A Physicist in the World of Vi

by WILLIAM ATWOOD

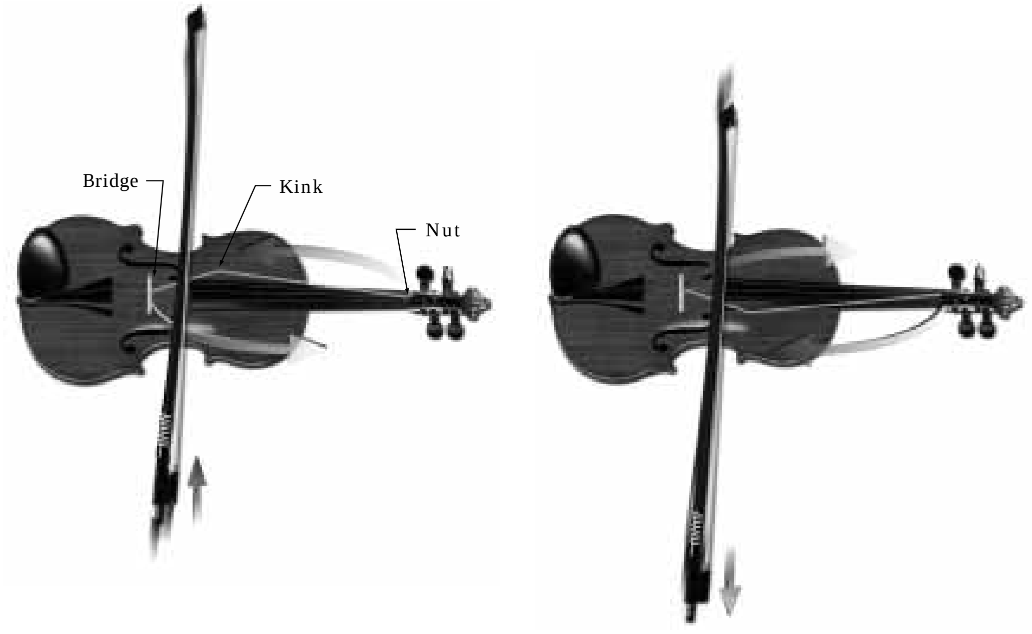
THE DEVELOPMENT AND HISTORY of the violin covers much the same time frame as contemporary physics. While Newton was developing his classical mechanics and formulating how the planets move in the sky, Antonio Stradivari was creating some of the world's greatest and most treasured violins in Cremona, Italy. This instrument had evolved from the viol family about one hundred years earlier and was given its final form by another Italian maker, Gasparo da Salo, and his student Giovanni Maggini. Like the modern-day electric guitar, this evolution was driven by the need for a louder sound to accommodate public functions. It was an instrument of the masses, unsophisticated and brash. The viol remained the refined favorite of the royal courts.

> *A physicist applies well-known principles to the construction of violins and in the process comes to a better understanding of techniques for shaping their sound.*

Salo's new design was seen and copied by a Cremonese maker, Andrea Amati, whose sons continued their father's lead and perfected the violin's design. The question of whether Stradivari was an apprentice in the Amati shop rests on a single label in one of his violins attributing its style to Amati. What we know for sure is that both lived in Cremona in the second half of the 1600s and that Stradivari began making violins in the Amati style. He soon branched out, however, and went on to a long and illustrious career during which he produced over a thousand instruments.

By 1750 the violin had gained wide acceptance. Indeed, most famous composers of the day produced works that featured this instrument. It was their demands that led to some last modifications. The definition of pitch was raised and the neck (and hence the string length) was lengthened, allowing

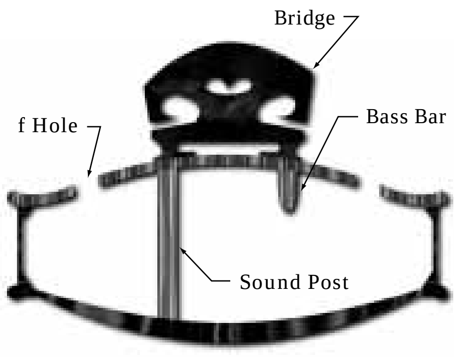
lins



An example of Helmholtz string motion where a kink appears to rotate around the string, alternately bouncing off the bridge and the nut.

a greater range in notes. Unfortunately, the original technique of simply nailing and gluing the neck onto the body proved too weak to counterbalance the pull of the strings. In response, in the late 1700s the maker Mantegazza developed a method of mortising the neck into the sides and top block for added strength. The modern violin had been born. However, violins made during the next two centuries failed to achieve the success of the earlier Italian makers, leading many to believe that a secret or secrets had somehow been lost.

Yet, progress was made during this period in understanding how bowed instruments produced their characteristic sound. The German physicist Hermann von Helmholtz is credited with explaining bowed string motion. The conditions of fixed length and tension and of continuous sound (periodicity) mean that the only frequencies present could be integral multiples of the fundamental (that is, the lowest allowed vibration mode). Musically this means that for a given note, its octave, the fifth above that, the next octave, the third above that, and so on are part of the sound. Helmholtz recognized that the timbre of the violin sound must be very rich in these harmonics, and he became curious as to how the moving string produced them. The string, he determined, does not just wag back and forth upon being agitated by the bow. Rather a sharp kink makes round-trips between the bridge and the nut, flipping over at each reflection (see the above illustration). As the kink passes by the contact point of the bow on its way to the bridge, it first releases the grip of the bow hairs on the string and then an instant later grabs them again. The result is a continuous plucking action often referred to as “slip stick.” The kink travels in an apparent



A cross section of the violin at the bridge.

clockwise direction for "down" bows and counter clockwise for "up" bows. As a result, when drawn across a level string, the bow tends to skate towards the bridge on an up-bow and towards the fingerboard on a down-bow.

Following the Second World War, a renaissance in violin making began with luthiers bent on recapturing the precision and artistry of former years and a new interest from scientists about how the violin works. Several new schools dedicated to the construction of stringed instruments now exist in the United States. And there are organizations with refereed journals focused mainly on the science of the instrument. In this stimulating environment, a young and enthusiastic group of makers is arguably producing violins that may in fact rival the old Italian instruments.

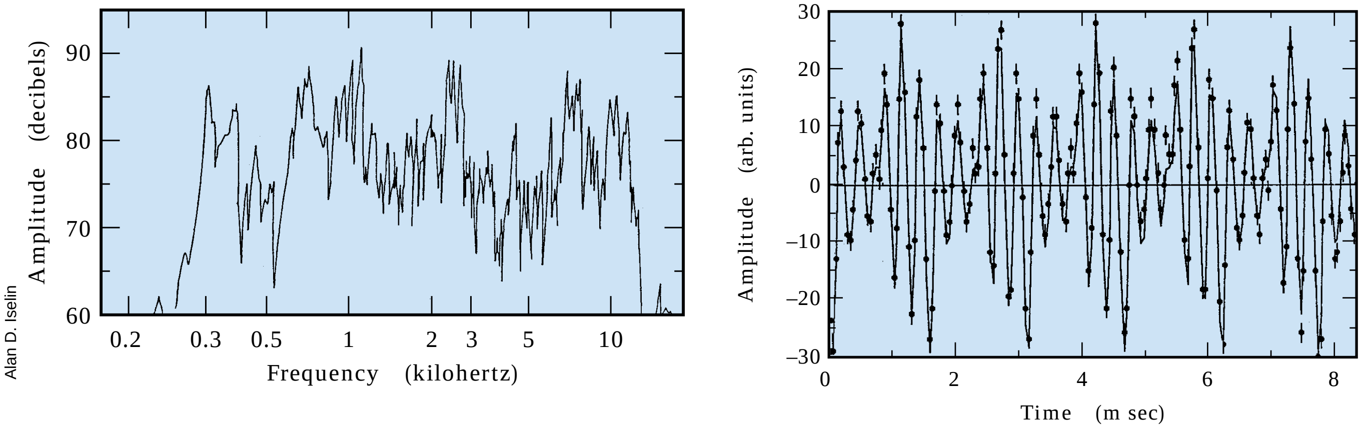
FUNDAMENTALS OF A WORKING VIOLIN

The illustration at the top of the page is a cross section of the violin at the location of the bridge. The body is essentially an empty, thin-walled box. The top plate, or "belly," that produces most of the sound is typically two to three millimeters thick while the back is similar in thickness around the edges but can be five millimeters or more near the center. The walls are approximately one millimeter thick except for the linings that reinforce the glue joint to the belly and the back. Under the foot of the bridge corresponding to the E string, a small dowel mechanically couples the top to the back. Under the other foot is a longitudinal stiffener called the bass bar. Both of these components not only have a profound influence on the sound but also are structurally critical as the downward force of the bridge owing to the tension of four strings is about twenty pounds.

The violin body is basically a resonator driven by the vibrations from the bridge induced by the aforementioned Helmholtz-type string motion. However, only those frequencies that approximately match the natural resonances of this system will result in any appreciable sound.

The frequency response can be measured by vibrating the bridge with a pure sine wave (no harmonics). This is accomplished by attaching a lightweight transducer to the bridge and sweeping the drive frequency while keeping the amplitude constant. The relative sound output can then be recorded (see left illustration on the opposite page). While the data in this plot are in detail complex, some overall features are easily understood. First there is a forest of resonances extending from about two hundred hertz up to about fifteen kilohertz. Although there are many sharp gaps, most notes generally sit sufficiently close to a resonance to be excited. The lower end of this range is quite curious: the bottom few notes on a violin (G at 196 Hz and G# at 208 Hz) fall off the edge of the response envelope. Somehow our ears are able to infer these notes from the higher harmonics they contain. Also, investing money in home audio equipment with responses above fifteen kilohertz won't make the violins sound better!

Each note on a violin has its own particular sound. The fundamental as well as its harmonics will fall in various places in the response



Alan D. Iselin

These two plots illustrate aspects of violin sound. On the left is the frequency response of a Guarnerius del Gesù violin. A dense "forest" of resonances covers the frequency band from approximately 220 Hz to 15 kHz. On the right, the waveform of a single note (E) played on a violin is shown. The pronounced rapid oscillations are due to the fifth and sixth harmonics, while the slow, overall envelope modulation is at fundamental frequency (660 Hz). (Courtesy Carleen Hutchins)

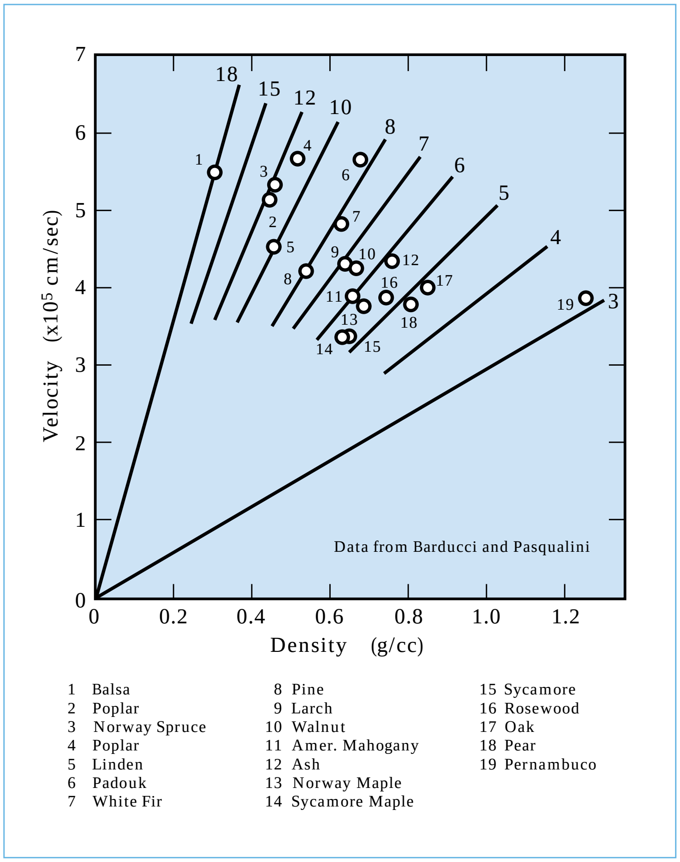
spectrum and be amplified accordingly. In the right-hand figure above the waveform of the sound of a violin being bowed on the open E string is shown fit to a harmonic series. For this note the fifth and sixth harmonics are three and two times larger than the fundamental. The apparent rapid oscillations are the result of these harmonics, slowly interfering with each other while the (slow) overall envelope modulation is at the fundamental frequency of 660 Hz. Yes, we hear E, but we also very quickly identify the note as coming from a violin. It is the presence of these harmonics that identifies the source of the sound for us. If we repeated the above exercise for other notes, similar results will be obtained; however, the harmonic content (which harmonics are important) will change.

To achieve good projection, one wants positive pressure waves to be produced simultaneously from the top and back of a violin. You might visualize this condition as the violin alternately swelling and shrinking. Jack Fry, another noted physicist, pointed out in the 1970s that the arrangement of sound post and bass bar inside a violin resulted in this type of radiator. But certainly not all of the resonances can be so simple. Various places on the surfaces of the two plates will be moving in and out of phase and hence form higher pole radiator patterns. For each note, various radiation patterns will emphasize various harmonics when measured at different locations. As Gabriel Weinreich recently observed, "perhaps this is part of the intriguing aspect of the violin sound. . .it seems to dance and sparkle, coming all at once from some place and at the same time from no place in particular!"

APPLYING PHYSICS TO MAKING VIOLINS

A physicist's approach to learning about a new physical system often starts with fundamentals such as how big, how small, what controls the overall behavior, etc. John Schelleng pursued this type of dimensional analysis and found an overall figure of merit for violin wood: the ratio of the density to the velocity of sound. One can intuitively understand this ratio is important by observing that if, for instance, the top weighs a lot, it will require a hard shove to get it moving. But, just being light is not sufficient. We want the entire top plate to move—not just the local area under the feet of the bridge. Hence, the stiffness is also of prime importance.

It is interesting to plot the density versus velocity for various types of wood (see illustration next page). Along straight lines emanating from



The velocity of sound versus the density is plotted for different types of wood. This ratio (velocity/density) is a good figure of merit for acoustical quality of a material, high values being best. The data points indicate typical values for a particular wood; however, a variation of 30 percent in both sound velocity and density is not uncommon.

the origin the ratio is constant: large values being “good.” Balsa wood seems to be the material of choice! Of course we’re not even suggesting this option since, as already mentioned, the static loading of twenty pounds from the downward force of the bridge would tax the strength of balsa. Not far behind, however, is spruce. This is the material of choice in practically all musical instruments that have a wooden sound board (pianos, guitars, harps, violins) and for the same reason (high stiffness per unit weight) was the material of choice for aircraft before aluminum was readily available (Howard Hughes’ famous Spruce Goose).

However only the top of a violin is made of spruce. The rest is usually made from curly maple, a high figured hard wood. The above chart reveals that this material wasn’t chosen on the basis of sound production but rather for its beauty and perhaps, more fundamentally, its strength (the axial pull of the four strings combine to over sixty pounds). It also matters how the wood is cut from the tree. Detailed experiments have revealed that the velocity of sound decreases as the alignment of the fibers deviates from being parallel to the surface. In fact not only does the sound velocity go down, but the rate at which vibrations are damped-out increases. This becomes a noticeable effect for deviations as small as a few degrees. Traditional makers have always preferred wood split from logs rather than sawed. Now we know why.

A modern mechanical engineer might view the violin as an example of thin shell technology. Violin plates begin as thick pieces of wood. The final cross-sectional shape shown in the figure on page 22 is arrived at through a two step-carving process. First the outside is shaped. The curve (called the arching pattern) is usually copied from one of the old Italian masters. While these arching patterns may appear the same to the untrained eye, there can be a wide variety in the maximum height and how full the curve is. “Height” is a measure of how much the center section puffs up from the plane of the sides (usually 14–16 mm). The back of the instrument can have a different height than the front. The “fullness” of the arch refers to how far this puff extends toward the edges. “Swoopy” arches start their descent closer to the center. Full arches (descent closer to the edge) tend to be stronger and more resistance to long-term deformation owing to the static loading produced by the strings.

After shaping the outside, the inside is scooped out. Here’s where the fun begins! How thick should the wood be and what thickness pattern should be used. This is complicated and still only poorly understood. Since the material is wood and therefore anisotropic with variations in physical properties from piece to piece, any recipes based purely on dimensions will fail. Somehow one must use the vibrational properties of the individual plate to guide the thinning (or graduating) process.

Traditionally makers have use a technique based on “tap” tones. Holding the free plate off center in the upper area with the thumb and forefinger and tapping near the center with the other hand produces a distinct pitch that can be perceived as the plate nears its final thicknesses. Keeping the same holding point and tapping at the center near the bottom reveals yet another pitch. When old Italian instruments are disassembled for repair, their tap tones are often examined. In particular, a famous maker/restorer named Simone Sacconi, who worked on many of the existing Stradivari violins, reported that all had tap tones between F and F#, strongly suggesting that this was part of the equation that evolved in Italy three hundred years ago.

Carleen Hutchins, who studied under Sacconi, developed a new method for measuring tap tone while making instruments. She showed that they corresponded to eigenmodes of the free plate, which can be seen clearly using the technique of Chladni patterns. Hutchins suspended a violin plate horizontally above a loud speaker connected to an audio signal generator. When black glitter is sprinkled on the surface, at resonance, the patterns shown in the drawing on the right appear. The nodal lines are the places where the plate is not moving and hence the glitter tends to accumulate. The traditional tap-tone technique required holding the plate on a nodal line and tapping at an anti-node. This simple process is in wide use today.

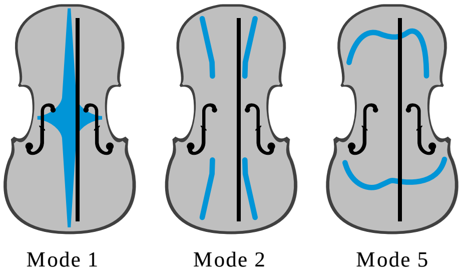
Tuning the tap tones has now been reduced to a “science,” but free violin plates are not an end in themselves. The plates are eventually glued to the sides, changing the boundary conditions from free to somewhere between hinged and clamped. Investigations into the vibration modes for plates both free and clamped have shown that the thickness of wood near the edges of a plate is critical in determining the eigenmodes for the clamped condition but has little effect on the free plate. Wood removal near the center controls the latter.

SETUP

Completing the violin after graduating the plates involves a great deal of precision woodworking and then comes the varnishing. The next phase in which physics can play a part is in the “setup.” This includes shaping the fingerboard and bridge, fitting the sound post, adjusting the tailpiece, and so on. Much of how the final product will sound is determined at this stage.

The bridge of the instrument conveys the vibrations from the strings

The three main eigenmodes for a free violin top plate are shown below. The frequencies associated with these modes are approximately an octave apart.



Both Helmholtz and Drell seem fascinated with bowed string motion. Here Drell is engaged in a real time analysis of the bow's slip-stick interaction with the string.

to the top. In some sense it may be viewed as an audio filter. Its traditional and intricate shape was most likely arrived at through trial and error with an eye on the esthetics and an ear on the resulting sound. Observations of which bridges sound good suggested an experiment using similar tuning techniques to those used for free plates. A high-power audio tweeter and audio generator revealed that the best bridges had a simple bending eigenmode pitched at high F (2800 Hz).

After a final fitting with strings, the vibrational modes of the complete instrument can be investigated and adjusted. When the lowest air cavity resonance and the lowest body-bending mode have similar frequencies, the violin comes alive. While there has been little quantitative work to detail what sound a mode-matched instrument makes as opposed to an unmatched one, both players and listeners prefer the former. The frequency of the air cavity mode is determined mostly by the volume of the box and the size and shape of the f holes. Little can be done to alter this “bottle” note and indeed one finds little variation from instrument to instrument. A closely related note can be found by humming into an f hole and finding the pitch at which the body resonates (usually between C to C#). The first bending mode has two transverse nodal lines running across the widest sections of the upper and lower areas. The pitch of this eigenmode can be determined by holding the instrument upside down at the widest point in the area next to the tailpiece and lightly tapping on the scroll. It turns out that the fingerboard can be modified to vary this note. By adding weight at the end near the bridge or by thinning the fingerboard in the area where it joins the neck, the note can be lowered.

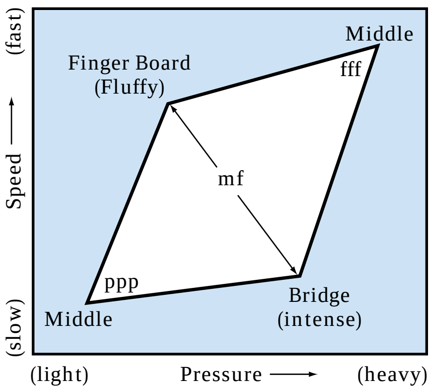
The tailpiece also plays an important role. The relative lengths of the strings between the bridge and the nut at the far end of the fingerboard and between the bridge and the tailpiece should be six to one. This length can be approximately tuned using a ruler; however, a knowledge of harmonics gives a simpler and more accurate technique. A string which is one-sixth the full length will sound a note at the fifth above the second octave. Given that violins are tuned in fifths means that by plucking the string behind the bridge and comparing it to the note of the next higher string plucked in front of the bridge an octave should be heard. The pitch of the note behind the bridge may be adjusted by changing the length of the cord (called the tailgut) fastening the tailpiece to the endpin of the instrument. But the tailpiece also has resonances which have been found to have optimal values. Changing the tail-gut length changes these pitches as well. The only remaining free parameter is to adjust the weight of the tailpiece: heavier lowers its resonance pitches while lighter raises them.

PLAYING

One of the appealing attributes of the violin is the variety of sounds. This sound palette is manipulated through the bowing technique. The main variables are the speed of the bow, the downward pressure on the string, and the point of contact on the string with the bow hair. (To a lesser extent the amount of hair which is in contact with the string also plays a role.) All this occurs essentially in a three-dimensional space. A two-dimensional projection of this bowing space is illustrated in the figure on the right. The softest sound is obtained with a slow, light bow halfway between the fingerboard and the bridge while the loudest notes are produced with a fast, heavy bow at a similar contact point. The most variation in speed and pressure can be found in the middle of the range. Here too is the largest range of possible contact points. Harsh, biting sounds may be produced with the bow near the bridge, while soft, fluffy sounds are obtained with the contact point nearer the fingerboard. By adjusting these three variables, the violinist is changing not only the music dynamic but also the timbre of the sound.

An obvious question is what happens if you go outside the “allowed” region in the bowing space? You produce either an annoying scratching sound or an equally unpleasant whistling. The edges of this space are given by the boundaries for the Helmholtz-type “slip stick” action as discussed in the introduction. Preferred violins have a large bowing space. These instruments allow the concert artist the greatest range in sound color and dynamic. A corollary is that cheap violins have a small bowing space. This explains how the beginning student, with little bow control, may sound atrocious on a $200 fiddle provided by the parents while the teacher (with a great deal of bow control) can sound reasonably good!

Physics or physics training does not give any intrinsic advantage to violin making. Music is essentially esthetic and producing an instrument necessitates highly developed woodworking skills. However, physics can be a tool for understanding what parameters of the instrument control various aspects of its sound, thereby providing a guide for shaping it. It can also inspire experiments with narrow focus aimed at resolving specific issues in violin making. And, finally, good experimental science, specifically the recording of detailed notes on each instrument, can pay off handsomely when trying to determine what works or doesn’t work.



The bowing space for the violin is illustrated above. The speed that the bow is drawn across the string together with the down pressure changes the musical dynamic (shown inside the white diamond). The point of contact between the bow and string changes the “color” of sound and has its largest range in the middle (shown outside the white diamond).