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ASYMPTOTIC BEHAVIOR OF FORM FACTORS OF COMPOSITE PARTICLES *†

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ABSTRACT

The observed rapid fall-off of the proton's electromagnetic form factor at high spacelike momentum transfer squared q^2 may result from compositeness. Here we examine simple models to determine upper bounds on the asymptotic behavior of the form factors derived from them. General requirements of non-relativistic quantum mechanics yield form factors of two-body composites which fall at least as fast as $1/q^2$; with Yukawa forces providing the binding, this becomes $1/q^4$. For three bodies we find, respectively, faster than $1/q^4$ and faster than $1/q^7$. Relativistic considerations are introduced via the Bethe-Salpeter formalism, for both ladder and non-ladder kernels, and we find the results (apart from logarithmic factors) of $1/q^4$ for two bodies and $1/q^8$ for three bodies. Comparison with other recent work is made briefly.

I. INTRODUCTION

It has become clear recently that one way to understand the rapid fall-off of the electromagnetic form factor of the nucleon may be the idea of compositeness of the nucleon.[1,2,3,4,5] In this note we summarize our own recent work on this problem. The central experimental fact to which all our comments refer is that the nucleon form factors appear to fall-off at large momentum transfer approximately as $1/q^4$ but in any event faster than $1/q^2$.

If the form factor satisfies a dispersion relation

$$G(q) = \frac{1}{\pi} \int d\, q'^2 \, \frac{\operatorname{Im} G(q'^2)}{q'^2 - q^2} \qquad \text{(I-1)}$$

then for large spacelike q^2

$$G(q) \underset{q \to -\infty}{\sim} \left[\frac{1}{\pi} \int dq'^2 \operatorname{Im} G(q'^2) \right] \frac{1}{-q^2} \qquad \text{(I-2)}$$

The only way to obtain a more rapid decrease of the form factor would be to require that

$$\int d\, q'^2 \; \operatorname{Im} G(q'^2) = 0 \qquad \text{(I-3)}$$

It is very difficult to develop a non-composite theory or model calculation which leads to this relation as a natural consequence. Thus in dispersion theory it is very difficult to obtain a more rapid decrease of $G(q^2)$ than $1/q^2$.

On the other hand, as shown in Section II, this rapid fall-off may be obtained very easily with a non-relativistic two-body model of the nucleon. Here as elsewhere we take all particles to be spinless.

In Section III we will consider the modifications arising from assuming that the nucleon consists of three non-relativistic particles (e.g. quarks). We will find there that the form factor will fall-off as $1/q^8$ for a two-body Yukawa potential between each of the three particles. This appears to be too rapid a decrease to be consistent with experiment.

Next, we turn to relativistic models. In Section IV, we present some general statements based on perturbation theory. In Section V we obtain the asymptotic behavior of $G(q^2)$ in a two-body Bethe-Salpeter formalism assuming the ladder approximation. This work largely parallels the recently published work of Ball and Zachariasen[3] and Amati et al.,[4,5] for this reason, we emphasize here only the parts of our approach which differ from published work. In the same section, we also discuss the effect of non-ladder terms in the Bethe-Salpeter kernel.

Finally we proceed in Section VI to an analogous Bethe-Salpeter approach to composites of three or more bodies.

SECTION II

NON RELATIVISTIC THEORY-TWO BODIES

If the composite particle consist of 2 equal-mass, scalar particles bound in an S-state, then, non-relativistically, the form factor is given by the Fourier transform of the charge distribution. (If one of the particles is charged this is just the absolute square of the wave function.)[2]

$$G(q^2) = \int d^3 r e^{i \underset{\sim}{q} \cdot \underset{\sim}{r}} \rho(r) = \int d^3 r \; e^{i \underset{\sim}{q} \cdot \underset{\sim}{r}} \psi^2(r) \; . \qquad (II-1)$$

It follows that

$$\rho(0) = \int d^3 q \, G(q^2) = \frac{1}{4\pi} \int d q \, q^2 \; G(q^2) \; ; \qquad (II-2)$$

hence if the charge density is finite at the origin, then $G(q^2)$ must fall off faster than $1/q^3$.

More precise information can be obtained by using the result that, asymptotically,

$$G(q^2) \sim \frac{g(0)}{q^2} - \frac{g''(0)}{q^4} + \frac{g^{iv}(0)}{q^6} \; . \qquad (II-3)$$

where $g(r) = r\,\rho(r)$. For a potential which goes as $1/r$ as $r \to 0$,

e. g. a Yukawa potential, $g(0)$ is 0 and $g'(0)$ is finite; for this type of potential, then, $G(q^2) \sim 1/q^4$ as $q^2 \to \infty$.

Thus it is extremely easy to reproduce the observed fall-off of the form factors with such a model.

SECTION III

THREE (OR MORE) BODIES

We have seen that very general considerations show that the form factor of a two-body composite particle falls off as $q \to \infty$. We can view this as a consequence of the difficulty of having the two particles stay together as one is given an enormous kick by the electron. If this is the case, one would expect that the form factor for three bodies would fall off even faster as the difficulty of staying together would be compounded. We shall now see that this is indeed the case.

The Schrodinger Eq. for three bodies is

$$\left[-\frac{3}{4M}\nabla_s^2 - \frac{1}{4M}\nabla_r^2 + V(|\underset{\sim}{s}+\underset{\sim}{r}|) + V(|\underset{\sim}{s}-\underset{\sim}{r}|) + V(2r)\right]\psi = E\psi \qquad \text{(III-1)}$$

where

$$\underset{\sim}{r} = \frac{1}{2} (\underset{\sim}{r}_3 - \underset{\sim}{r}_2) \;, \; \underset{\sim}{s} = \frac{1}{2}(2 \underset{\sim}{r}_1 - \underset{\sim}{r}_2 - \underset{\sim}{r}_3)$$

Equation (II-3) still holds, with the replacements $r \to s$ and $g(r) \to g(s)$ where, now

$$g(s) = s \int d^3 r \left| \Psi(\underset{\sim}{r}, \underset{\sim}{s}) \right|^2 . \qquad \text{(III-2)}$$

Now, for two bodies, Hermiticity of the Hamiltonian requires that

$$r \Psi \to 0 \quad \text{as} \quad r \to 0$$

The generalization for three bodies is

$$r \Psi \to 0 \quad \text{as} \quad r \to 0 \qquad \text{(III-3a)}$$

$$\left| \underset{\sim}{r} - \underset{\sim}{s} \right| \Psi \to 0 \quad \text{as} \quad \left| \underset{\sim}{r} - \underset{\sim}{s} \right| \to 0 \qquad \text{(III-3b)}$$

and

$$\left| \underset{\sim}{r} + \underset{\sim}{s} \right| \Psi \to 0 \quad \text{as} \quad \left| \underset{\sim}{r} + \underset{\sim}{s} \right| \to 0 \; ; \qquad \text{(III-3c)}$$

in addition

$$s \Psi \to 0 \quad \text{as} \quad s \to 0 \, , \qquad \text{(III-3d)}$$

Using this information, we show in the Appendix that $g''(0) = 0$, in other words, from Eq. (II-3) for 3 bodies, $G(q^2)$ falls faster than $1/q^4$ with any two-body potential whatsoever.

If we limit ourselves to a superposition of Yukawas which goes as $1/r$ as $r \to 0$, then the most singular part of the Schrodinger equation is solved by

$$\Psi(r, s) = \sum_{mnp} C_{mnp} (2r)^m \left|r-s\right|^n \left|r+s\right|^p \qquad \text{(III-4)}$$

where m, n and p range over the positive integers and 0. From this it follows that

$$\int_0^\infty d\,qq^6\, G(q) = \left[(\nabla^2)^2 \int d^3\, r\, \rho(r, s) \right]_{s=0}$$

$$\propto \left[\int d^3 r \left\{ \delta(\underset{\sim}{r}-\underset{\sim}{s}) + \delta(\underset{\sim}{r}+\underset{\sim}{s}) + \text{less singular terms} \right\} \right] < \infty \qquad \text{(III-5)}$$

hence $G(q^7)$ falls to 0 faster than $1/q^7$ as $q^2 \to -\infty$. This fall-off is more rapid than that given by experiment.

SECTION IV

RELATIVISTIC CALCULATION - INTRODUCTION

In this section we shall derive the asymptotic behavior of the form factor of two-body scalar composites by means of the Bethe-Salpeter formalism, but before we do this we shall discuss a simple perturbation model which may help clarify the essential point of the relativistic result.

Consider first the most simple possible perturbation theory treatment of a composite form factor; since the system does not interact with the photon as a whole , this is given by the Feynman diagram in Fig. 1 or

$$G(q^2) = \int d^4p \frac{\Gamma_0^2}{\left[(P-p)^2 - m^2 + i\epsilon\right]} \frac{1}{\left[p^2 - m^2 + i\epsilon\right]} \frac{1}{\left[(P'-p)^2 - m^2 + i\epsilon\right]} \quad \text{(IV-1)}$$

$q = P' - P$ where for simplicity we ignore the spin of the photon.

The resultant asymptotic behavior in the space-like region is $G(q^2) \sim \frac{\ln q^2}{q^2}$, $q^2 \to -\infty$. Thus, the fact that the system is composite is sufficient to cause some fall-off of $G(q^2)$.

A more realistic model would have variable rather than constant coupling at the vertex of the composite and its constituent. This would lead, in many cases to faster fall-off; the essential feature being, in perturbation theory, that an additional propagator is needed to carry large space-like momentum to the constituent particle which does not couple directly to the photon. To illustrate this point further, we examine an over simplified model in which the behavior of the vertex is simulated by an extra propagator as shown in Fig. 2. Here we have added a new particle of mass $\sigma^2 > m^2 > \frac{1}{4} M^2$ as shown. The asymptotic behavior of this diagram can be proved[6] to be

$$G(q^2) \sim \frac{\ln q^2}{(q^2)^2} \quad .$$

An extension of this model would be the inclusion of higher order diagrams of a similar type. A typical example with n m and r propagators (of particles of mass-squared greater than $\frac{1}{4} M^2$) on the left and right legs and base of the triangle respectively, is shown in Fig. 3.

The behavior associated with this diagram is found to be $G(q^2) \sim \frac{\ln q^2}{(q^2)^N}$ where $N = \min(n, m)$, or, in other words, the contribution of each diagram will go as $\ln(q^2)$ times $(\frac{1}{q^2})$ raised to the power given by the smallest number of lines along which large space-like momentum must pass.[6]

Similar considerations would hold had we considered more realistic theories (such as a renormalized ϕ^3 Lagrangian) provided they were not too singular. We shall consider such theories in the Bethe-Salpeter formalism.

SECTION V

BETHE-SALPETER FORMALISM FOR TWO BODIES[6]

Ladder Approximation. In ladder approximation, the form factor $G(q^2)$ is given by

$$G(q^2)\ (2P+q)_\mu = \int d^4p\ [\phi^*(p+\frac{q}{2}, \underset{\sim}{P}+\underset{\sim}{q})\frac{1}{[(\frac{P}{2}+q+p)^2-m^2]}$$

$$\frac{(P+2p+q)_\mu}{[(\frac{P}{2}-p)^2-m^2]}\ \frac{1}{[(\frac{P}{2}+p)^2-m^2]}\quad \phi\ (p, \underset{\sim}{P})] \qquad (V\text{-}1)$$

Where ϕ represents the Bethe-Salpeter vertex function

$$\phi(p, P) = [(\frac{P}{2}+p)^2-m^2]\ [(\frac{P}{2}-p)^2-m^2]\chi(p, P) \qquad (V\text{-}2)$$

(χ is the usual Bethe-Salpeter wave function in momentum-space).

To extract the asymptotic behavior of $G(q^2)$ from Eq. (V-1). or Fig. 4, we must first extract the asymptotic behavior of ϕ from the Bethe-Salpeter equation:

$$\phi(p, \underset{\sim}{P}) = \int d^4p'\ \frac{1}{[(p-p')^2-\mu^2]}\ \frac{1}{[(p'+\frac{P}{2})^2-m^2]} \qquad (V\text{-}3)$$

$$\frac{1}{[(p'+\frac{P}{2})^2-m^2]}\quad \phi\ (p, \underset{\sim}{P})$$

either by solving the equation or somehow extracting the asymptotic behavior of ϕ by extracting denominators from under the integral sign on the right hand side of Eq. (V-3), a process which only makes sense if there is a sufficient knowledge of the behavior of ϕ from some other source to make sure that the integral remaining afterwards will still converge. The knowledge of the asymptotic behavior of ϕ, thus obtained, enables one to find the asymptotic behavior of G from Eq. (V-1).

Such analyses for two body composites in ladder approximation have recently been carried out independently by a number of different groups[3,4,5] (including the present authors). Since the others' work has already been published, and since our results agree completely we shall merely outline our own method and state the result, then, before proceeding to the discussion of more complicated kernels, we shall discuss the differences between our method and those of the other workers.

In order to obtain a bound on the asymptotic behavior of ϕ we impose the requirement that the self-mass correction of the bound state stemming from the type of diagram show in Fig. 5 be finite. This implies

$$\int d^4 p \, \phi^*(p, \underset{\sim}{P}) \frac{1}{[(\frac{P}{2} + p)^2 - m^2]} \frac{1}{[(\frac{P}{2} - p')^2 - m^2]} \phi(p, \underset{\sim}{P}) < \infty \qquad (V - 4)$$

The integral here is the coefficient of the double pole term in the second order correction to the bound state propagator.

The resultant upper bound on the asymptotic behavior of $\phi(p, P)$ is

$$\frac{1}{(\log (p^2))^{\frac{1}{2}}} \quad \text{as } p^2 \to \infty .$$

Inserting this behavior in the Bethe-Salpeter eq. allows us to extract the first denominator for p^2 large. We thus find $\phi\,(p, P)\ \frac{1}{p^2}$, up to logarithmic factors, as $p^2 \to \infty$.

Now the integral indicated by Fig. 4 (Eq. V-1) is done by choosing the momentum of the bottom line, say p_1, as the integration variable and performing a contour integral in the p_1^0 plane by closing the contour below. This yields an integral over the discontinuity of the cuts in ϕ starting at $p_1^0 = (p_1^2 + (m + \mu)^2 - i\epsilon)^{\frac{1}{2}}$ and the residue at the one particle pole at $p_1^0 = (p_1^2 + m^2)^{\frac{1}{2}} - i\epsilon$. The result is found to be $G(q^2)\ \frac{1}{(q^2)^2}$ up to logarithms.

One difference between the above and the other work cited is in the input assumptions about the asymptotic behavior of ϕ (corresponding to Eq. V-4). Our method for finding the asymptotic behavior of ϕ (Eq. V-4) is closely related to that of Ball and Zachariasen[3], who, however made use of a stronger input assumption, essentially

$$\int d^4 p \quad \frac{1}{[(p' + \frac{P}{2})^2 - m^2]} \quad \frac{1}{[(p' - \frac{P}{2})^2 - m^2]} \quad \phi(p', P) < \infty . \qquad (V-5)$$

(Since the quantity in curly brackets correspond to the wave function, this condition is analogous to the condition that the spatial wave function remain finite at the origin) Amati et. al.[5], on the other hand, obtain a bound on the behavior of ϕ by manipulating the partial wave Bethe-Salpeter equation into a Volterra integral equation, a bound on the solution of which is given by the solution of a simpler equation which they proceed to solve. The only input assumption is that needed for their form of Wick rotation to leave no contribution from ∞; this is assured by the condition ϕ (p, P) $\rightarrow \infty$ less rapidly than p_μ as $p_\mu \rightarrow \infty$, a much weaker condition than ours. The only difficulty with this method is in its extension to more complicated diagrams.

Both Ball and Zachariasen and Amati et. al. take up the case of one constituent particle having spin; Amati et. al. also treat arbitrary orbital angular momentum. We extend our discussion instead to non-ladder kernels and three-particle composites.

<u>Non-Ladder Contributions</u>.

If non-ladder kernels of the type shown in Fig. 6a are included then essentially the same arguments as above hold for the contribution of Fig. 4. But now the requirements of gauge invariance demand that we also include additional corresponding terms in G.

Consider Fig. 6b. The effect of the additional particles in the type of argument used above is to change the one particle pole to a three particle cut. Explicit examination shows that this yields the same type of behavior as Fig. 4. We conjecture that this also holds for Fig. 6c etc. on the basis that it is the minimum number of lines which must carry large space-like momentum which seems to determine the number of powers of $\frac{1}{q^2}$ in G.

SECTION VI

THREE BODIES: BETHE-SALPETER TREATMENT

It is convenient to introduce not the ladder kernel shown in Fig. 7, but a connected kernel as seen in Fig. 8. (Since no disconnected parts in fact appear in the bound state wave function, nothing is lost by keeping only the connected kernel.)

We now keep only the lowest order contribution in the Bethe-Salpeter equation (inspection suggests that the higher order terms will yield similar behavior). Then the right-hand side of the B-S equation will contain terms like

$$\phi(p_{12}, p_3, P) = \int d^4 p'_3 \, d^4 p'_{12} \frac{1}{\left(p_{12} - p'_{12} - \frac{p_3 - p'_3}{2}\right)^2 - \mu^2} \times$$

$$\times \frac{1}{\left(\frac{P}{3} - p_3 + p'_{12} - \frac{p'_3}{2}\right)^2 - m^2} \frac{1}{(p_3 - p'_3)^2 - \mu^2} \frac{1}{\left(p'_{12} + \frac{P}{3} - \frac{p'_3}{2}\right)^2 - m^2} \times$$

$$\times \frac{1}{\left(\frac{P}{3} - p'_{12} - \frac{p'_3}{2}\right)^2 - m^2} \frac{1}{\left(\frac{P}{3} + p'_3\right)^2 - m^2} \phi(p'_{12}, p'_3, P) \qquad \text{(VI-1)}$$

+ similar terms

Now using a condition analogous to (V-5) enables us to extract three denominators (apart from logarithms) from the above: thus we obtain the following asymptotic behavior in the various limits shown

$$\left.\begin{array}{lll} \phi \sim \frac{1}{p_3^6} & p_3^2 \to \infty & \text{(VI-2a)} \\ \phi \sim \frac{1}{p_{3\mu}^{\;3}} & p_3^2 \text{ finite } p_{3\mu} \to \infty, \ \frac{P_\mu}{3} - p_{3\mu} \to \infty & \text{(VI-2b)} \\ \phi \sim \frac{1}{p_{3\mu}^{\;2}} & p_{3\mu} \to \infty, \ \frac{P_\mu}{3} - p_3 \text{ finite} & \text{(VI-2c)} \end{array}\right\} p_{12} \text{ finite}$$

or with same limits for a, b, and c respectively,

$$\left.\begin{array}{ll} (2a) \to \phi \sim \frac{1}{p_3^4} & \text{(VI-3a)} \\ (2b) \to \phi \sim \frac{1}{(p_{3\mu})^2} & \text{(VI-3b)} \\ (2c) \to \phi \sim \frac{1}{(p_{3\mu})} & \text{(VI-3c)} \end{array}\right\} 2P_{12} - P_3 \text{ finite}$$

To obtain the three body form factor G, we now must evaluate Fig. 9.

After the integral over the relative momentum of particles 1 and 2 has been carried out, there remains one more 4 dimensional momentum integral. We take this momentum to be the total momentum of particles 1 and 2.

Again, the asymptotic behavior of this integral is found by converting the integration along the zero component to an integral over the discontinuities of cuts extending to $+\infty - i\epsilon$. These cuts are either cuts in

ϕ itself or simply the two body cut corresponding to particles 1 and 2 being on mass shell. In either case, the asymptotic behavior established for ϕ enables one to ascertain the behavior $G_{3\text{-body}}(q^2) \sim \frac{1}{(q^2)^4}$ (up to logarithms).

Again this corresponds to $(\frac{1}{q^2})$ raised to the power given by the smallest number of lines which must carry large space-like momentum in order that all three particles in the bound state enter and leave the diagram together. If we now add non-ladder terms of the type shown in Fig. 10a to the kernel then we again expect the same sort of behavior both for ϕ and for non-ladder terms in G itself: (Fig. 10b).

Hence, we conclude that for three particle bound states, the form factor should fall as $\frac{1}{(q^2)^4}$, up to logarithms.

N Particle Composites

The formalism for three bodies may be extended directly to N. The result will then be $G(q^2) \sim \frac{1}{(q^2)^{2N-2}}$ up to logarithms, since it it possible to extract 2N-3 denominators from ϕ and there is an additional denominator in the leg of the triangle connecting to the photon.

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APPENDIX I

PROOF THAT $G(q^2)$ FALLS FASTER THAN $\frac{1}{q^4}$ FOR 3 BODIES IN NON-RELATIVISTIC POTENTIAL

Note first that for $r > s$ the potential term in Eq. III - 1 is analytic and even in $z \equiv \hat{\underset{\sim}{r}} \cdot \hat{\underset{\sim}{s}}$. From this it follows that $|\psi(\underset{\sim}{r}, \underset{\sim}{s})|^2$ is even and analytic in z, $(|r| > |s|)$. But the wave function can depend only on the three inter-particle vectors $2\underset{\sim}{r}$, $\underset{\sim}{r}-\underset{\sim}{s}$ and $\underset{\sim}{r}+\underset{\sim}{s}$, and since it is a scalar the only s dependence is on s^2 and $\underset{\sim}{r} \cdot \underset{\sim}{s} = rsz$. Hence, $\psi(\underset{\sim}{r}, \underset{\sim}{s})^2$ is even and analytic in s for $r > s$. Thus $g^{(2n)}(0)$ would be 0 if we kept only the portion of the integral in Eq. (III - 2) in which $r > s$.

It follows immediately from the above observations and Eqs. (III-2) and (III - 3) that $g(0) = 0$ and $g'(s)$ is integrable; also we can write

$$g''(0) = g''_1(0) + g''_2(0) \tag{A - 1}$$

where

$$g_1(s) = s \int_{ks}^{\infty} r^2 \left[f_1(r) + f_2\left(r, \frac{s^2}{r^2}\right) \right] dr \tag{A - 2}$$

where $k > 1$, and the quantity in square brackets is the result of carrying out the z integration; $f_2(r, (s^2/r^2))$ is analytic everywhere in its second argument and $f_2(r, 0) = 0$. Also,

$$g_2(s) = s \int_0^{ks} r^2 dr \int_{-1}^{1} |\psi(r,s)|^2 \, dz \qquad (A-3)$$

Since the volume of integration in (3) is proportional to s^3 and since ψ^2 is less singular than $\frac{1}{r^2}$ (or $1/|r-s|^2$ or $1/|r+s|^2$), $g_2(s) \rightarrow 0$ faster than s^2 as $s \rightarrow 0$. Hence $g_2''(0) = 0$.

As for g_1'', by explicit differentiation

$$g_1''(s) = 4(k^3 s^2)\left[f_1(ks) + f_2(ks, \frac{1}{k^2})\right] - k(ks)^3\left[f_1'(sk) + f_2'(ks, \frac{1}{k^2})\right],$$

$$-ks^2 f_{2,1}(ks, \frac{1}{k^2}) + 3s \int_{ks}^{\infty} f_{2,1}(r, \frac{s}{r^2})\, dr + s^3 \int_{ks}^{\infty} \frac{1}{r^2} f_{2,2}(r, \frac{s^2}{r^2})\, dr, \qquad (A-4)$$

where $f_2'(x,y) \equiv \frac{d}{dx} f_2(x,y)$; $f_{2,n}(x,y) \equiv \left(\frac{d}{dy}\right)^n f_2(x,y)$.

Since both $f_1(r)$ and $f_2(r, \frac{s^2}{r^2})$ are less singular than $1/r^2$ as $r \rightarrow 0$, $(s < r)$, by direct examination, term by term

$$g_1''(0) = 0 \rightarrow g''(0) = 0. \qquad (A-5)$$

Thus, we have shown that for <u>any</u> acceptable wave function the three-body form factor $\rightarrow 0$ faster than $1/q^4$ as $q \rightarrow \infty$.

FOOTNOTES AND REFERENCES

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6. For details see Michael H. Goldhaber, Doctoral Dissertation Submitted to Stanford University (1968)

FIGURE CAPTIONS

Figure 1 Simplest possible form factor of a composite particle.

Figure 2 Form factor in composite model with bound-state vertex function replaced by additional propagator of mass .

Figure 3 Generalized diagram for same type of theory as Figure 2.

Figure 4 $G(q^2)$ in Bethe-Salpeter formalism (Ladder approximation).

Figure 5 Self-mass correction to bound state propagator.

Figure 6 a) A non-ladder contribution to the kernel

b) Additional term which must be added to $G(q^2)$ to insure gauge invariance when (a) is included in kernel.

c) Another non-ladder contribution to $G(q^2)$.

Figure 7 Disconnected three-body kernel

$\left[P \equiv k_1 + k_2 + k_3,\ p_{12} \equiv \frac{1}{2}(k_1 - k_2),\ \text{and}\ p_3 \equiv \frac{2}{3} k_3 - \frac{1}{3}(k_1 + k_2)\right]$

Figure 8 Connected three-body kernel.

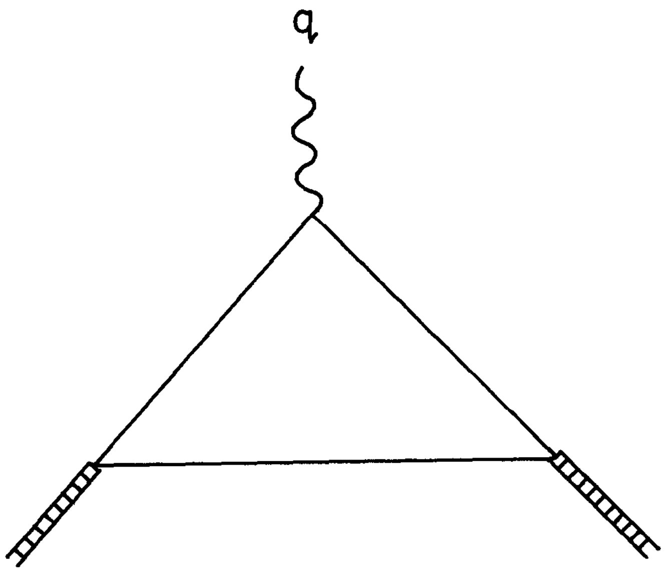

Fig. 1

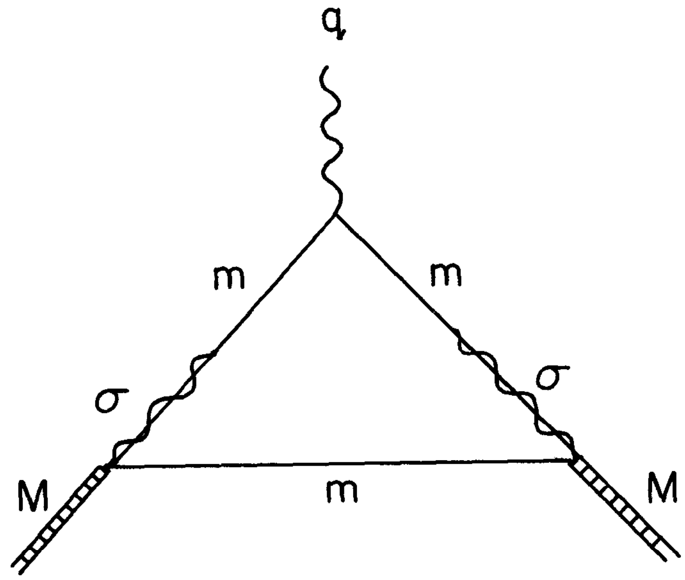

Fig. 2

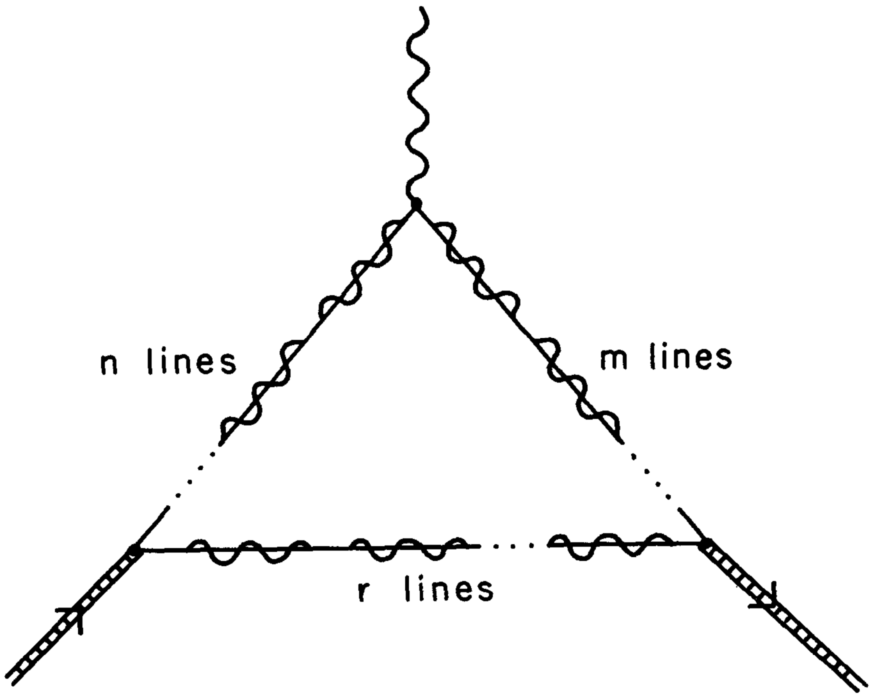

Fig. 3

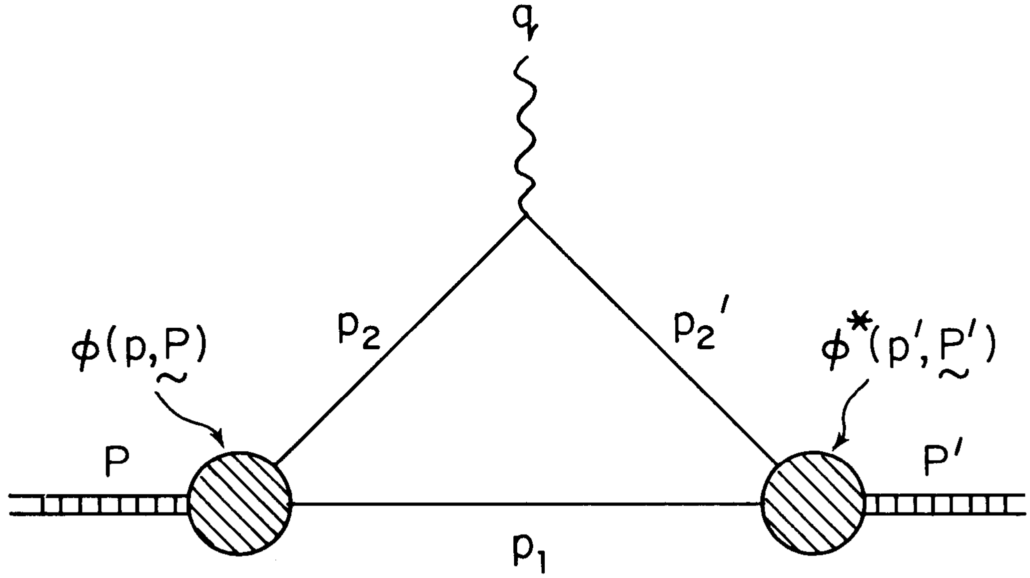

Fig. 4

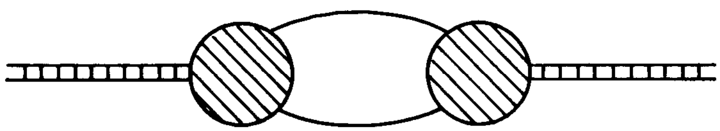

Fig. 5

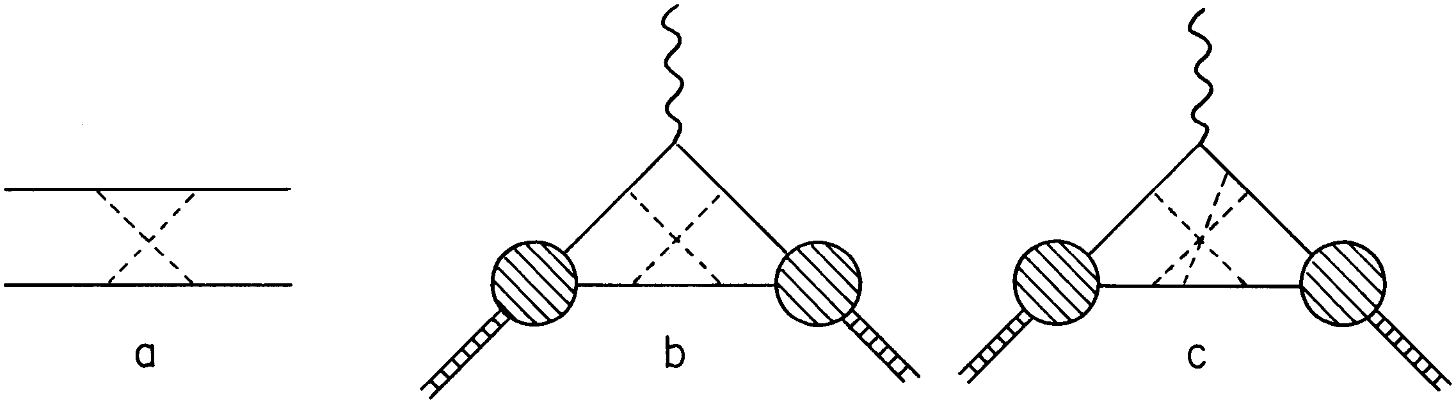

Fig. 6

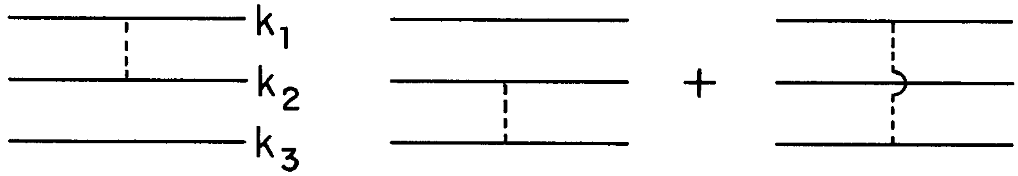

Fig. 7

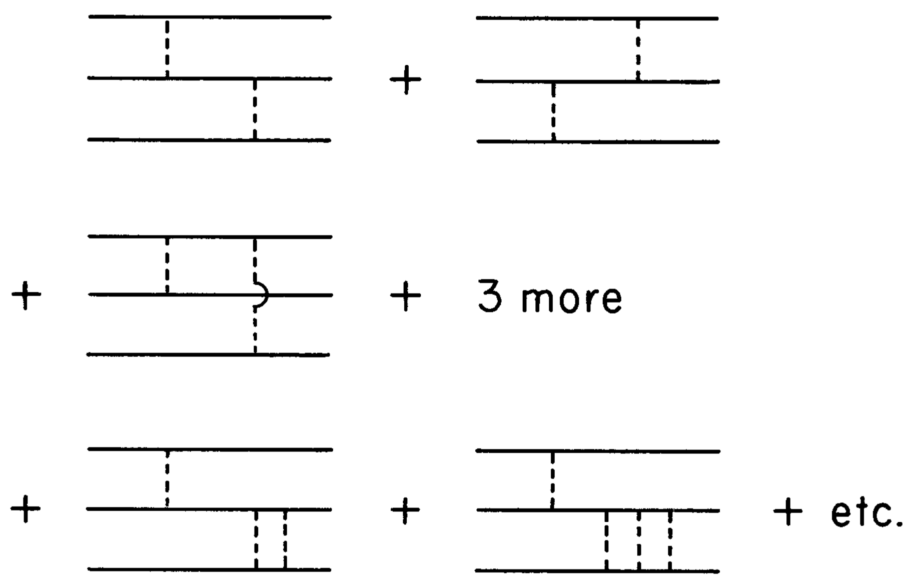

Fig. 8