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ABSTRACT

The cross section for the reaction $\gamma + C \rightarrow e^+ + e^- + C$ has been measured for a series of kinematic points giving nuclear momentum transfers in the region $0.2 - 0.7 F^{-2}$, with incident photon energies in the region for photoproduction of the first two pi-nucleon resonances. Measurements were made, using asymmetric detection geometry, of electron and positron cross sections for each point. The charge independent yields are shown to be in agreement with predictions of quantum electrodynamics for coherent and quasi-elastic production on carbon. The charge asymmetric results are shown to be consistent with recent estimates of the interference between Bethe-Heitler and Compton pair production amplitudes with excitation of an intermediate π -N state in the Compton diagram.

I. INTRODUCTION

The results of recent wide angle electron pair production experiments,^{1,2} performed to verify the predictions of quantum electrodynamics (QED), have been in general agreement with theoretical expectations³ based on the point interaction hypothesis of QED. These symmetric experiments detected both electron and positron at equal momenta (p_{\pm}) and at equal angles (θ) with respect to the incident photon beam. As a means of detecting possible deviations from the point interaction theory, the symmetric geometry has the advantages that (1) momentum transfer to the virtual electron is the same for both Bethe-Heitler graphs of Fig. 1(a), $q^2 = (k-p_{\pm})^2 \approx -2k \cdot p_{\pm}$, (2) momentum transfer at the nuclear vertex is relatively small, varying as $-p^2\theta^4$, such that the process is insensitive to the details of the nuclear interaction, and (3) contribution from interference between Bethe-Heitler and Compton amplitudes, Figs. 1(b) and 1(c) is small.⁴

The asymmetric detection scheme treated by BDF³ considered the detection of only one member of the electron pair; the unobserved lepton is produced preferentially in the forward direction. In this case, momentum transfer to the virtual electron is physically interesting only for the Feynman diagram in which the detected lepton proceeds directly from the incoming photon vertex, in which case it varies as $q^2 = -2k \cdot p$. If the detected lepton originates at the nuclear vertex, the virtual lepton is almost on the mass shell with $q^2 \approx \mu^2$, and this diagram contributes an estimated 80% of the yield. Thus an experiment of relatively high accuracy is required to place a reliable limit on the off-the-mass-shell behavior of the electron propagator. A second feature

of the asymmetric geometry is that contributions from interference between Bethe-Heitler and Compton amplitudes do not vanish and can lead to a charge asymmetric yield since the interference terms are odd under charge conjugation. Finally, for the asymmetric geometry, the momentum transfer to the target nucleus is relatively large, varying as $-p^2\theta^2$, which leads to the additional possibility of charge asymmetric contributions from interference between first and second Born Bethe-Heitler amplitudes, Fig. 1(d).

The first asymmetric pair production experiment⁵ observed only the 70 MeV positron produced at 90° in the laboratory by a bremsstrahlung beam with a peak energy of 137 MeV. The ratio of experimental to point interaction theoretical cross section was found to be (0.96 ± 0.14) , and no attempt was made to measure interference contributions. These were estimated by BDF, based on a Compton amplitude valid for incident photon energies well below threshold for excitation of the first pi-nucleon resonance, to be a factor of 10^{-4} lower than the direct yield from the square of the Bethe-Heitler amplitude.

More recently, asymmetric pair production^{2,6} has been used as a means of obtaining the relative phase of ρ^0 amplitude at high energies by measuring the interference between Bethe-Heitler and Compton amplitudes for e^+ , e^- masses in the region of the rho resonance. In these experiments both members of the pair are detected at nearly symmetric angles, momenta and transverse momenta, so that momentum transfer at the nuclear vertex is close to that of symmetric detection.

However, in the intermediate energy range, where π -N resonances may dominate the Compton interference contributions, there have been no previous experimental or theoretical investigations.

The purpose of the present work was to measure these terms below threshold for ρ^0 production by studying the reaction



using asymmetric detection geometry with one member of the pair momentum analyzed by a magnetic spectrometer and detected by a counter telescope consisting of gas and plastic Cerenkov counters. Carbon electromagnetic form factors are well known from the extensive electron scattering work at Stanford;⁷ the nucleus has spin zero which simplifies calculations and the low Z value minimizes second Born contributions to the experimental yield. The actual target material was polystyrene (CH) necessitating a small (5%) correction for pair production on protons in the plastic polymer chain.

Spurious events from protons, mesons and muons were effectively eliminated by the Cerenkov counters but appreciable background electrons from "double scattering" and from π^0 decays were measured and subtracted from the raw yields. Double scattering events, comprising 30 - 40% of the total target-in yield, arise from pair production in the forward direction followed by a second elastic scattering into the detector acceptance. Events from neutral pion production followed by either internal Dalitz decay or external conversion of one of the decay photons from the more common decay mode constituted less than 10% of the total target-in yield. For both types of background, the fraction of events was determined experimentally. Loss of real positron events from annihilation-in-flight was a calculated correction

to the experimental data.

Parts II and III describe the experimental apparatus and procedures. Part IV outlines the theoretical interpretations of the results in terms of recent calculations of Compton interference and first and second Born Bethe-Heitler interference contributions, and Part V presents a discussion of the results and conclusions.

II. EXPERIMENTAL APPARATUS

The energy analyzed electron beam of the Stanford Mark III Linear accelerator⁸ was used with an energy spread of 1%. The beam transport and energy defining systems have been described elsewhere.⁹ The primary energy calibration has been performed previously by floating wire measurements and could be set to an estimated accuracy of 0.5% using precision shunts monitoring analyzing magnet currents.

The primary electron beam emerged from the accelerator drift tube and first passed through the secondary emission monitor (SEM) shown by Fig. 2. This device was the primary beam current monitor and consisted of five 0.25 mil aluminum foils, had an efficiency of approximately 8% with a small energy dependence, and was mounted on a wobbling base which prevented fatigue effects on the efficiency by moving the foils so that the roughly 0.25 in. diameter beam spot swept over an area of about 1 square inch.

Following production of the bremsstrahlung beam in 0.0495 radiation lengths of copper foil, the residual electron beam was swept horizontally 20° to the right, passing through an air Cerenkov monitor signal into the beam catcher house. In order to minimize contamination of the proton spectrum, the sweeping magnet gap was filled with a

container of helium at atmospheric pressure. The air Cerenkov monitor signal provided a monitor of beam current profile; it was periodically photographed for use in estimating dead time corrections, and was supplied to the accelerator operator as an aid of obtaining a beam pulse as long as possible and reasonably free of violent fluctuations in electron density.

Positrons or electrons produced in a target of 0.995 gm/cm^2 of polystyrene plastic were collimated by a $2.5 \times 5 \text{ in.}^2$ lead mask at the entrance to the spectrometer vacuum chamber, and the residual photon beam was absorbed by approximately 35 radiation lengths of lead stacked on one side of the spectrometer face. Lead and tungsten collimation was used along the air gap between the sweeping magnet and target to prevent possible scattering of charged secondaries into the spectrometer acceptance.

SEM efficiency was measured by comparison with a Faraday cup (not shown in Fig. 2) which could be rotated into the electron beam line at about the position of the spectrometer. When not in use, the Faraday cup was moved well out of the path of the photon or swept electron beams. Intercalibrations of the Faraday cup with other Stanford and SLAC beam monitors, described by several authors,¹⁰ have indicated efficiency differences of less than 0.3%.

The spectrometer was an $n = 0$, 90° bend magnet,¹¹ with a mean orbital radius of 44 inches, mounted on a rotatable mount with provisions for remote angular adjustment and monitoring. The mean polar angle of the observed leptons was reproducible to within less than 0.05° and the absolute calibration was verified with reference to the primary beam line fiducials to the same precision. The measured momentum

dispersion of the spectrometer is $\Delta p/p = 0.01$ per inch at the focal plane and the length of the defining counters gave a total momentum acceptance of about 8% (FWHM). Polar angle acceptance, determined by the counter width of 4 inches due to lack of magnet focusing in this direction, was about 0.02 radians.

Spectrometer fields were monitored by a lithium chloride nuclear magnetic resonance (NMR) probe with only moderate success. The proton resonance signal was usable at low momenta, but at momenta greater than 300 MeV/c, the low signal-to-noise ratio did not provide a reliable field monitor. In this case, the shunt voltage readout of the magnet current was used as a field monitor, and the shunt settings calibrated against the NMR signal with accelerator rf off. The NMR-shunt voltage correlations were verified to within 0.2% and it was found that the polarity of the magnet could be reversed, using shunt readout only, with the same degree of accuracy.

The detection system consisted of two Plexiglas Cerenkov counters, C2 and C3, and a double chamber threshold gas Cerenkov counter, shown in Fig. 2. The gas Cerenkov counter is shown in detail in Fig. 3. Each plastic Cerenkov counter was made from $4 \times 8 \times 1$ in. thick ultraviolet transmitting Plexiglas with three Amperex XP1110 phototubes attached with epoxy resin to the 4×1 in. face of the plastic. The 4×8 in.² surface of the counter was oriented normal to the incident beam direction to match the sensitive area of the gas counter. The anode pulse output of the three tubes was added directly in a linear fan-in circuit and the single output was amplified to obtain a usable signal level. The measured efficiency of the counters for detection of minimum ionizing

cosmic rays was 0.75 , and, for the ranges of incident momenta, were sensitive to electrons and pions but rejected protons.

The gas Cerenkov counter (Fig. 3) was designed to provide a coincidence output from two optically independent chambers enclosed in a single gas envelope. The momentum analyzed beam entered the counter through a window laminated from 15×0.0075 in. thick Mylar film; the details of construction are described elsewhere.¹² The two chambers have identical optics and the spherical mirrors are fitted to the glass light pipe to eliminate light leaks between chambers. The gas dielectric is Freon 13 (CClF_3) at a pressure of 200 psig and a regulated temperature of 85°F . Under these conditions, the index of refraction is approximately 1.02 , giving threshold for pions of 700 MeV/c and for muons of 525 MeV/c . By reducing gas pressure, the pion and muon thresholds may be increased with a loss in Cerenkov light yield from electron events. The optimum operating conditions were obtained by jointly varying pressure and phototube voltages to obtain a plateau in the electron detection efficiencies for the two chambers. Repeated measurements of individual chamber efficiencies for electron detection gave momentum independent values of (0.94 ± 0.03) and (0.92 ± 0.03) for the first and second chambers, respectively, and the measured average pion/^{or}muon detection efficiency for both chambers was $(2 \pm 4) \times 10^{-4}$.

As stated above, the SEM was used as the primary monitor of incident electron current. The output of the SEM was integrated by a conventional feed-back type integrator. The integrator capacitors have been calibrated previously against 0.1% precision standards, and the capacitors used for the present experiment were rechecked and the values verified to within 0.2% .

III. EXPERIMENTAL PROCEDURES

A. Photoproduction Experiment

Measurements were made of electron and positron events at seven kinematic points defined by spectrometer angle and momentum and peak bremsstrahlung energy, which gave nuclear momentum transfers in the range 0.2 to 0.7 F^{-2} . An event was defined by the coincidence C3C5C7; including C2 in the logic requirement did not increase the detection efficiency of the telescope. Hence, it was included only during efficiency and rejection ratio measurements. For a fixed peak photon energy and kinematic setting of the spectrometer, three to five yield measurements were made at each polarity, and target-out yields were measured after each target-in run. Following this, a series of copper foils, varying from 0.01 to 0.04 radiation lengths in thickness, were inserted in the photon beam ahead of the target in order to measure the linear increase in double scattering events. Similarly, a series of aluminum foils was placed at the entrance of the spectrometer vacuum chamber to measure the contributions from multiple scattering and π^0 decay electrons or positrons.

SEM efficiency was calibrated against the Faraday cup at approximately six hour intervals or more frequently if primary electron energy had been altered. At least two measurements were made for each value of peak photon energy.

At the beginning and end of runs at one kinematic point, the sweeping magnet was turned off, and the primary electron beam allowed to strike the target with the spectrometer angle increased to a point

at which pion electroproduction was kinematically forbidden. Under these conditions, momentum analyzed electrons from inelastic scattering events in the target were used to measure gas Čerenkov counter efficiencies.

The efficiency of the gas counter for pions and muons was measured using the standard photoproduction experimental configuration and included the signal from the plastic counter C2 in the coincidence logic requirement. Using the electron detection efficiencies obtained from the procedure described above, values of efficiency for pion and muon detection were obtained which ranged from $(5 \pm 4) \times 10^{-4}$ to $(0 \pm 5) \times 10^{-4}$.

B. Data Reduction

The raw data consisting of C3C5C7 coincidence events were normalized to the integrated primary beam current, corrected for accidental coincidence rates of about 1% measured by delayed logic channels, and for dead time losses (< 4%) induced by the electronic logic. Target-out background was subtracted, and the result expressed as events per volt, or equivalently, as events/(1.003 μ Coulomb) of primary beam current. The three to five measurements at each polarity and each kinematic point were then averaged by Maximum Likelihood methods to obtain the values of "observed yield" listed in Table I.

Subtraction of double scattering and π^0 decay events was done in the following manner: In the absence of additional radiator foils in the photon beam, the observed lepton yield, after the corrections described above, may be expressed in the form,

$$Y_o = \sigma_p t + \sigma'_p \sigma_s (t/2 + t_u)t + \sigma_o t_d$$

In this expression,

- σ_p = electron pair production probability for asymmetric production at angle, θ ;
- σ'_p = electron pair production probability at forward angles;
- σ_s = elastic scattering probability at angle, θ ;
- σ_o = neutral pion production probability;
- t_u = material ahead of the target;
- t_d = material between the point of neutral production and the spectrometer vacuum chamber entrance, plus 1/80 for Dalitz decay probability;
- t = target thickness.

All production probabilities are in units of (radiation length)⁻¹.

A simple calculation shows that the fraction of double scattering events is given by the quantity,

$$\frac{Y - Y_o}{Y_o} \frac{t/2 + t_u}{\tau} ,$$

where Y is the observed event rate with τ radiation lengths of foil added in front of the target. Similarly, the fraction of π^0 decay events is given by,

$$\frac{Y - Y_o}{Y_o} \frac{t_d}{\tau} ,$$

where Y is the observed event rate with τ radiation lengths of foil added between the target and the spectrometer entrance. For most of the kinematic points investigated, the observed lepton energy was within 100 MeV or less of the peak energy of the bremsstrahlung spectrum which suppressed neutral pion production as compared with multiple

scattering when radiators were added between the target and the spectrometer vacuum entrance window. In these cases, it was assumed that the relatively small negative slope of yield with radiator thickness was due entirely to multiple scattering losses; the Dalitz decay probability was dropped from the sum of equivalent radiator thicknesses and the radiator subtractions carried out as usual with the appropriate change of sign. The resulting fractional values of π^0 or multiple scattering loss for the two spectrometer polarities have been averaged, since there is no physical reason to suppose a charge asymmetry, and the data are statistically consistent with a single value. The double scattering fractions, however, have not been averaged between polarities to allow for the possibility that an electron excess may be due to Compton electrons. The experimentally determined fractions for these background processes are listed in Table I.

Corrections for positron annihilation-in-flight in the target or in material ahead of the spectrometer entrance are implicit in the radiator subtraction procedure, however, annihilation in the counter telescope has been computed from the Heitler total cross section¹³ and the estimated thickness presented by the Cerenkov counters. The correction to the positron yields as given by Table I amounts to less than + 2% for all kinematic points.

C. Normalization

The combined acceptance and detection efficiency of the counter telescope was determined by a measurement of the yield of electrons scattered inelastically from the same target as used for pair production.

With a primary electron beam energy of 390 MeV, the spectrometer was adjusted to observe 170 MeV/c electrons scattered at 30° from the standard polystyrene target. Events originated from inelastic scattering and from double scattering, and, as in photoproduction runs, a sequence of copper radiators was inserted in the primary beam upstream of the target to measure the double scattering component.

The yield remaining after subtraction of double scattering events was due to wide angle bremsstrahlung (WAB) events in which a real photon is emitted either before or after an elastic scattering event. For the present experimental conditions, a photon of about 220 MeV is radiated and the largest part of the yield of WAB events (97%) comes from radiation by the incoming electron followed by scattering at 170 MeV. The Berg-Lindner¹⁴ cross section for this process, differential in the variables of the three final state particles, has been integrated numerically by Allton¹⁵ over the unobserved particle parameters; it has also been integrated by Hand¹⁶ in the "peaking approximation". In the latter, it was assumed that the principal contribution to the yield arises from events in which photon emission is approximately collinear with the incident or final electron. With this simplification, the WAB cross section factors into products of the Rosenbluth elastic scattering cross section and kinematic factors. If E_1 (E_2) is the incident (final) electron energy,

$$\frac{d\sigma}{d\Omega dE_2} = \frac{2}{k_1} \frac{x_1}{k_1} \frac{d\sigma}{d\Omega}(\theta, E_1') + \frac{x_2}{k_2} \frac{d\sigma}{d\Omega}(\theta, E_1) \quad , \quad (2)$$

in which,

$$\begin{aligned} \eta_1 &= E'_1/E_2 = [1 - E_2(1 - \cos \theta)/M]^{-1}, \\ \eta_2 &= E_1/E'_2 = 1 + E_1(1 - \cos \theta)/M, \\ \chi_i &= \left(\frac{\sigma}{\pi}\right)\left(\frac{q_i}{\eta}\right) \left[\ln\left(\frac{|q_i|^2}{\mu^2}\right) - 1 + \frac{k_i^2}{E_1 E'_1} \ln \frac{2E_i}{\mu} \right], \\ i &= 1, 2 \\ \eta &= E_1/E_2, \\ k_1 &= E_1 - E'_1, \\ k_2 &= E'_2 - E_2. \end{aligned}$$

For c.m. energies below the first resonance, the peaking approximation has been found by Allton¹⁵ to differ by less than 2% from a numerical integration of the exact Berg-Lindner formula over the phase space of the unobserved photon and lepton.

The WAB cross section has been calculated from Eq. (2) for interactions with C¹² (95%) and with protons, both quasi-elastically on protons in carbon and on free protons in the target (5%). The percentages indicate the relative contributions. Radiative corrections, following Berg and Lindner, have been calculated from the expression given by BDF for the closely analogous process of electron pair production. Including these corrections of + 5.6% the acceptance of the spectrometer was estimated as,

$$\frac{\Delta\Omega\Delta p}{p} = 1.71 \times 10^{-4} (1 \pm 0.04)$$

with errors due to statistics alone.

Since lepton momenta of the various experimental points extended from 170 MeV/c to 625 MeV/c, corrections to the acceptance for multiple scattering in the counter telescope were calculated from the known thickness of Plexiglas and Freon 15 in the counter telescope. The analysis assumed that one half of the gas counter material plus the thickness of counter C2 was concentrated at the location of C2, and a multiple scattering distribution for this geometry was constructed from the work of Nigam¹⁷. A Monte Carlo computer simulation was used to calculate the average efficiency of the telescope for each value of spectrometer momentum. The calculated efficiencies are shown in Table II, along with the effective spectrometer acceptance obtained by normalizing the multiple scattering efficiency to unity at the 170 MeV/c momentum point. Thus the multiple scattering correction had the effect of increasing the relative acceptance of the telescope by as much as 28% at higher momenta.

The second Born approximation single scattering cross section used by Nigam allows the computation of multiple scattering distributions which differ depending on lepton charge. The difference was apparent in the derived distributions to the extent of one part in 10^4 but produced negligible difference between multiple scattering corrections for electron and positron detection.

E. Experimental Results and Estimated Errors

A listing of the experimental results for the seven kinematic points is shown in Table I in which the observed event rate has been corrected for dead time losses, accidental coincidences,

since the multiple scattering efficiencies were normalized to unity at the 170 MeV/c normalization point.

In the pair production experiment, uncertainties in target density and thickness have been omitted since the same target was used for both pair production and normalization. Furthermore, since the Bethe-Heitler cross section, including carbon form factors varies approximately as $p^{-4.5} \theta^{-6}$ the uncertainty induced by spectrometer angular and momentum tolerances are 3% and 2.2%, respectively, considering the $\frac{1}{2}\%$ uncertainty in absolute momentum calibration and 0.05° angular tolerance in the absolute accuracy of the spectrometer alignment.

The error in the bremsstrahlung spectrum comes from uncertainty in the copper radiator foil (1.5%), a correction for contributions from the SEM foils (2.5%) and a small uncertainty due to drift in the end point energy of the photon spectrum (0.5%). Similarly, the error in radiator subtractions is due to uncertainties in the estimates of the thickness of extraneous material in the beam line and of the thickness of added radiators used to determine the fractional contributions from double scattering and π^0 decay contributions.

Errors due to beam current monitors and integrators have been assigned the value of 0.5%, the observed fluctuation in measured values of the SEM efficiency. Linearity and stability of the integrators was about 0.1% and the observed fluctuations may be attributed to slight missteering of the primary beam during calibrations or to other minor experimental errors.

The major uncertainty in estimates of dead time corrections came from the assumption that the primary beam pulse length was stable.

Long term fluctuations of pulse length varied between 0.4 and 1.2 nsec and dead time corrections for variations of this magnitude were based on the photographic records of beam pulse shape. Short term fluctuations of $\pm 10\%$ were common under the most ideal accelerator operating conditions, however, causing an estimated 0.4% uncertainty in the final results.

The quadratic combination of the various errors gives a total systematic uncertainty of less than 7.3% in the case of the pair production yields and the value of 3% from the reversibility of the spectrometer polarity applicable to the electron-positron asymmetry measurements.

IV. THEORETICAL ESTIMATES OF PAIR PRODUCTION

A. General Information

In this section, it is convenient to denote the amplitudes associated with the graphs of Fig. 1 by the following:

B_1 = first Born Bethe-Heitler amplitude

C = Compton amplitude for an intermediate Dirac proton

C^* = Compton amplitude with excitation of a resonant
pi-nucleon intermediate state of a target nucleon

B_2 = Second Born Bethe-Heitler amplitude

Then, as examples, the contribution to the cross section from the square of the first Born Bethe-Heitler amplitude is labeled as $B_1 \times B_1$, and the interference term between first and second Born Bethe-Heitler amplitudes as $B_1 \times B_2$. The notation is merely a means of labeling the process or the resulting contribution to the cross section.

B. Contributions From B_1 and C Amplitudes

The BDF calculation of pair production represented an improvement on the original Bethe-Heitler¹⁸ work by the inclusion not only of the Compton diagrams but also the effect of dynamical corrections from the proton current, the inclusion of general nuclear form factors, an estimate of radiative corrections and estimates of the $B_1 \times C$ and $C \times C$ contributions. The $B_1 \times B_1$ contribution was carried out in sufficient generality to allow the use of arbitrary nuclear form factors, but in the construction of the Compton amplitude, the target was considered merely as a point Dirac particle with an anomalous magnetic moment. With this model, the $B_1 \times C$ and $C \times C$ terms are expected to be valid only for incident photon energies well below the first pi-nucleon resonance.

The $B_1 \times B_1$ cross section differential in electron and positron variables, was integrated by BDF over the variables of the unobserved lepton to obtain the cross section for the asymmetric experiment. The integration required an expansion of the nuclear form factors to second order in q^2/M^2 . However, this expansion was not sufficient in the present calculations to represent the carbon form factors, since momentum transfers extended beyond the first diffraction minimum of the C^{12} form factors even for the lowest value of q^2 investigated. Hence, the BDF coincidence cross section has been used as a basis for estimating yields and was integrated numerically over unobserved lepton parameters.

The pair production differential cross section is

$$\frac{d\sigma}{d\Omega_1 d\Omega_2 dp_1 dp_2} = \frac{\alpha^3 M}{4\pi^2} \frac{p_1^2}{E_1} \frac{p_2^2}{E_2} \frac{G(k)}{Q \cdot (p_1 + p_2) - p_1 \cdot p_2} (\lambda_{11} + \lambda_{12} + \lambda_{22}) \quad (3)$$

in which $G(k)$ is the laboratory photon spectrum and λ_{11} , λ_{12} and λ_{22} are the contributions from $B_1 \times B_1$, $B_1 \times C$ and $C \times C$, respectively. The photon energy is given by

$$k = \frac{Q \cdot (p_1 + p_2) - p_1 \cdot p_2}{M - k \cdot (p_1 + p_2)/|k|},$$

and

$$\lambda_{11} = \frac{1}{2q^4} f_1(q^2) \left\{ \left(\frac{\mu}{k \cdot p_1} \right)^2 q^2 - 2 \left(\frac{k \cdot p_1}{k \cdot p_2} + \frac{k \cdot p_2}{k \cdot p_1} + \frac{q^2 p_1 \cdot p_2}{k \cdot p_1 k \cdot p_2} \right) \right\} + \frac{1}{2q^4} f_2(q^2) \left\{ 2 \left(\frac{\mu p_1 \cdot p_2}{k \cdot p_2} \right)^2 - q^2 \frac{(p_1 \cdot p_2)^2 + (p_2 \cdot p_2)^2}{k \cdot p_1 k \cdot p_2} \right\}. \quad (4)$$

The remaining terms are defined by Table IV. The subscript 1 (2) denotes positron (electron) variables, and, with the exception of terms of order μ^2 which arise from simplifications made by BDF and not from inherent asymmetry of the $B_1 \times B_1$ terms, the expression is symmetric under charge conjugation: $1 \leftrightarrow 2$. Similarly, the $C \times C$ terms are even under charge conjugation and the $B_1 \times C$ terms are odd; for the conditions of the present experiment, these contributions are factors of 10^{-5} and 10^{-3} lower than the $B_1 \times B_1$ yield and are omitted from further consideration.

The form factors in Eq. (4) are given by BDF as,

$$(1/Z^2) f_1(q^2) = \frac{q^2}{M^2} (F_1 + \kappa F_2)^2 + \frac{P^2}{2M^2} F_1^2 - \frac{\kappa^2 q^2}{4M^2} F_2^2 \quad (5)$$

$$(1/Z^2) f_2(q^2) = \frac{1}{M^2} F_1^2 - \frac{\kappa^2 q^2}{4M^2} F_2^2 \quad ,$$

in which $F_1 \equiv F_1(q^2)$ and $F_2 \equiv F_2(q^2)$ are the Dirac and Pauli form factors. The form factors F_1 and F_2 are related to other form factors commonly used in describing electron-proton elastic scattering by the following:

$$F_{ch} = G_E = F_1 - r\kappa F_2 \quad ,$$

$$F_{mag} = G_M / (1 + \kappa) = (F_1 + \kappa F_2) / (1 + \kappa)$$

$$r = |q^2| / (4M^2)$$

Using these relations, the approximation that $P^2 \approx 4M^2$, and the fact that $G_M^2 = 0$ for carbon, Eqs. (5) become,

$$(1/Z^2) f_1(q^2) = 2G_E^2 / (1 + r) \quad (6)$$

$$(1/Z^2) f_2(q^2) = (G_E/M)^2 / (1 + r)$$

Form factors for proton targets are obtained by defining,

$$G \equiv G_{E_P} = G_{M_P} (1 + \kappa) \quad , \text{ from which the following form factors applicable}$$

to Eq. (4) are obtained:

$$f_{1P}(q^2) = \frac{2G^2}{1+r'} [1 - r'(1+\kappa)^2] \quad (7)$$

$$f_{2P}(q^2) = \frac{G^2}{m^2(1+r')} [1 + r'(1+\kappa)^2]$$

in which $r' = q^2/(4m^2)$, and terms of order r'^2 have been neglected.

Form factors for quasi-elastic production have been based on the expression obtained by Drell and Schwartz¹⁹ and experimentally verified by Faissler and co-workers²⁰ in electron-carbon scattering. The quasi-elastic form factors reflect the relative probability that an electromagnetic interaction takes place with an individual nucleon of the target rather than coherently with the nucleus. The Drell-Schwartz sum rule for elastic and quasi-elastic electron interactions modifies the elastic form factor by the replacement:

$$Z^2 F^2 \rightarrow G^2 [Z + Z(Z-1)F^2 + O(Z/m^2)]$$

in which $F^2(q^2)$ is the elastic nuclear form factor, $G^2(q^2)$ is the free proton form factor and $O(Z/M^2)$ represents the largest of three correction terms for the conditions of this experiment. It amounts to about 1% of the preceding terms and has been neglected. Subtracting the carbon elastic term, $Z^2 F^2$, and substituting from Eqs. (6) and (7), the quasi-elastic form factors for the $B_1 \times B_1$ cross section are:

$$f_{1Q}(q^2) = f_{1P} [Z + (Z-1) f_1/2Z] - f_1$$

$$f_{2Q}(q^2) = f_{2P} (m/M)^2 [Z + (Z-1) M^2 f_2/Z] - f_2 \quad (8)$$

The second of Eqs. (3) does not follow directly from (6) and (7). The additional factor of $(m/M)^2$ is required by the limiting form of the BDF form factors, $f_2(0) = Z^2 M^{-2}$ and $f_{2p}(0) = m^{-2}$. If the proton and carbon contributions to the quasi-elastic yield are computed separately from Eq. (4), the appropriate mass term in the form factor cancels a similar term in the quantities $(p_i \cdot P)^2$ which appear in Eq. (4) and are closely approximated by $4E_i^2 M^2$ or $4E_i^2 m^2$. Addition of the factor $(m/M)^2$ simplifies computations by allowing the use of the carbon mass in Eqs. (3) and (4). Then $f_{1Q}(0) = f_{2Q}(0) = 0$.

The estimate of $B_1 \times B_1$ production in the plastic target is given by an integration of Eq. (3) over the unobserved lepton variables and the spectrometer acceptance. For events arising from carbon elastic, quasi-elastic and from protons in the CH target the yield has been obtained by evaluation of the integral:

$$Y(B_1 \times B_1) = Q \int dp_1 dp_2 d\Omega_1 d\Omega_2 d\sigma(B_1 \times B_1; p_1, p_2, \theta_1, \theta_2, \phi_2) \quad (9)$$

using form factors from Eqs. (6), (7) and (8), respectively. The factor Q contains target and beam monitor parameters, and the spectrometer resolution was assumed to be sharp enough to allow the substitution $dp_2 d\Omega_2 \rightarrow \Delta p_2 \Delta \Omega_2$ for the detected lepton. The effect of a finite photon beam has been suppressed in the integration since carrying out the integration using a realistic photon density distribution over the target produced an error of less than 0.01%.

The integrals were evaluated to a 1% convergence tolerance by Monte Carlo methods²¹ and the results are listed in Table V. Column 4 gives the expected yield per volt of integrated primary beam current

into 1.003 μf capacitor for $B_1 \times B_1$ production on carbon nuclei, and the remaining columns list other contributions as a fraction of the $B_1 \times B_1$ yield on carbon. Yields from quasi-elastic and proton production are given in columns (5) and (6) respectively.

C. Contributions from $B_1 \times B_2$ Interference

The interference between first and second Born Bethe-Heitler pair production amplitudes is expected to be a small correction to the $B_1 \times B_1$ contribution. The relevant diagrams are shown in Figs. 1(a) and 1(d). Calculation of this term, while straightforward in principle, leads to a number of integrals for which exact solution is extremely difficult for spin $\frac{1}{2}$ pair production. In the case of spin 0, however, calculations by Brodsky and Gillespie²² have shown that the principal contribution from the three B_2 amplitudes derives from the first two graphs on the left of Fig. 1(d). Then by analogy with the arguments concerning the relative yield from the two B_1 graphs, the principal electron yield from $B_1 \times B_2$ interference will be the result of interference between the middle graph of Fig. 1(d) and the right-hand graph of Fig. 1(a). In both graphs, the virtual electron coming off the incident photon vertex is almost on the mass shell, so that the ratio of yields, $B_1 \times B_2 / B_1 \times B_1$ is given to a good approximation by the ratio of second Born interference to first Born elastic electron scattering cross sections. Similarly, the principal contribution to asymmetric positron production arises from interference between the left-hand graphs of Figs. 1(a) and 1(d), and the ratio of $B_1 \times B_2 / B_1 \times B_1$ for this case is approximated by the ratio of second Born interference to first Born elastic positron scattering cross sections. Thus an

estimate of the $B_1 \times B_2$ pair production yield has been obtained by assuming,

$$\sigma(B_1 \times B_2) = (\pm R) \sigma(B_1 \times B_1), \quad (10)$$

where $\pm R$ = ratio of second Born/first Born elastic scattering cross sections; positive for electron detection, negative for positron.

Calculations of the ratio R have been carried out by Lewis²³ assuming a static, spherically symmetric nucleus with a Yukawa charge distribution. Using Lewis' notation,

$$R = 2Z\alpha [K^2 A_2 / (a^2 P^2)] \left\{ [k/(KA_1)](P^2 + 4k^2 + 2a^2) \arctan [aK/(2A_1)] \right. \\ \left. + (2k/K) \arctan[2a^3/(K^3 + 3Ka^2)] \right. \\ \left. - [(P^2 + 4k^2 + a^2) / A_2] \arctan [a/(2K)] \right\} \quad (11)$$

with

$$A_1 = (k^2 K^2 + 4k^2 a^2 + a^4)^{\frac{1}{2}},$$

and

$$A_2 = K^2 + a^2.$$

In the context of elastic electron scattering, \underline{K} = three-momentum transfer to the nucleus, \underline{P} = sum of initial and final electron three-momenta, and k = absolute value of the final electron three-momentum. For pair production, assuming detection of the electron, the momentum of the almost real virtual lepton coming from the real photon vertex assumes the role of the initial electron in the elastic scattering calculation. That is, $\underline{K} \rightarrow \underline{q}$, $\underline{P} \rightarrow \underline{q} + 2\underline{p}_2$ and $K \rightarrow |\underline{p}_2|$. The

terms in Eqs. (11) become:

$$\begin{aligned}
 K^2 &\approx -q^2 \\
 P^2 &\approx 4E_2^2 - 2(k \cdot p_2 - k \cdot p_1 - p_1 \cdot p_2) \\
 k &\approx E_2
 \end{aligned}
 \tag{12}$$

and those on the right are consistent with the definitions of Table IV. The Yukawa parameter has been assigned the value, $a = 173 \text{ MeV}/c$, by an empirical fit of the analytic form, $F(q^2) = (1 + q^2/a^2)^{-1}$, to the experimental carbon electromagnetic form factor.

The fractional contribution from this process has been calculated from the ratio of the integral,

$$\begin{aligned}
 Y(B_1 \times B_2) &= Q \int dp_1 dp_2 d\Omega_1 d\Omega_2 \times d\sigma(B_1 \times B_1; p_1, p_2, \theta_1, \theta_2, \phi_2) \\
 &\quad \times R(p_1, p_2, \theta_1, \theta_2, \phi_2)
 \end{aligned}
 \tag{13}$$

to the value of the $B_1 \times B_1$ yield of Eq. (9) using $d\sigma$ from Eq. (3), carbon form factors and R from Eqs. (11) and (12). The ratios, listed in Table V, turn out to be relatively small compared with the experimental charge asymmetries and, above a nuclear momentum transfer of $0.3\text{-}0.4 \text{ F}^{-2}$, have a sign opposite to that observed.

D. Contributions from $B_1 \times C^*$ Interference

For the region of incident photon energies used in the present experimental study, the BDF calculations of $B_1 \times C$ are no longer valid; the interference term must consider formation of virtual states of the pi-nucleon system. Preliminary results of a calculation of this contribution by Brodsky, Hearn and Parsons²⁴ have been used for

the present analysis. It is assumed that the Compton interaction takes place on a single nucleon of carbon, exciting a virtual pi-nucleon state which decays by virtual photon emission leaving the nucleus in the ground state. The phenomenological form of the C^* amplitude has been assumed to follow the Breit-Wigner single level resonance formula, and for the preliminary result given below, it has been assumed that terms in electron mass squared and nucleon magnetic moment may be neglected and that the crossed diagram of Fig. 1(c) is small compared with the direct term. For detected electron energies much greater than that of the unobserved positron, the ratio, R^* , is defined as the ratio of cross sections differential in angles and momenta of both members of the pair:

$$\begin{aligned}
 R^* = \frac{d\sigma(B_1 \times C^*)}{d\sigma(B_1 \times B_1)} &= (8/3)(A/Z) [(m^3 + 2mM_3^2 + 3M_3^3 + 2m^2E_2)/M_3^3] \\
 &\times (E_2/m) \frac{(P^{*2} - M_3^2) \underline{q}^2}{(P^{*2} - M_3^2)^2 + \Gamma^2 M_3^2} \\
 &\times [p_1 \cdot p_2 - (\theta_2^2 - \theta_1^2)E_1E_2/2] / (p_1 + p_2)^2, \quad (14)
 \end{aligned}$$

in which $P^{*2} = (k + Q)^2 = 2km + m^2$,

M_3 = mass of the resonant state, MeV,

Γ = resonance width, MeV, and

\underline{q}^2 = square of nuclear three-momentum transfer.

For incident photon energies up to about 800 MeV, analysis²⁵ of experimental proton Compton scattering ($C^* \times C^*$) is in reasonable agreement with a superposition of non-interfering P_{33} and D_{13}

states of the pi-nucleon system associated with the $\Delta(1236)$ and $N^*(1525)$. Calculation of the P_{33} contribution has been carried out with the assumptions of: (a) a constant width²⁶ of $\Gamma = 120$ MeV, (b) an energy dependent width of the form²⁷,

$$\Gamma(P^*) = 120 \left(\frac{P_\pi^*}{P_R^*} \right)^3 \text{ MeV} \quad (15)$$

and (c) a variable width of the form²⁸,

$$\Gamma(P^*) = 127.5 \left(0.85 \frac{P_\pi^*}{m_\pi} \right)^3 / [1 + (0.85 \frac{P_\pi^*}{m_\pi})^2] \text{ MeV}, \quad (16)$$

in which P_π^* is the pion momentum in the rest system of the $\Delta(1236)$,

$$P_\pi^{*2} = (km + m_\pi^2/2)^2 / (2km + m_\pi^2) - m_\pi^2, \quad (17)$$

and P_R^{*2} is obtained by setting $km = (M_3^2 - m_\pi^2)/2$ in Eq. (17).

The estimate of the D_{13} state contributions to the interference used only the experimental value²⁶ of the resonance width, $\Gamma = 105$ MeV, and in view of the preliminary nature of the theoretical work, the calculation of R^* for the two virtual states differed only by use of the respective masses and widths. The fractional contributions were calculated from the ratio of the integral:

$$Y(B_1 \times C^*) = Q \int dp_1 dp_2 d\Omega_1 d\Omega_2 \times d\sigma(B_1 \times B_1; p_1, p_2, \theta_1, \theta_2, \Phi_2) \\ \times R^*(p_1, p_2, \theta_1, \theta_2, \Phi_2), \quad (18)$$

divided by the $B_1 \times B_1$ result from Eq. (9). The resulting values obtained for each of the experimental points is given in Table V for the case of the cubic variation of the P_{33} resonance width which

turned out to provide a better fit to the experimental data than either the experimental value of 120 MeV or the Dalitz and Sutherland estimate. The sign of the interference contribution is clearly dominated by the term $(P^{\star 2} - M_3^2)$, although at the 170 MeV/c, 30° experimental point, the negative value has been suppressed since the range of the integration over photon energies contains the zero of R^{\star} . This is illustrated by Fig. 4 which shows the relative variation of the resonant factor of Eq. (14) with incident photon energy; the bars indicate the range of integration required for the four detected lepton momenta.

E. Radiative Corrections

The BDF calculations give radiative corrections to the $B_1 \times B_1$ process in the form

$$\sigma_{\text{RAD}} / \sigma(B_1 \times B_1) = \frac{\alpha}{\pi} \left[\ln \frac{2p_1 \cdot p_2}{\mu^2} \left(13/6 - \ln \frac{E_1 E_2}{(\Delta E)^2} \right) \right], \quad (19)$$

in which $\Delta E = k_{\text{max}} - k$, and k_{max} is the peak energy of the incident bremsstrahlung spectrum. The correction factors are given in Table V, column 7 as the ratio of the integral,

$$Q \int dp_1 dp_2 d\Omega_1 d\Omega_2 \sigma(B_1 \times B_1; p_1, p_2, \theta_1, \theta_2; \varphi_2) \frac{\sigma_{\text{RAD}}}{\sigma(B_1 \times B_1)},$$

to the $B_1 \times B_1$ yield given by Eq. (9).

F. Analysis and Comparison with Experimental Results

For each of the kinematic points listed by Table V, the estimated

theoretical yields may be written in the form,

$$Y(e^\pm) = Y(B_1 \times B_1) [1 + R_{\text{even}}^\pm + (R_{12} + a_1 R_{33} + a_2 R_{13})] \quad (20)$$

in which $Y(e^\pm)$ is the theoretical positron (electron) yield, R_{even} represents a sum of the ratios of quasi-elastic, proton and radiative correction terms, even under charge conjugation, and R_{12} is the $B_1 \times B_2$ interference term, odd under charge conjugation. R_{33} and R_{13} are the odd contributions from the P_{33} and D_{13} pi-nucleon resonance interference terms, and the a_i are constants determined by comparison of experimental and theoretical results. The expected charge asymmetry of the data is,

$$A = \frac{Y(e^-) - Y(e^+)}{Y(e^-) + Y(e^+)} = (R_{12} + a_1 R_{33} + a_2 R_{13}) / (1 + R_{\text{even}}), \quad (21)$$

and the a_i are found by Maximum Likelihood methods for each of the three forms of the P_{33} resonance width. The values of the a_i , shown in Table VI, use all of the available experimental data to obtain a best fit. Discarding the least reliable of the seven kinematic points, on the basis of non-reproducibility of the data, increases the Chi-square probability of the fit to approximately 0.8, but does not change the essential result that the cubic variation of the resonance width, Eq. (15), has a slightly higher probability of representing the data. The case for the cubic variation is reinforced by theoretical arguments²⁷ that the width should vary as $2L + 1$, where $L = 1$ for the P_{33} state.

In all cases, the coefficient of the second resonance contribution is consistent with zero and for cubic variation of the P_{33} width, the value of a_1 is essentially unity. Thus for the calculation of the

estimated theoretical yields, given in Table VII, the values $a_1 = 1$ and $a_2 = 0$ have been used. Table VII also shows a comparison between experimental and theoretical values of charge asymmetry, the latter calculated from Eq. (21) with the a_i as given above. The stated errors in both tables are due to counting statistics only.

V. DISCUSSION AND CONCLUSIONS

The ratios of experimental to theoretical results shown in Table VII indicate general agreement of the experiment and theory. The values of $R(\text{exp}/\text{thy})$ essentially represent the consistency of the data with the predictions of QED for the case of asymmetric detection geometry, and, assuming that the contributions from the "bad" diagram of Fig. 1(a) agree with the point interaction hypothesis of QED, the ratio of experiment to theory may be written as:

$$\text{Experiment/Theory} = (0.95 \pm 0.06) - (0.3 \pm 0.82) |q^2|/5$$

from which a 68% level of confidence limit on the cut off parameter give $\Lambda < 1.06$ Fermi. In addition to the statistical errors given in this expression, the systematic errors have been estimated to be less than 7.3% as given in Table III. Without consideration of the validity of QED, the ratio may be written as,

$$\text{Experiment/Theory} = (0.95 \pm 0.06) - (0.06 \pm 0.16) |q^2|$$

indicating the lack of evidence for q^2 dependent systematic errors, a verification of the BDF formulation of asymmetric pair production and the Drell-Schwartz quasi-elastic sum rule contributions which become an appreciable fraction of total production at the larger nuclear momentum transfers.

The experimental charge asymmetry has been explained almost entirely by interference between Bethe-Heitler and Compton amplitudes with excitation of the $\Delta(1236)$ resonance. Based on the preliminary theoretical work, the distinguishing observable feature of this cross section would be a sharp reversal of the sign of the charge asymmetry at the zero of the P_{33} amplitude corresponding to an incident photon energy of about 340 MeV. Unfortunately the only experimental point obtained in this region corresponded to an average of photon energies over the interval, 170 - 390 MeV, resulting in a small, but statistically unreliable positron excess as expected from the theoretical analysis. For the remaining experimental points, an electron excess was expected and observed, and the conclusions about the charge asymmetric production are weighted heavily by these results due not only to their number but to their smaller statistical errors. On this basis, the observed charge asymmetry is consistent with the Brodsky et al.²⁴ estimate, Eq. (14), of the interference contribution, using a cubic variation of the resonance width normalized to the experimental value of 120 MeV at the zero of the $\Delta(1236)$ interference term. In general this interference term is a factor of about 10^2 greater than that based on the low energy Thomson limit for the Compton process, valid below the first resonance.

Contributions from the $N^*(1525)$ to the experimental yield appear to be suppressed, although the absolute values predicted from Eq. (14) with the N^* mass and width are of the same absolute order of magnitude as those for the $\Delta(1236)$. The uncertainties in present theoretical predictions and experimental numbers make any conclusions regarding contributions from the $N^*(1525)$ purely conjectural. The contribution

from the D_{13} amplitude may be suppressed by a factor of two from the Clebsch-Gordan coefficients if the incoming photon is predominantly isovector. Additionally, since the Bethe-Heitler amplitude is real, the interference terms measure the real part of the pi-nucleon amplitude and the D_{13} suppression may indicate the phase difference between P_{33} and D_{13} amplitudes, in qualitative agreement with the apparent lack of interference between these states in the results of Compton scattering²⁵.

In conclusion, it has been shown that (1) asymmetric electron pair production in the region of the first two pi-nucleon resonances is in general agreement with known QED processes, (2) charge asymmetric production is consistent with recent estimates of interference between Bethe-Heitler and Compton processes with excitation of pi-nucleon resonant states which contribute interference terms approximately 10^2 greater than those expected from the low energy Thomson limit, and (3) that interference between first and second Born approximation pair production processes is a relatively small contribution to the charge asymmetric production in this region. Further measurements of charge asymmetric production may provide a means of increasing the present degree of understanding of the baryon resonances. For example, the magnitude of the asymmetry is sensitive to the assumed form of the presently poorly defined energy dependence of the resonance width. Secondly, the sharp variation of the asymmetry at the zero of the Compton interference may provide an independent means of measuring the mass of the $\Delta(1236)$ if the asymmetry from the non-resonant background is as small as indicated in this work.

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TABLE 1

EXPERIMENTAL RESULTS AND CORRECTIONS

| E_0 (MeV) | P (MeV/c) | θ (deg) | POLARITY | EXPER. YIELD ^(a) | FRACTIONAL CORRECTION FACTORS | | | CORR. NET YIELD ^(e) |
|----------------|--------------|-------------------|-------------|--------------------------------|-------------------------------|---------------------|-----------------------------|-----------------------------------|
| | | | | | PI DECAY ^(b) | D.S. ^(c) | ANNIHILATION ^(d) | |
| 120.0 | 170.0 | 10.00 | + | 16.29 ± 0.31 | -0.063 ± 0.052 | -0.410 ± 0.024 | +0.018 | 3.74 ± 0.25 |
| | | | - | 16.43 ± 0.28 | | -0.456 ± 0.025 | | 7.90 ± 0.16 |
| | 180.0 | 13.10 | + | 4.86 ± 0.10 | +0.077 ± 0.034 | -0.416 ± 0.025 | +0.0095 | 1.24 ± 0.11 |
| | | | - | 6.94 ± 0.14 | | -0.368 ± 0.039 | | 4.92 ± 0.17 |
| 150.0 | 150.0 | 14.85 | + | 9.23 ± 0.20 | +0.125 ± 0.032 | -0.441 ± 0.032 | +0.0075 | 6.16 ± 0.11 |
| | | | - | 10.66 ± 0.29 | | -0.448 ± 0.045 | | 7.21 ± 0.17 |
| | 20.40 | + | 1.74 ± 0.04 | +0.058 ± 0.044 | -0.415 ± 0.050 | +0.0075 | 1.13 ± 0.12 | |
| | | - | 2.00 ± 0.05 | | -0.377 ± 0.046 | | 1.36 ± 0.12 | |
| 125.0 | 625.0 | 10.15 | + | 15.94 ± 0.36 | (f) | -0.512 ± 0.057 | +0.006 | 7.82 ± 0.38 |
| | | | - | 20.16 ± 0.46 | | -0.326 ± 0.108 | | 13.59 ± 2.20 |
| | | 13.00 | + | 4.63 ± 0.12 | +0.053 ± 0.040 | -0.428 ± 0.042 | +0.006 | 2.91 ± 0.18 |
| | | | - | 5.33 ± 0.13 | | -0.466 ± 0.037 | | 3.16 ± 0.10 |
| | | 14.95 | + | 2.19 ± 0.06 | +0.058 ± 0.046 | -0.438 ± 0.053 | +0.006 | 1.36 ± 0.16 |
| | | | - | 2.69 ± 0.07 | | -0.334 ± 0.048 | | 1.94 ± 0.19 |

(a) Observed events/volt corrected for accidentals, deadtime losses and target out background.

(b) Radiator subtraction for fractional yield due to π^0 decay leptons or multiple scattering losses.

(c) Radiator subtraction for fractional yield from double scattering events.

(d) Calculated fraction of positron annihilation in flight.

(e) CORR. NET YIELD equals EXPER. YIELD corrected for the total fractional losses of items (b), (c) and (d).

(f) Data not recorded. Calculation assumes zero contribution.

TABLE II

EFFECTIVE SPECTROMETER ACCEPTANCE

| MOMENTUM (MeV/c) | MULTIPLE SCATTERING EFFICIENCY | EFFECTIVE ACCEPTANCE ($\Delta\Omega \Delta p$) _{eff} |
|---------------------|--------------------------------------|---|
| 170 | 0.61 | 0.0290(1 \pm 0.04) |
| 340 | 0.74 | 0.0708 |
| 450 | 0.77 | 0.0976 |
| 625 | 0.78 | 0.1376 |

TABLE III

ESTIMATED SYSTEMATIC UNCERTAINTIES

| | |
|--|------|
| Normalization | 6.7% |
| Spectrometer angle | <3.0 |
| Spectrometer momentum | 2.2 |
| Bremsstrahlung spectrum | 3.0 |
| Radiator subtractions | 2.0 |
| Beam current monitors and integrators | 0.5 |
| Dead time corrections | 0.4 |

TABLE IV

SYMBOLS

| | | |
|------------|---|--|
| α | = | 1/137 |
| M | = | Carbon mass |
| m | = | Proton mass |
| μ | = | Electron mass |
| κ | = | Proton anomalous magnetic moment |
| E_1, p_1 | = | Energy, four-momentum of outgoing positron |
| E_2, p_2 | = | Energy, four-momentum of outgoing electron |
| Q | = | Four-momentum of initial target nucleus |
| Q' | = | Four-momentum of final target nucleus |
| P | = | Q + Q' |
| k | = | Four-momentum of incident photon |
| q | = | $k - p_1 - p_2$ |

A metric is used such that $p_1 p_2 = E_1 E_2 - \underline{p_1} \cdot \underline{p_2}$

TABLE V

THEORETICAL ESTIMATES OF ASYMMETRIC PRODUCTION

| E_o (MeV) | P_{\pm} MeV/c | θ (deg) | $B_1 \times B_1$ | | | | $B_1 \times c^{(a)}$ | | $B_1 \times B_2^{(a)}$ |
|----------------|--------------------|-------------------|-------------------|-------------------|--------|----------------|----------------------|----------|------------------------|
| | | | Yield/V Eq.(9) | Quasi- Elastic | Proton | Rad. Corrns | P_{33} | D_{13} | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 390 | 170 | 30.0 | 7.28 | 0.049 | 0.048 | 0.056 | -0.029 | -0.003 | 0.008 |
| | 340 | 18.1 | 3.12 | 0.080 | 0.048 | -0.010 | 0.037 | -0.029 | 0.003 |
| 550 | 450 | 14.8 | 6.44 | 0.102 | 0.056 | -0.002 | 0.092 | -0.057 | 0.001 |
| | 450 | 20.4 | 0.91 | 0.316 | 0.100 | -0.003 | 0.136 | -0.086 | -0.002 |
| 715 | 625 | 10.2 | 11.70 | 0.087 | 0.052 | -0.014 | 0.072 | -0.155 | 0.001 |
| | 625 | 13.0 | 2.83 | 0.196 | 0.078 | -0.015 | 0.103 | -0.226 | -0.003 |
| | 625 | 15.0 | 1.16 | 0.340 | 0.101 | -0.014 | 0.128 | -0.273 | -0.007 |

(a) Sign corresponds to electron detection.

TABLE VI

EFFECT OF $\Lambda(1236)$ RESONANCE WIDTH ASSUMPTIONS

| | Constant Width $\Gamma = 120$ MeV | Energy Dependent Widths | |
|-------------|--------------------------------------|-------------------------|------------------|
| | | Eq. 15 | Eq. 16 |
| a_1 | 0.33 ± 0.32 | 1.21 ± 0.67 | 0.56 ± 0.47 |
| a_2 | -0.38 ± 0.55 | -0.16 ± 0.47 | -0.23 ± 0.61 |
| $P(\chi^2)$ | 0.004 | 0.012 | 0.005 |

TABLE VII

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

| E_c (MeV) | p_{\pm} MeV/c | θ (deg) | Polarity | Exper. Yield/V | Theoret. Yield/V | $R\left(\frac{\text{Exp}}{\text{Thy}}\right)$ | A_{thy} Eq.(21) | A_{exp} | $A(\text{exp/thy})$ |
|----------------|--------------------|-------------------|-----------------|-------------------|---------------------|---|-----------------------------|-------------------|---------------------|
| 390 | 170 | 30.00 | + | 8.74 ± 0.95 | 8.55 | 1.02 ± 0.11 | -0.018 | -0.05 ± 0.08 | 2.8 ± 4.1 |
| | | | - | 7.90 ± 0.96 | 8.24 | 0.96 ± 0.12 | | | |
| | 340 | 18.10 | + | 3.24 ± 0.22 | 3.36 | 0.96 ± 0.07 | 0.036 | 0.21 ± 0.05 | 5.8 ± 1.4 |
| | | | - | 4.92 ± 0.37 | 3.61 | 1.36 ± 0.10 | | | |
| 550 | 450 | 14.85 | + | 6.36 ± 0.44 | 6.84 | 0.93 ± 0.06 | 0.080 | 0.063 ± 0.053 | 0.79 ± 0.66 |
| | | | - | 7.21 ± 0.57 | 8.04 | 0.90 ± 0.07 | | | |
| | 20.40 | + | 1.13 ± 0.12 | 1.16 | 0.97 ± 0.10 | 0.095 | 0.092 ± 0.072 | 0.97 ± 0.76 | |
| | | - | 1.36 ± 0.13 | 1.41 | 0.96 ± 0.09 | | | | |
| 715 | 625 | 10.15 | + | 7.82 ± 0.92 | 12.31 | 0.64 ± 0.08 | 0.065 | 0.270 ± 0.112 | 4.2 ± 1.7 |
| | | | - | 13.59 ± 2.2 | 14.02 | 0.97 ± 0.16 | | | |
| | 13.00 | + | 2.91 ± 0.28 | 3.28 | 0.89 ± 0.09 | 0.080 | 0.041 ± 0.063 | 0.51 ± 0.85 | |
| | | - | 3.16 ± 0.30 | 3.84 | 0.82 ± 0.08 | | | | |
| | 14.95 | + | 1.36 ± 0.16 | 1.52 | 0.90 ± 0.11 | 0.085 | 0.176 ± 0.076 | 2.1 ± 0.9 | |
| | | - | 1.94 ± 0.19 | 1.80 | 1.08 ± 0.11 | | | | |

FIGURE CAPTIONS

- Fig. 1 Electron pair production diagrams.
- Fig. 2 Experimental apparatus.
- Fig. 3 Cerenkov counter cross section in the median plane of the incident beam: (A) 10 cm \times 20 cm light pipe, 7.5 mm thick front surface glass mirror, (B) 22 cm focal length aluminized Plexiglas spherical mirror, 2.0 mm thick, (C) aluminized Plexiglas light pipe, (D) Plexiglas II UVT window, 1.9 cm thick, (E) Mylar window, 15 \times 0.19 mm thick, (F) light port assembly, (G) 30.0 cm diameter aluminum tube, (H) Amperex 58AVP photomultiplier tube.
- Fig. 4 Relative variation of the Compton interference term R^* in the region of the $\Delta(1236)$ resonance. The horizontal bars indicate the range of photon energies contributing to the observed yield and the numbers attached give the momentum of the detected lepton in MeV/c.