu(0)\, n\rangle \langle n|j_\nu(0)|P\rangle (2\pi)^3 \delta^4(P_n - P - q) = \frac{P_\mu P_\nu}{M_p^2} W_2(q^2, \nu) - g_{\mu\nu} W_1(q^2, \nu) + \ldots$$

3. See, e.g., L. Hand, Proceedings of the 1967 Symposium on Electron and Photon Interactions at High Energies. The definitions are:

$$W_1 = \frac{(\nu - Q^2/2M_p)\,\sigma_t}{4\pi^2 \alpha} \qquad W_2 = \frac{(\nu - Q^2/2M_p)(\sigma_t + \sigma_\ell)}{4\pi^2 \alpha\,(1 + \nu^2/Q^2)} \quad .$$

4. J. Bjorken, Proceedings of the International School of Physics "Enrico Fermi," Course XLI; J. Steinberger, ed. (Academic Press, New York, 1968), see also J. Bjorken in Ref. 3.

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12. S. Berman and W. Schmidt, unpublished.

13. H. Harari, Weizmann Institute preprint.

14. D. Ritson, (unpublished).

15. J. Bjorken, Phys. Rev. Letters 16, 408 (1966).

16. S. Adler, Phys. Rev. 143, 1144 (1966).

17. Since the New York meeting, Adler and Tung (IAS preprint) and also Jackiw and Preparata (Harvard preprint) have cast some doubt on the Callan-Gross arguments for σ_t/σ_ℓ, as well as the arguments based on the "divergent" sum-rules for W_1.

18. Sakurai (EF1-69-16) has recently presented arguments, of a kinematical nature, that for the vector-dominant model $\sigma_\ell/\sigma_t = Q^2/m_\rho^2$. This restores scale-invariance at high Q^2, and allows one (a la Berman-Schmidt) to fit the data reasonably well. This work was done since the New York meeting.

19. D. Gross, (private communication).