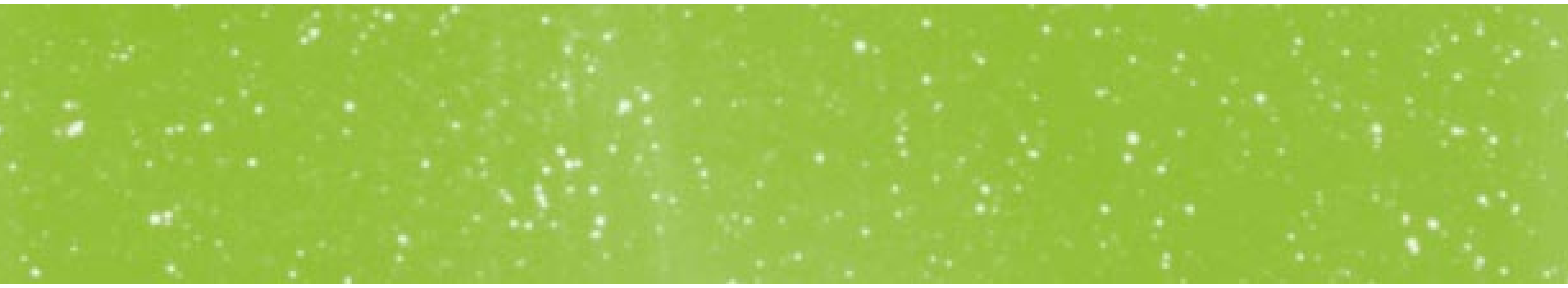
by VIRGINIA TRIMBLE

The Ratios of Small Whole Numbers: Misadventures in Astronomical Quantization

This is also Part III of "Astrophysics Faces the Millennium," but there are limits to the number of subtitles that can dance on the head of an editor.

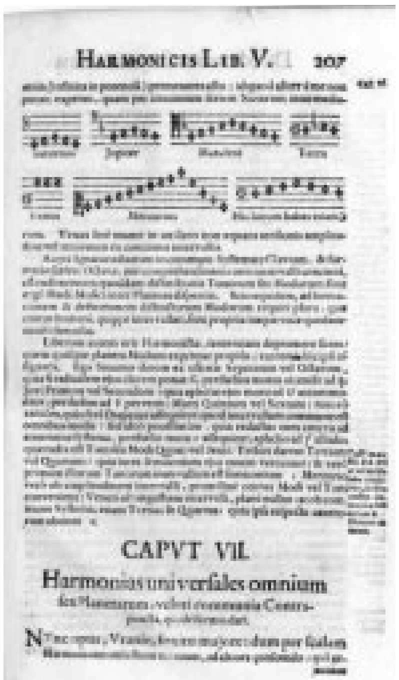
GOD MADE THE INTEGERS; all else is the work of Man. So said Kronecker of the delta. And indeed you can do many things with the integers if your tastes run in that direction. A uniform density stretched string or a pipe of uniform bore will produce a series of tones that sound pleasant together if you stop the string or pipe into lengths that are the ratios of small whole numbers. Some nice geometrical figures, like the 3-4-5 right triangle, can also be made this way. Such things were known to Pythagoras in the late sixth century BCE and his followers, the Pythagoreans. They extended the concept to the heavens, associating particular notes with the orbits of the planets, which therefore sang a harmony or music of the spheres, finally transcribed in modern times by Gustav Holtz.

A fondness for small whole numbers and structures made of them, like the regular or Platonic solids, persisted for centuries. It was, in its way, like our fondness for billions and billions, a fashion. George Bernard Shaw claimed early in the twentieth century that it was no more rational for his contemporaries to think they were sick because of an invasion by millions of germs than for their ancestors to have attributed the problem to an invasion by seven devils. Most of us would say that the germ theory of disease today has a firmer epistomological basis than the seven devils theory. But the insight that there are fashions in numbers and mathematics, as much as fashions in art and government, I think stands.



KEPLER'S LAWS AND LAURELS

Some of the heroes of early modern astronomy remained part of the seven devils school of natural philosophy. We laud Johannes Kepler (1571–1630) for his three laws of planetary motion that were later shown to follow directly from a Newtonian, inverse-square law of gravity, but he had tried many combinations of small whole numbers before showing that the orbit periods and orbit sizes of the planets were connected by $P^2 = a^3$. And the same 1619 work, Harmony of the Worlds, that enunciated this relation also showed the notes sung by the planets, in a notation that I do not entirely understand. Venus hummed a single high pitch, and Mercury ran up a glissando and down an arpeggio, while Earth moaned mi(seria), fa(mes), mi(seria). The range of notes in each melody depended on the range of speeds of the planet in its orbit, which Kepler, with his equal areas law, had also correctly described.



The music of the spheres according to Kepler. Saturn (upper left) is scored on the familiar bass clef, with F on the second line from the top, and Venus (beneath Saturn) on a treble clef with G on the second line from the bottom. The others are F, G, and C clefs with the key notes on other lines, not now generally in use. (Copyright © 1994, UK ATC, The Royal Observatory, Edinburgh.)

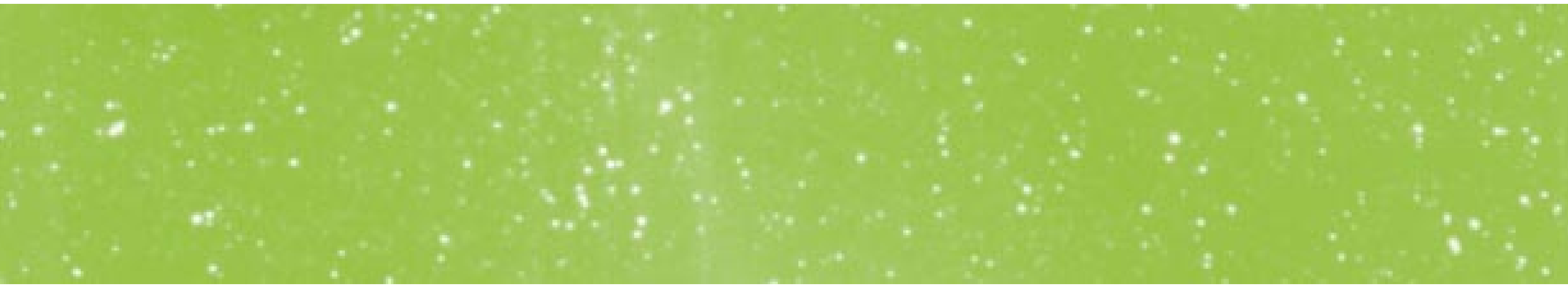
Kepler's planets were, however, six, though Copernicus and Tycho had clung to seven, by counting the moon, even as they rearranged Sun and Earth. Why six planets carried on six spheres? Why, because there are precisely five regular solids to separate the spheres: cube, pyramid, octahedron, dodecahedron, and icosahedron. With tight packing of sequential spheres and solids in the right order, Kepler could approximately reproduce the relative orbit sizes (semi-major axes) required to move the planets to the positions on the sky where we see them. Though this scheme appears in his 1596 Cosmographic Mystery and so belongs to Kepler's geometric youth, he does not appear to have abandoned it in his algebraic maturity.

EXCESSIVELY UNIVERSAL GRAVITATION

Newton (1643–1727) was born the year after Galileo died and died only two years before James Bradley recognized aberration of starlight, thereby demonstrating unambiguously that the Earth goes around the Sun and that light travels about 10^4 times as fast as the Earth's orbit speed (NOT a ratio of small whole numbers!) But the SWN ideal persisted many places. Galileo had found four moons orbiting Jupiter (which Kepler himself noted also followed a $P^2 = a^3$ pattern around their central body), and the Earth had one. Mars must then naturally have two. So said Jonathan Swift in 1726, attributing the discovery to Laputian* astronomers, though it was not actually made until 1877 by Asaph Hall of the U.S. Naval Observatory.

Attempts to force-fit Newtonian gravity to the entire solar system have led to a couple of ghost planets, beginning with Vulcan, whose job was to accelerate the rotation of the orbit of Mercury ("advance of the perihelion") until Einstein and General Relativity were ready to take it on. Planet X, out beyond Pluto, where it might influence the orbits of comets, and Nemesis, still further out and prone to sending comets inward to impact the Earth every 20 million years or so, are more recent examples. They have not been observed, and, according to dynamicist Peter Vandervoort of the University of Chicago, the Nemesis orbit is dynamically very interesting; it just happens not to be occupied.

It is, I suspect, nothing more than bad diction that leads to the frequent false attribution of the discovery to Lilliputian astronomers, when their culture must surely have specialized in microscopy.



THE GREGORY-WOLFF-BONNET LAW

Meanwhile, the small whole numbers folk were looking again at the known planetary orbits and trying to describe their relative sizes. Proportional to 4, 7, 10, 15, 52, and 95, said David Gregory of Oxford in 1702. The numbers were published again, in Germany, by Christian Wolff and picked up in 1766 by a chap producing a German translation of a French volume on natural history by Charles Bonnet. The translator, whose name was Johann Titius (of Wittenberg) added the set of numbers to Bonnet's text, improving them to 4, 7, 10, 16, 52, and 100, that is 4+0, 4+3, 4+6, 4+12, 4+48, and 4+96, or 4+ (3×0,1,2,4,16,32). A second, 1712, edition of the translation reached Johann Elert Bode, who also liked the series of numbers, published them yet again in an introduction to astronomy, and so gave us the standard name Bode's Law, not, as you will have remarked, either a law or invented by Bode.

Bode, at least, expected there to be a planet at 4 + 3×8 units (2.8 AU), in the region that Newton thought the creator had left empty to protect the inner solar system from disruption by the large masses of Jupiter and Saturn. Closely related seventeenth century explanations of the large separation from Mars to Jupiter included plundering of the region by Jupiter and Saturn and collisional disruptions. Don't forget these completely, since we'll need them again. Great, however, was the rejoicing in the small whole numbers camp when William Herschel spotted Uranus in 1781 in an orbit not far from 4 + 192 units from the sun.

Among the most impressed was Baron Franz Xaver von Zach of Gotha (notice the extent to which this sort of numerical/positional astronomy seems to have been a German enterprise). After a few years of hunting the skies where an a = 28 unit planet might live, he and few colleagues decided to enlist a bunch of European colleagues as "celestial police," each to patrol a fixed part of the sky in search of the missing planet. Hegel, of the same generation and culture, on the other hand, objected to potential additional planets, on the ground that Uranus had brought us back up to seven, equal to the number of holes in the average human head, and so the correct number.*

Even as von Zach was organizing things, Italian astronomer Giuseppe Piazzi, working independently on a star catalog, found, on New Year's Day 1801, a new moving object that he at first took to be a comet before losing it behind the sun in February. Some high powered mathematics by the young Gauss (whom astronomers think of as the inventor of the least squares method in this context) provided a good enough orbit for Zach to recover the wanderer at the end of the year. Sure enough, it had a roughly circular orbit of about the size that had been Boded. But it was smaller than our own moon and, by 1807, had acquired three stablemates. These (Ceres, Juno, Pallas, and Vesta) were, of course, the first four of many thousands of asteroids, whose total mass is nevertheless still not enough to make a decent moon. I do not know whether Piazzi made his discovery in the sky sector that von Zach had assigned to him or whether, if not, the person who was supposed to get that sector was resentful, though the latter seems likely.

Neptune and Pluto entered the planetary inventory with orbits quite different and smaller than the next two terms in the Titius-Bode sequence. Strong expectation that they should fall at the sequence positions probably retarded the searches for them (which were based on irregularities in the motion of Uranus).

In recent years, the orbits of triplets of planets around one pulsar and one normal star (Ups And) have also been claimed as following something like a Gregory-Wolff-Titius sequence. Notice however that it takes two orbits to define the sequence, even in completely arbitrary units.

What is the status of Bode's Law today? The table on the next page shows the actual orbit semi-major axes and the fitted number sequence. All agree to about five

**It is left as an exercise for the reader to formulate an appropriate snide remark about Hegel having thus physiologically anticipated the discovery of Neptune, which he did not in fact live to see.*

Planetary Orbit Sizes

Astronomical Object	Mean Orbit Size in Titius-Bode sequence	Actual Semi-Major Axis in AU×10
Mercury	4	3.87
Venus	7	7.32
Earth	10	10.0 (definition)
Mars	16	15.24
Asteroid belt (mean)	28	26.8
Jupiter	52	52.03
Saturn	100	95.39.
Uranus	196	191.9
Neptune	388	301
Pluto	722	395

percent until you reach Neptune. But current theory comes much closer to what Newton and his contemporaries said than to orbit quantization. The process of formation of the planets and subsequent stability of their orbits are possible only if the planets further from the sun, where the gravitational effect of the sun is progressively weaker, are also further from each other.

BI- AND MULTI-MODALITIES

The minimum requirement for quantization is two entities with different values of something. With three you get eggroll, and twentieth century astronomy has seen a great many divisions of its sources into two discrete classes. Examples include galactic stars into populations I and II; star formation processes into those making high and low mass stars; binary systems into pairs with components of nearly equal vs. very unequal masses; stellar velocities into two streams; and supernovae into types I and II (occurring in populations II and I respectively).

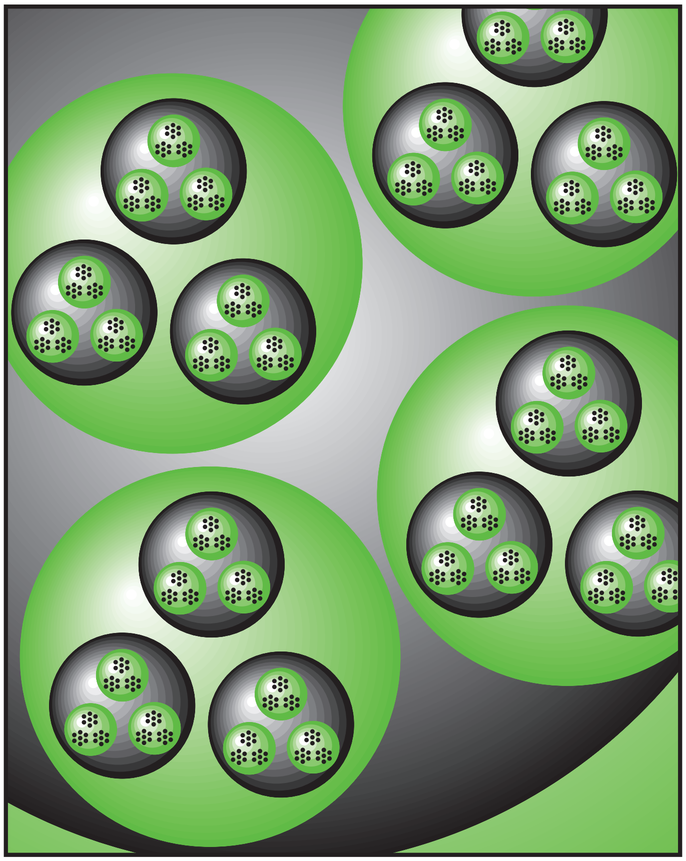
None of these has achieved the status of true crankiness. Rather, most have eventually either smoothed out into some kind of continuum, from which early investigations had snatched two widely separated subsets (stellar populations, for instance), or *n*-furcated into "many," each with honorably different underlying physics (including supernova types). A few, like the distinction between novae and supernovae, turned out to be wildly different sorts of things, initially perceived as similar classes because of some major misconception (that their distances were similar, in the nova-supernova case).

Thus to explore the extremities of weirdness in astronomical quantization, we must jump all the way from the solar system to the wilder reaches of quasars and cosmology. Here will we find fractals, universes with periodic boundary conditions, non-cosmological (and indeed non-Doppler and non-gravitational redshifts), and galaxies that are permitted to rotate only at certain discrete speeds.

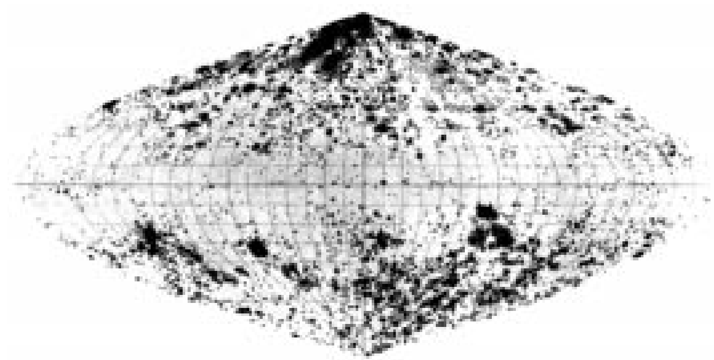
FRACTAL, TOPOLOGICALLY-COMPLEX, AND PERIODIC UNIVERSES

In the cosmological context, fractal structure would appear as a clustering heirarchy. That is, the larger the scale on which you looked at the distribution of galaxies (etc.), the larger the structures you would find. Many pre-Einsteinian pictures of the Universe were like this, including ones conceived by Thomas Wright of Durham, Immanuel Kant, and Johann Lambert (all eighteenth century). Since we have global solutions for the equations of GR only for the case of homogeneity and isotropy on average, it is not entirely clear what a general relativistic fractal universe ought to look like.

But it probably doesn't matter. Observations of distances and locations of galaxies currently tell us that there are clusters and superclusters of galaxies (often sheet-like or filamentary) with voids between, having sizes up to 100–200 million parsecs. This is less than 10 percent of the distance to galaxies with large redshifts. No honest survey of galaxy positions and distances has found structure larger than this, though a couple of groups continue to reanalyze other people's data and report fractal or heirarchical structure out to billions of parsecs. A naive plot of galaxies on the sky indeed appears to have a sort of quadrupole, but the cause is dust in our own galaxy absorbing the light coming from galaxies in the direction of the galactic plane, not structure on the scale of the whole observable Universe.



A non-artist's impression of a hierarchical or fractal universe in which observations on ever-increasing length scales reveal ever-larger structure. In this realization, the first order clusters each contain seven objects, the next two orders have three each, and the fourth order systems (where we run off the edge of the illustration) at last four. Immanuel Kant and others in the eighteenth century proposed this sort of structure, but modern observations indicate that there is no further clustering for sizes larger than about 150 million parsecs. (Adapted from Edward R. Harrison's Cosmology, The Science of the Universe, *Cambridge University Press, second edition, 2000. Reprinted with the permission of Cambridge University Press)*



A plot of the distribution of nearby galaxies on the sky, dating from before the confirmation that other galaxies exist and that the Universe is expanding. Most of the small clumps are real clusters, but the concentration toward the poles and dearth at the equator (in galactic coordinates) result from absorption of light by dust in the plane of our own Milky Way. [From C. V. L. Charlier, Arkiv. for Mat. Astron., Fys *16, 1 (1922)]*

Coming at the problem from the other side, the things that we observe to be very isotropic on the sky, including bright radio sources, the X-ray background, gamma ray bursts, and, most notoriously, the cosmic microwave radiation background have been used by Jim Peebles of Princeton to show that either (a) the large scale fractal dimension $d = 3$ to a part in a thousand (that is, in effect, homogeneity over the distances to the X-ray and radio sources), or (b) we live very close indeed to the center of an inhomogeneous but spherically symmetric (isotropic) universe.

Several of the early solvers of the Einstein equations, including Georges Lemaitre, Alexander Friedmann, and Willem de Sitter, considered at least briefly the idea of a universe with topology more complex than the minimum needed to hold it together.

Connecting little, remote bits of universe together with wormholes, Einstein-Rosen bridges, or whatever is not a good strategy, at least for the time and space scales over which we observe the cosmic microwave background. If you peered through such a connection in one direction and not in the patch of sky next to it, the

photons from the two directions would have different redshlfts, that is different temperatures. The largest temperature fluctuations we see are parts in 10^5, so the light travel time through the wormhole must not differ from the travel time without it by more than that fraction of the age of the Universe, another conclusion expressed clearly by Peebles.

A periodic universe, in which all the photons go around many times, is harder to rule out. Indeed it has been invoked a number of times. The first camp includes people who have thought they had seen the same object or structure by looking at different directions in the sky and those who have pointed out that we do not see such duplicate structures, though we might well have, and, therefore, that the cell size of periodic universe must be larger than the distance light travels in a Hubble time. The second camp includes people who want to use periodic boundary conditions to explain the cosmic microwave background (it's our own galaxy seen simultaneously around the Universe in all directions), high energy cosmic rays (photons which have been around many times, gaining energy as they go), or the centers of quasars (bright because they illuminate themselves).

Descending a bit from that last peak of terminal weirdness, we note that a periodic universe, with cell size much smaller than the distance light goes in 10–20 billion years, automatically guarantees that the radio and X-ray sources will be seen quite isotropicalIy on the sky. It also has no particle horizon, if that is the sort of thing that bothers you.

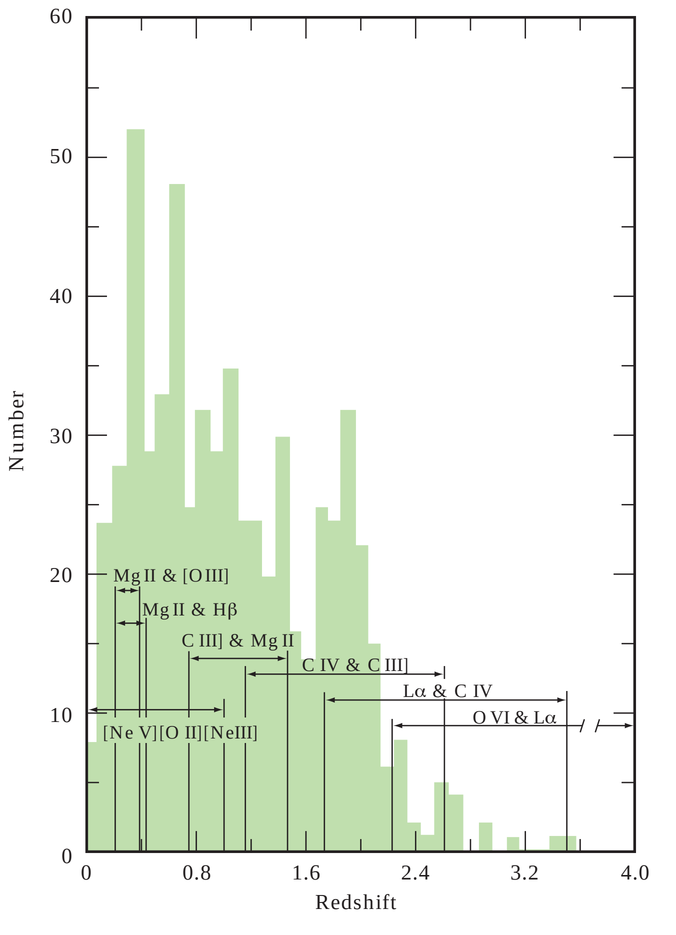
Each year, the astronomical literature includes a few cosmological ideas about which one is tempted to say (with Pauli?) that it isn't even wrong. A couple of my favorites from the 1990s are a universe with octehedral geometry (the advertised cause is magnetic fields) and one with a metric that oscillates at a period of 160 minutes (accounting for fluctuations in the light of the Sun and certain variable stars).

QUANTIZED VELOCITIES AND REDSHIFTS

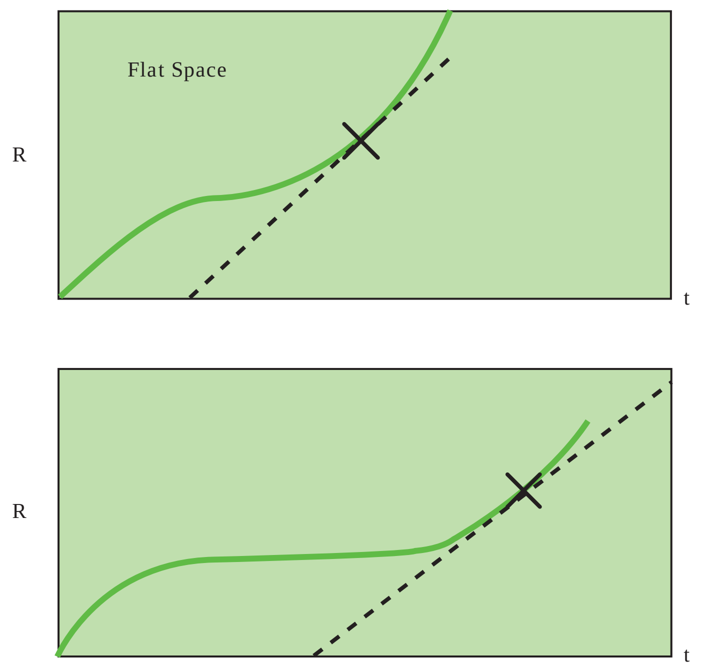
The ideas along these lines that still bounce around the literature all seem to date from after the discovery of quasars, though the quantization intervals are as small as 6 km/sec. The topic is closely coupled (a) with redshifts whose origins are not due to the expansion of the Universe, strong gravitation, or relative motion, but to some new physics and (b) with alternatives to standard hot Big Bang comology, especially steady state and its more recent offspring. The number of people actively working on and supporting these ideas does not seem to be more than a few dozen (of a world wide astronomical community of more than 10,000), and most of the papers come from an even smaller group. This is not, of course, equivalent to the ideas being wrong. But they have been out in the scholarly marketplace for many years without attracting vast numbers of buyers.

The underlying physics that might be responsible for new kinds of wavelength shifts and/or quantized wavelength shifts have not been worked out in anything like the mathematical detail of the Standard Model and processes. One suggestion is that new matter is added to the Universe in such a way that the particles initially have zero rest mass and gradually grow to the values we see. This will certainly result in longer wavelengths from younger matter, since the expression for the Bohr energy levels has the electron rest mass upstairs. Quantized or preferred redshifts then require some additional assumption about matter not entering the Universe at a constant rate.

The first quasar redshift, $\Delta\lambda/\lambda = 0.16$ for 3C 273, was measured in 1963 by Maarten Schmidt and others. By 1968–1969, about 70 were known, and Geoffrey and Margaret Burbidge remarked, first, that there seemed to be a great many objects with $z = 1.95$ and, second, that there was another major peak at $z = 0.061$ and smaller ones at its multiples of 0.122, 0.183, and so on up to 0.601. The "$z = 1.95$" effect was amenable to two semi-conventional



Histogram of redshifts of quasars initially identified as radio sources. This removes many (not all) of the selection effects associated with the apparent colors of sources at different z's and with the present or absence of strong emission lines in the wavelength ranges usually observed from the ground. The redshift ranges in which various specific lines are usable is indicated. C IV, for instance (rest wavelength 1909 A) disappears into the infrared at z = 3.4. Geoffrey Burbidge concludes that the sharp peaks in the distribution are not the result of selection effects arising from availability of lines to measure redshifts. (Courtesy G. R. Burbidge)



The scale factor, R(t), for universes with non-zero cosmological constant. The case (upper diagram) with flat space, now thought to be the best bet, goes from deceleration to acceleration through an inflection point (which probably happened not much before z = 1) and so has no coasting phase to pile up redshifts at a preferred value, as would happen in a universe with negative curvature (lower diagram). In either case, we live at a time like X, so that the Hubble constant (tangent to the curve) gives the impression of the Universe being younger than it actually is. The early expansion has R proportional to $t^{2/3}$ and the late expansion has exponential R(t).

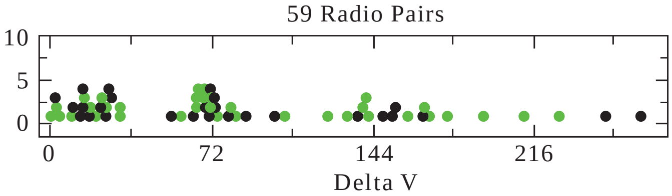
explanations. The first was that $z = 2$ was the maximum that could be expected from a strong gravitational field if the object was to be stable. The second was that $z = 1.95$ might correspond to the epoch during which a Friedmann-Lemaitre universe (one with a non-zero cosmological constant) was coasting at almost constant size, so that we would observe lots of space (hence lots of objects) with that redshift.

In the interim, gravitational redshifts have been pretty clearly ruled out (you can't keep enough low density gas at a single level in your potential well to emit the photons we see), and the peak in $N(z)$ has been diluted as the number of known redshifts has expanded beyond 2000. A cosmological constant is back as part of many people's favorite universes, but (if space is flat) in a form that does not produce a coasting phase, but only an inflection point in $R(t)$.

No conventional explanation for the periodicity at $\Delta z = 0.061$ has ever surfaced, and the situation was not improved over the next decade when, as the data base grew, Geoffrey R. Burbidge reported additional peaks at $z = 0.30$, 0.60, 0.96, and 1.41 and a colleague showed that the whole set, including 1.95, could be fit by $\Delta\log(1+z) = 0.089$. Meanwhile, and apparently independently, Clyde Cowan, venturing out of his usual territory, found quite different peaks separated by $\Delta z = 1/15$ and 1/6. As recently as 1997, another group found several statistically significant peaks in $N(z)$ between 1.2 and 3.2.

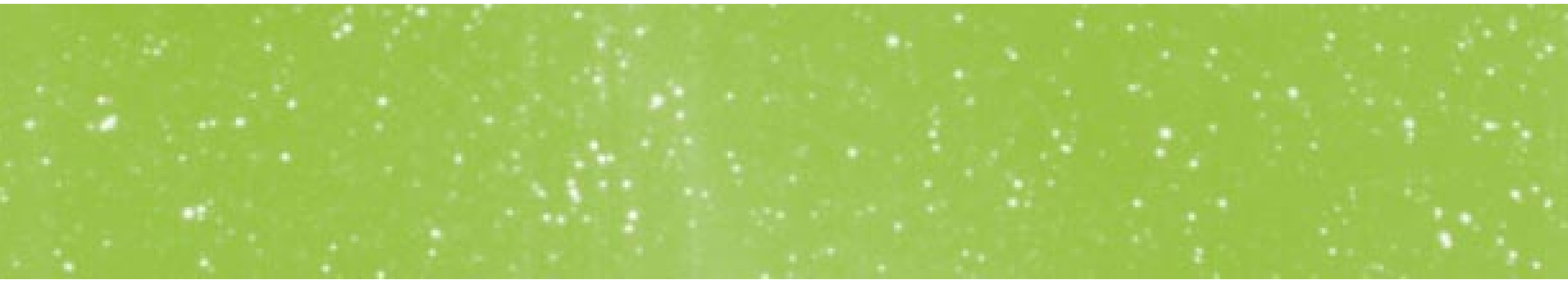
Any attempt to assign statistical significance to these apparent structures in $N(z)$ must take account of two confounders. First, the observed sample is not drawn uniformly from the real population. Quasars have only a few strong emission line from which redshifts can be measured, and these move into, through, and out of observable ranges of wavelength as redshift increases. Second, you must somehow multiply by the probability of all the other similar effects that you didn't find but that would have been equally surprising. The 0.061 effect still appears in the larger data bases of the 1990s, at least when the analysis is done by other supporters of noncosmological redshifts and quasi-steady-state universes. It does, however, need to be said (and I say it out from under a recently-resigned editorial hat) that the analyses require a certain amount of work, and that it is rather difficult to persuade peopIe who are not violently for or against an unconventional hypothesis to do the work, even as referees.

At the same time the Burbidges were reporting $z = 1.95$ and 0.061 structures, another major player was sneaking up on quantization with much finer intervals. William



A data sample showing redshift quantization on the (roughly) 72 km/sec scale. The points represent differences in velocities between close pairs of galaxies. An extended discussion of the development of the observations and theory of quantization at intervals like 72 and 36 km/sec is given by William Tifft in Journal of Astrophysics and Astronomy 18, 415 (1995). The effects become clearer after the raw data have been modified in various ways. [Courtesy William G. Tifft, Astrophysics and Space Science 227, 25 (1997)].

Tifft of the University of Arizona had started out by trying to measure very accurately the colors of normal galaxies as a function of distance from their centers (one way of probing the ages, masses, and heavy element abundances of the stars that contribute most of the light). In the process, he ended up with numbers he trusted for the total apparent brightnesses of his program galaxies. He proceeded to graph apparent brightness vs. redshift as Hubble and many others had done before him. Lo and behold! His plots were stripy, and, in 1973 he announced that the velocities of ordinary nearby galaxies were quantized in steps of about 72 km/sec ($z = 0.00024$). Admittedly, many of the velocities were uncertain by comparable amounts, and the motion of the solar system itself (about 30 km/sec relative to nearby stars, 220 km/sec around the center of the Milky Way, and so forth) was not negligible on this scale. In the intervening quarter of a century, he has continued to examine more and more data for more and more galaxies, ellipticals as well as spirals, close pairs, and members of rich clusters, as well as nearly isolated galaxies, and even the distributions of velocities of stars within the galaxies. He has also devoted attention to collecting data with uncertainties less than his quantization interval and to taking out the contribution of our own many motions to velocities seen in different directions



in the sky. Some sort of quantization always appears in the results. The interval has slid around a bit between 70 and 75 km/sec and some submultiples have appeared in different contexts, including 36 and 24 km/sec and even something close to 6 km/sec. But the phenomenon persists.

A pair of astronomers at the Royal Observatory Edinburgh looked independently at some of the samples in 1992 and 1996. They found slightly different intervals, for example, 37.22 km/sec, but quantization none the less. A 1999 variant from another author includes periodicity (described as quantization) even in the rotation speeds of normal spiral galaxies.

Tifft's Arizona colleague W. John Cocke has provided some parts of a theoretical framework for the 72 (etc.) km/sec quantization. It has (a) an exclusion principle, so that identical velocities for galaxies in close pairs are at least very rare if not forbidden and (b) some aspects analogous to bound discrete levels on the atomic scale, with continua possible only for velocity differences larger than some particular value. The quantization is, then, responsible for the large range of velocities observed in rich clusters of galaxies which would then not require additional, dark matter to bind them.

Am I prepared to say that there is nothing meaningfuI in any of these peaks and periods? No, though like the race being to the swift and the battle to the strong, that's how I would bet if it were compulsory. There is, however, a different kind of prediction possible. Most of the major players on the unconventional team(s) are at, beyond, or rapidly approaching standard retirement ages. This includes Cocke and Tifft as well as the Burbidges, H. C. (Chip) Arp, a firm exponent of non-cosmological (though not necessarily quantized) redshifts, and the three founders of Steady State Cosmology, Fred Hoyle, Thomas Gold, and Hermann Bondi. Meanwhile, very few young astronomers are rallying to this particular set of flags. Whether this reflects good judgment, fear of ostracism by the mainstream astronomical community, or something else can be debated. But it does, I think mean that periodic, quantized, and non-cosmological redshifts will begin to disappear from the astronomical literature fairly soon.

QUANTUM GRAVITY, QUANTUM COSMOLOGY, AND BEYOND

The further back you try to look into our universal past, the further you diverge from the regimes of energy, density, and so forth at which existing physics has been tested. General relativity is said to be non-renormalizable and nonquantizable, so that some other description of gravity must become necessary and appropriate at least when the age of the Universe is comparable with the Planck time of 10^{-43} sec, and perhaps long after. One prong of the search for new and better physics goes through attempts (via string theory and much else) to unify all four forces, arguably in a space of many more than three dimensions.

A very different approach is that of quantum cosmology, attempting to write down a Schrödinger-like (Klein-Gordon) equation for the entire Universe and solve it. It is probably not a coincidence that the Wheeler (John Archibald)-DeWitt (Bryce S.) equation, governing the wave function of the Universe, comes also from 1967–1968, when the first claims of periodic and quantized redshifts were being made. The advent of quasars with redshifts larger than one and of the microwave background had quickly made the very large, the very old, and the very spooky all sound exciting.

The difference between the quanta in this short section and in the previous long ones is that quantum gravity and quantum cosmology are respectable part-time activities for serious physicists. The papers appear in different sorts of journals, and the authors include famous names (Hawking, Linde, Witten, Schwartz, Greene, Vilenkin, and others) some of whose bearers are considerably younger than the velocity quantizers and the present author.

This last adventure in astronomical quantization still has most of its bright future ahead of it.

Where to find more:
P. J. E. Peebles, **Principles of Physical Cosmology**, Princeton University Press, 1993 is the repository of how the conventional models work and why some plausible sounding alternatives do not. Some early fractal cosmologies are discussed by E. R. Harrison in **Cosmology: The Science of the Universe**, 2nd edition, Cambridge University Press, 2000. The lead up to Bode's law appears in M. A. Hoskin, **The Cambridge Concise History of Astronomy**, Cambridge University Press, 1999.