M-297

Stanford Linear Accelerator Center
Stanford University
Stanford, California

Internal Memorandum
M Report No. 297
April 1962

DEFLECTION MAGNET SYSTEMS FOR THE PRIMARY BEAM

By
H. S. Butler and J. J. Muray

I. INTRODUCTION

In order to use effectively the electron beam from the Stanford two-mile linear accelerator, an elaborate beam switchyard will be needed. The magnets in this switchyard will provide the 20° to 40° deflections necessary to allow the various experimental areas to be reasonably well shielded and separated from each other. Two basic components will constitute the switchyard: a switching magnet and a deflecting magnet system. Figure 1 shows the switchyard schematically. Present plans call for a second deflection system (also shown) that will be serviced by the same switching magnet.

The switching magnet will serve to divert the beam into the deflecting magnet system. As presently envisioned, the switching magnet will be designed for pulsed operation. This provides the capability of switching only occasional pulses into the experimental areas.

The deflection magnet system, a dc system containing several individual magnets, will bend the beam, momentum analyze it and direct it toward the end station. This system should be of the zero-dispersion type

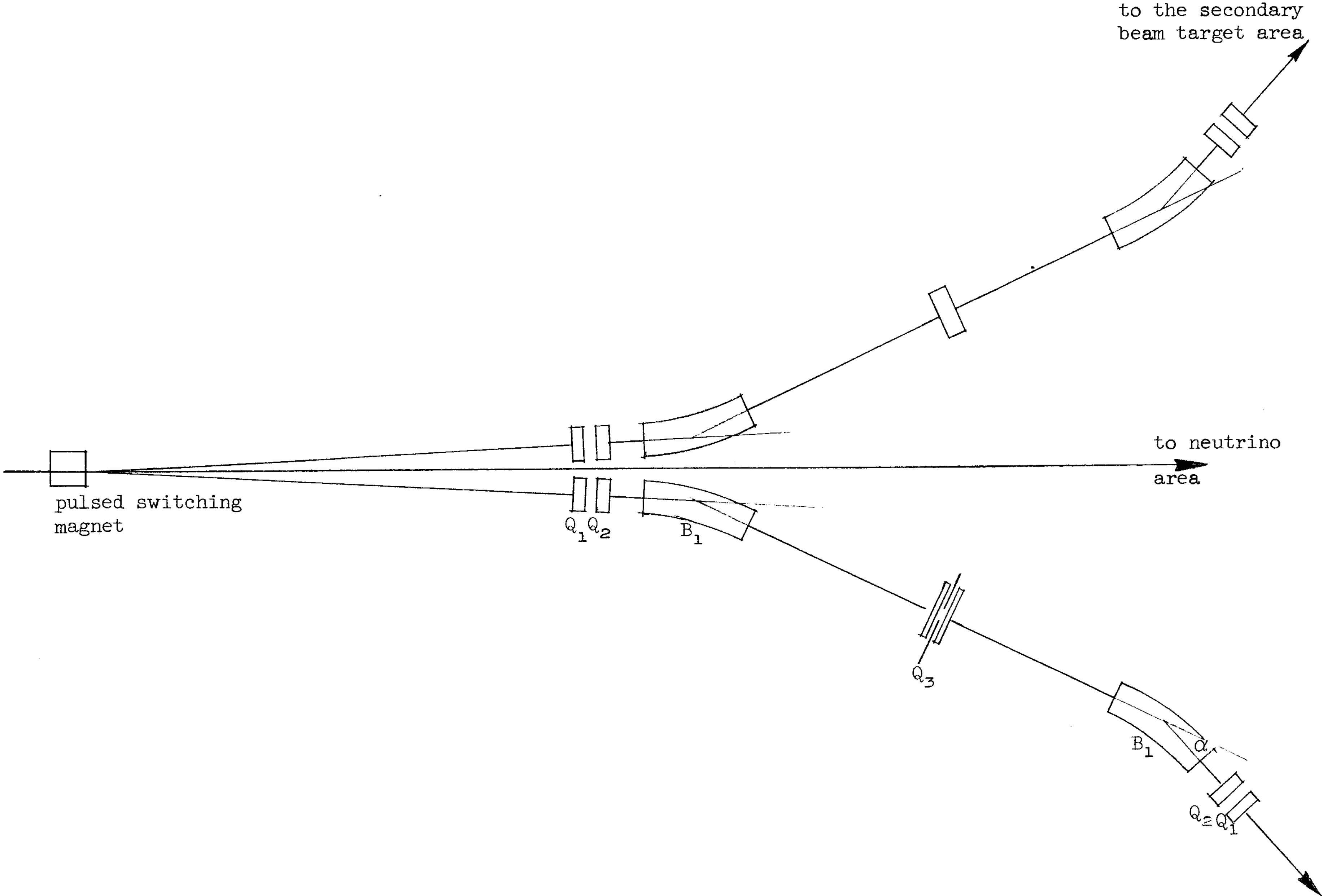

FIG. 1--Beam switchyard.

because one may wish to conduct the beam several hundred feet from the deflection system to an experiment.

The purpose of this report is to present a comparison between two different deflection systems that could be utilized to transport the primary beam to the target area. Consideration of the switching magnet is left for a later report. In this comparison the limit of momentum resolution as a function of system length was of prime consideration. To conform with current beam parameters the electron energies were assumed to lie in the range of 10-50 Bev, while the angular divergence was varied from 10^{-3} to 10^{-5} radians. Bending magnets with a field strength of 18,000 gauss and quadrupole magnets with a 2,000 gauss/cm gradient were taken as the building blocks of the system.

The analysis of the two systems was made using the matrix formulation described by Penner.[1] Familiarity with this method is assumed, and no discussion of it is included in this report.

II. THE TRI-WEDGE SYSTEM

An analysis of the first of the two deflection systems to be considered was presented by Penner[2] as an example of the application of the matrix method. The analysis will be reproduced here for completeness. The proposed system is one that deflects a parallel beam of charged particles through a finite angle, preserves parallelness and produces no momentum dispersion in the final beam. The system consists of three identical uniform-field bending magnets whose radius of curvature for the central momentum is ρ and whose deflection angle is α, as shown in Fig. 2a. Since there is no vertical focusing in this system (the beam enters and leaves perpendicular to each magnet face) an initially parallel beam in the vertical plane will remain parallel. However, because there is a finite divergence the beam will spread vertically. The final half-height

[1] S. Penner, "Calculations of Properties of Magnetic Deflection Systems," Rev. Sci. Instr., 32, 150-160 (1961).

[2] *Ibid.*

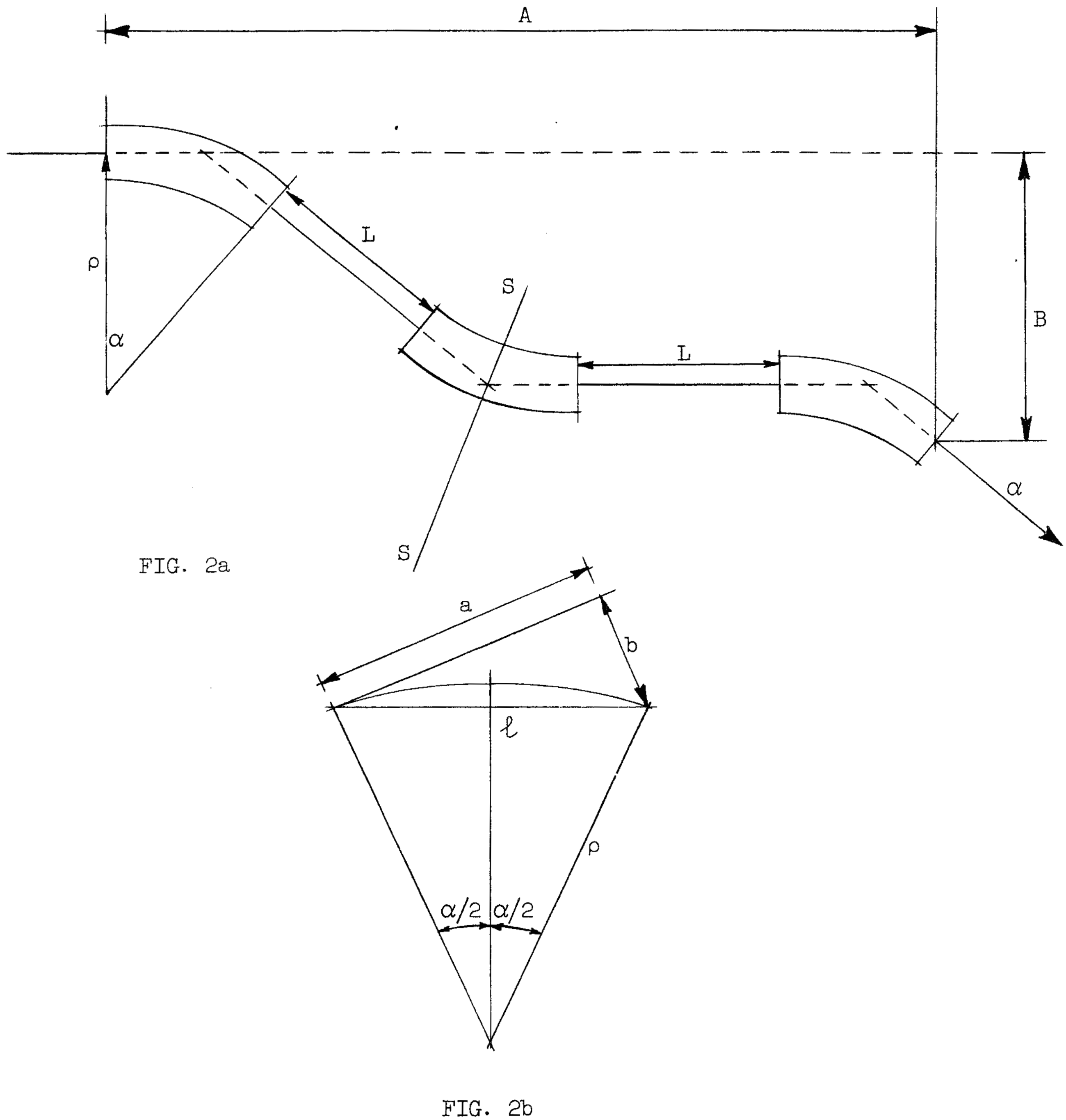

FIG. 2--Schematic diagram of tri-wedge magnet system.

of the beam is given by

$$y = y_o + (3\rho\alpha + 2L)\varphi_o \tag{1}$$

If the distance L between magnets can be chosen such that the horizontal trajectories of any fixed momentum come to focus on the symmetry axis SS, and the central ray ($x_o = 0$, $\theta_o = 0$) of any momentum deviation $\Delta p/p$ crosses the symmetry axis perpendicularly (i.e., with $\theta_s = 0$), then by symmetry the beam emerging from the system will be parallel and undispersed (i.e., θ and x independent of $\Delta p/p$). The first condition is satisfied by requiring that, at the symmetry axis, the displacement x_s be independent of the initial displacement x_o. The second condition requires that the angle at the symmetry axis θ_s be independent of $\Delta p/p$.

The transformation from the entrance of the first magnet to the symmetry axis is obtained by applying to the initial coordinates (x_o, θ_o, $\Delta p/p$) successively: the matrix for a uniform-field magnet with no vertical focusing, the matrix describing a translation through field-free space by a distance L, and the matrix that describes an inverted magnet with bending angle $\alpha/2$.

The transformation is

$$\begin{pmatrix} x_s \\ \theta_s \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} \cos\frac{\alpha}{2} & \rho\sin\frac{\alpha}{2} & -\rho\left(1-\cos\frac{\alpha}{2}\right) \\ -\frac{1}{\rho}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} & -\sin\frac{\alpha}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & \rho\sin\alpha & \rho(1-\cos\alpha) \\ -\frac{1}{\rho}\sin\alpha & \cos\alpha & \sin\alpha \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_o \\ \theta_o \\ \frac{\Delta p}{p} \end{pmatrix}$$

Multiplication of the matrices results in

$$\begin{pmatrix} x_s \\ \theta_s \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} M \end{pmatrix} \begin{pmatrix} x_o \\ \theta_o \\ \frac{\Delta p}{p} \end{pmatrix}$$

where

$M_{11} = \cos 3/2\ \alpha - L/\rho \sin \alpha \cos \alpha/2$

$M_{21} = -1/\rho(\sin 3\alpha/2 - L/\rho \sin \alpha/2 \sin \alpha)$

$M_{12} = \rho(\sin 3\alpha/2 + L/\rho \cos \alpha/2 \cos \alpha)$

$M_{22} = \cos 3/2\ \alpha - L/\rho \sin \alpha/2 \cos \alpha$

$M_{13} = \rho(2 \cos \alpha/2 - 1 - \cos 3\alpha/2) + L \cos \alpha/2 \cos \alpha$

$M_{23} = \sin 3\alpha/2 - 2 \sin \alpha/2 - L/\rho \sin \alpha/2 \sin \alpha$

$M_{31} = M_{32} = 0$

$M_{33} = 1$

The focus condition requires that $M_{11} = 0$. That is

$$\frac{L}{\rho} = \left(\cos \frac{3\alpha}{2}\right) \Big/ \left(\sin \alpha \cos \frac{\alpha}{2}\right)$$

The second condition requires that $M_{23} = 0$. This yields

$$\frac{L}{\rho} = \left(\sin \frac{3\alpha}{2} - 2 \sin \frac{\alpha}{2}\right) \Big/ \left(\sin \frac{\alpha}{2} \sin \alpha\right)$$

The equating of the right-hand members of these two relations yields an identity that defines L in terms of the parameters of the bending magnets.

$$\frac{L}{\rho} = \cot \alpha - \tan \frac{\alpha}{2} \tag{2}$$

From this equation we see that L is negative for $\alpha > 60^\circ$, so that this arrangement is practical only for deflection angles somewhat less than 60°. Substituting for L/ρ in the matrix M leads to a simplified form of the complete transformation from the entrance of the system to the symmetry axis.

$$\begin{pmatrix} x_s \\ \theta_s \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} 0 & \frac{\rho}{2 \sin \alpha/2} & \rho\left(2 \cos \frac{\alpha}{2} - 1\right) \\ -\frac{2}{\rho} \sin \frac{\alpha}{2} & 2 \cos \frac{\alpha}{2} - \frac{3/2}{\cos \alpha/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_o \\ \theta_o \\ \frac{\Delta p}{p} \end{pmatrix}$$

Since we are dealing with a beam that has a finite phase space ($\theta_o \neq 0$), it is not possible to achieve infinite energy resolution. The obtainable energy resolution is determined by the relative importance of the θ_o and the $\Delta p/p$ terms in x_s. The minimum energy resolution that can be obtained by placing energy-defining slits on the symmetry axis is

$$\left(\frac{\Delta p}{p}\right)_{min} = \frac{\theta_o}{2 \sin \alpha/2 \ (2 \cos \alpha/2 - 1)} \tag{3}$$

In this expression $\Delta p/p)_{min}$ (corresponding to zero slit width) is the half-width at half-maximum intensity of the momentum interval for the case of 100% transmission of the central momentum ($\Delta p/p = 0$).

The transformation matrix for the second half of the complete system is found by applying the reflection operation discussed by Penner[3] to the

[3]S. Penner, "Calculations of Properties of Magnetic Deflection Systems," Rev. Sci. Instr., 32, 150-160 (1961).

transformation matrix for the first half of the system. Their product then yields the transformation through the complete system. It is

$$\begin{pmatrix} x_f \\ \theta_f \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} -1 & \rho\left(\frac{2}{\tan \alpha/2} - \frac{3}{\sin \alpha}\right) & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_o \\ \theta_o \\ \frac{\Delta p}{p} \end{pmatrix}$$

where x_f and θ_f are the trajectory coordinates at the exit of the last magnet. It is easily seen that this transformation does accomplish the desired purposes of parallel-to-parallel beam transformation $\left(\theta_f = 0 \text{ if } \theta_o = 0\right)$, and the zero dispersion $\left(x_f \text{ and } \theta_f \text{ independent of } \Delta p/p\right)$.

The total length and offset distance of this system can be calculated from the geometry shown in Figs. 2a and 2b. We have

$$\ell = 2\rho \sin \alpha/2$$

from which we calculate

$$a = \ell \cos \alpha/2 = 2\rho \sin \alpha/2 \,(\cos \alpha/2) = \rho \sin \alpha$$

and

$$b = \ell \sin \alpha/2 = 2\rho \sin^2 \alpha/2$$

The total length of the system is then

$$A = a + L \cos \alpha + a + L + a = 3a + L(1 - \cos \alpha)$$

Using the value of L from Eq. (2) gives

$$A = 3\rho \sin \alpha + \rho(\cot \alpha - \tan \alpha/2)(1 - \cos \alpha) \tag{4}$$

Similarly, we obtain the following expression for the offset distance of the system.

$$B = b + L \sin \alpha + b + b = 6\rho \sin^2 \alpha/2 + L \sin \alpha \qquad (5)$$

This completes ths analysis of the tri-wedge system. Given the electron energy of interest, the field strength of the magnets and the bending angle, one can calculate from Eq. (2) the drift length L, which will give the system the required optical properties. The corresponding total length A and offset distance B for the system can be calculated from Eqs. (4) and (5), respectively. Finally, the minimum energy resolution that can be obtained with the system for a given beam divergence is calculated from Eq. (3).

In Figs. 3 and 4 the size of the system (A and B) is shown as function of the energy for representative bending angles.

Figure 5 shows the minimum energy resolution as function of the beam divergence, Eq. (3), for various deflection angles.

Although the resolution and other properties of the tri-wedge magnet system are acceptable, the size of the system as measured by the distances A and B is prohibitively large. It could be compressed to an acceptable length by the judicious use of quadrupole magnets. However, bending magnets for high-energy particles are very expensive, and any design that eliminates the need for even one bending magnet is to be looked upon with favor. Such a design is the KLB system.

III. THE KLB SYSTEM

A second system that satisfies the design criteria for the deflection system is one proposed by K. L. Brown. It is depicted schematically in Fig. 6 and was discussed in a semi-quantitative manner by Penner,[4] who concurred that it would be an acceptable system subject to more precise calculations. In this system the electron beam incident on the bending magnet B_1 is brought to a focus in both the horizontal and vertical planes on the symmetry axis SS in the center of the quadrupole Q.

[4]S. Penner, "Electron Beam Deflection Systems for the Monster," M Report No. 200-13, Stanford Linear Accelerator Center, Stanford University, Stanford, California, Summer 1960.

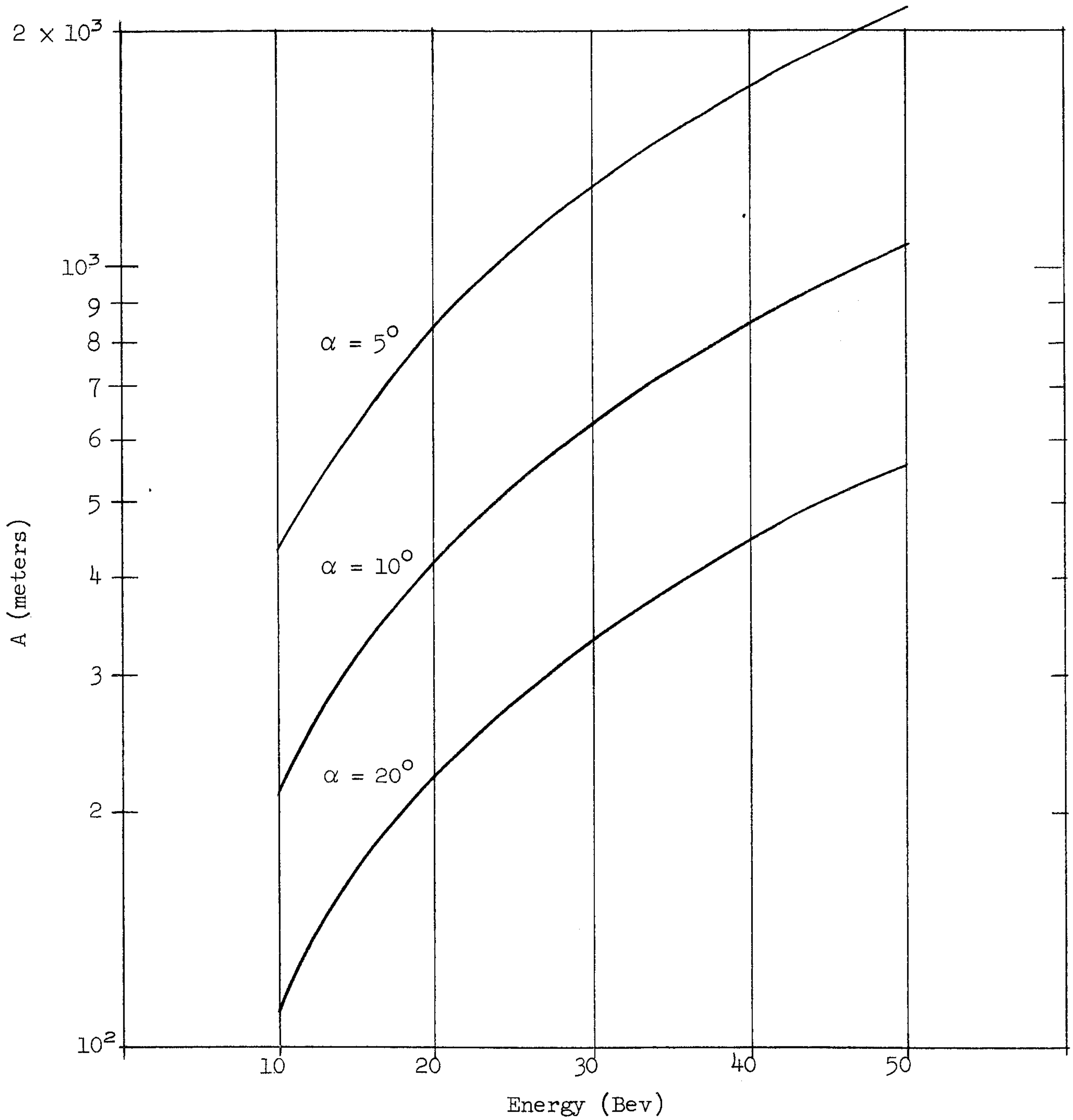

FIG. 3--The total length of the tri-wedge system vs energy, with the deflection angle α as a parameter.

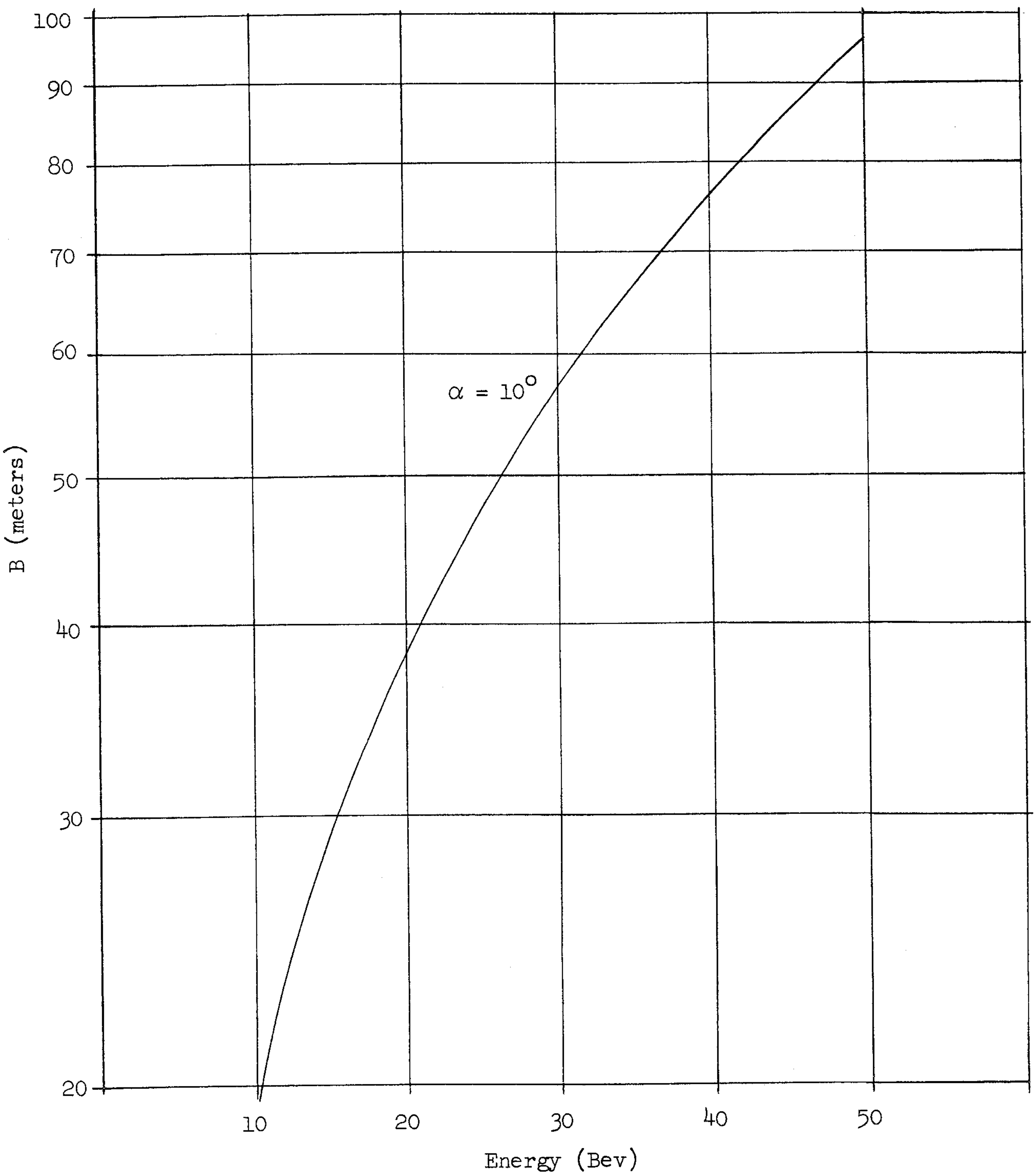

FIG. 4--The offset distance B vs energy for fixed deflection angle α.

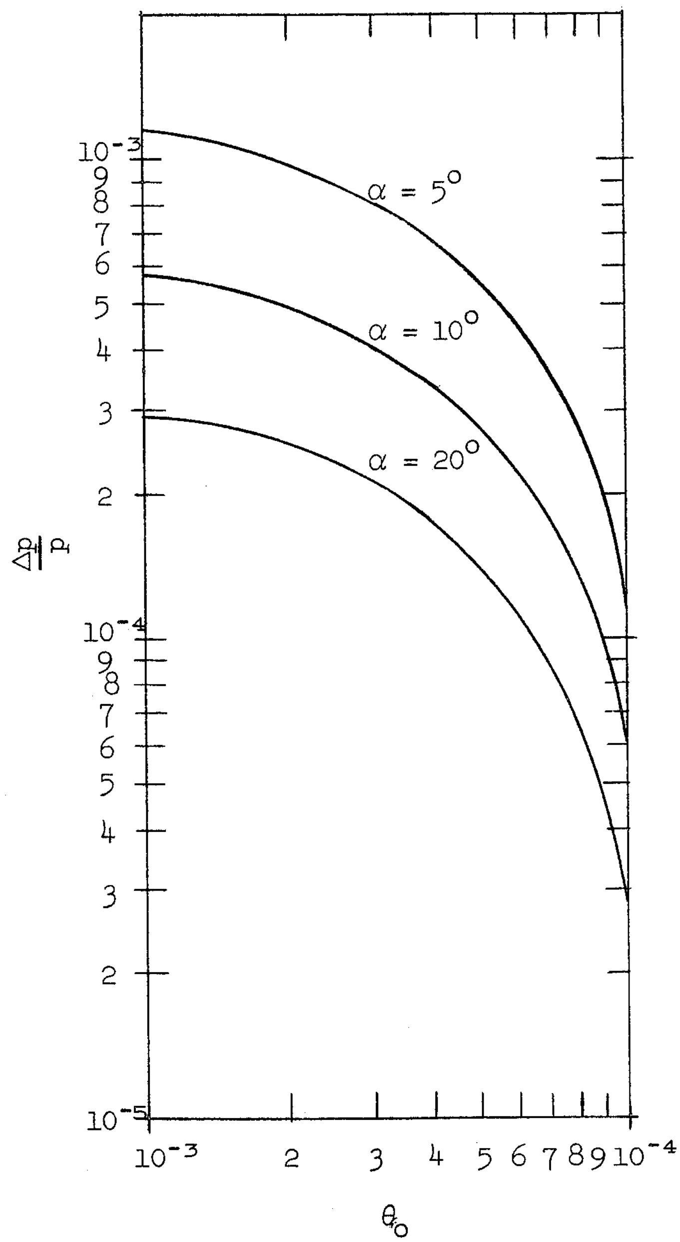

FIG. 5--$\Delta p/p$ vs θ_o for various fixed deflection angles.

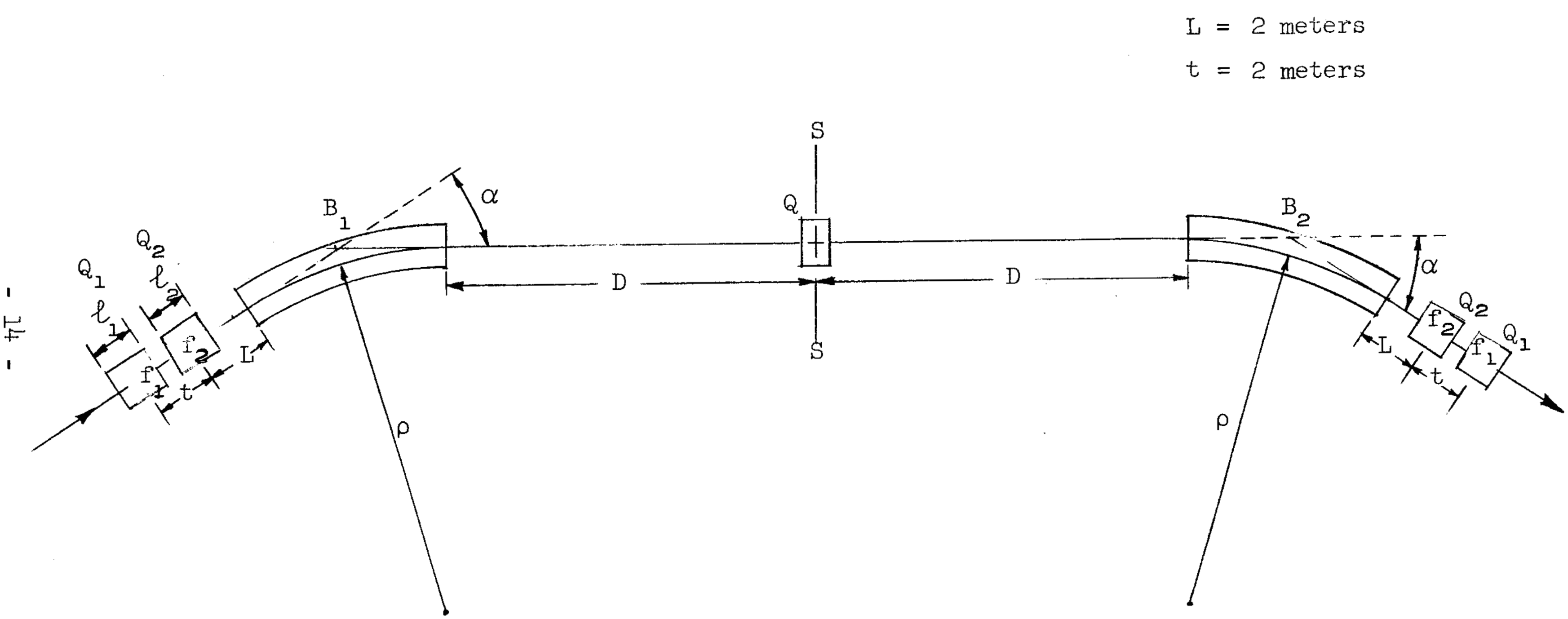

FIG. 6--A schematic diagram of the KLB magnetic deflection system.

This quadrupole, which focuses in the horizontal plane, acts as a field lens; i.e., the beam is focused at Q and Q images the aperture of B_1 on B_2. Viewed as a thin lens, Q will have no effect on trajectories with $\theta_o = 0$ and $\Delta p/p = 0$, since these trajectories pass through the center of the lens. However, the lens does serve to recollect trajectories of different momenta (which have been dispersed by the first bending magnet) in a symmetrical manner so that there will be zero dispersion of the final beam.

In the vertical plane the transformation relating the vertical trajectories at the center of Q_1, the entrance of the system, to those at the symmetry plane is given by

$$\begin{pmatrix} y_s \\ \varphi_s \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} 1 & d & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 + \frac{t}{f_1} & t & 0 \\ -\frac{1}{F_y} & 1 + \frac{t}{f_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_o \\ \varphi_o \\ \frac{\Delta p}{p} \end{pmatrix}$$

where

$$d = L + \rho\alpha + D$$

and

$$\frac{1}{F_y} = -\frac{1}{f_1} - \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

Because the vertical trajectories will be made to cross over at the center of Q, there is no net vertical force on the electrons from Q. Hence there is no matrix for Q in the vertical transformation. The requirement for vertical focusing at the symmetry plane is satisfied by making y_s independent of y_o; that is, the (1,1) element of the complete transformation matrix must vanish.

This yields the equation

$$1 + \frac{t}{f_1} - \frac{d}{F_y} = 0$$

Rewriting this we obtain

$$\frac{1}{F_y} = \frac{1}{d}\left(1 + \frac{t}{f_1}\right) \qquad (6)$$

Because of the relation between F_y and f_2 we may rewrite this in still another form which will prove useful

$$\frac{1}{f_2} = -\frac{1}{d} - \frac{1}{(t + f_1)} \qquad (7)$$

The horizontal plane transformation from the center of the first magnet to the symmetry plane is

$$\begin{pmatrix} x_s \\ \theta_s \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} & & \\ & f & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & D & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & B & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & L & \\ & & \end{pmatrix} \begin{pmatrix} & & \\ & F & \\ & & \end{pmatrix} \begin{pmatrix} x_o \\ \theta_o \\ \frac{\Delta p}{p} \end{pmatrix}$$

where the non-zero elements of the above matrices are

$f_{11} = f_{22} = f_{33} = 1$ $\qquad$ $f_{21} = -\frac{1}{2}f$

$D_{11} = D_{22} = D_{33} = 1$ $\qquad$ $D_{12} = D$

$B_{11} = B_{22} = \cos\alpha$ $\qquad$ $B_{12} = \rho \sin\alpha$ $\qquad$ $B_{33} = 1$

$B_{13} = \rho(1 - \cos\alpha)$ $\qquad$ $B_{21} = -\frac{\sin\alpha}{\rho}$

$$L_{11} = L_{22} = L_{33} = 1 \qquad L_{12} = L$$

$$F_{11} = 1 - t/f_1 \quad F_{22} = 1 - t/f_2 \qquad F_{12} = t \quad F_{33} = 1$$

and

$$F_{21} = - 1/F_x$$

where

$$1/F_x = 1/f_1 + 1/f_2 - t/f_1 f_2$$

To obtain focusing in the horizontatl plane we require that the (1,1) element of the total transformation matrix vanish. This yields the equation

$$\left(\cos \alpha - \frac{D \sin \alpha}{\rho}\right) \left(1 - \frac{t}{f_1} - \frac{L}{F_x}\right) - \frac{1}{F_x} \left(\rho \sin \alpha + D \cos \alpha\right) = 0$$

Solving for $1/F_x$ we obtain

$$1/F_x = a\left(1 - t/f_1\right) \tag{8}$$

where

$$a = \frac{\cos \alpha - (D \sin \alpha)/\rho}{(L + D) \cos \alpha + (\rho - LD/\rho) \sin \alpha}$$

Assuming that we are given the system parameters t, L, D, α and ρ, Eqs. (6) and (8) are sufficient to define the focal length of the quadrupoles f_1 and f_2 required to produce double focusing at the symmetry plane. One way of obtaining an explicit expression for f_1

is to add together Eqs. (6) and (8) and then substitute for $1/f_2$ from Eq. (7). This gives

$$\frac{2t}{f_1}\left(\frac{1}{d} + \frac{1}{t + f_1}\right) = \frac{1}{d}\left(1 + \frac{t}{f_1}\right) + a\left(1 - \frac{t}{f_1}\right)$$

Straightforward solution of this equation leads to the following expression for f_1..

$$f_1 = \left(t^2 + \frac{2t}{a + 1/d}\right)^{\frac{1}{2}} \tag{9}$$

Having solved for f_1 it is a simple matter to calculate f_2 using Eq. (7). Knowing the two focal lengths, it is possible to calculate the effective length of the two quadrupoles from

$$\ell_1 = \frac{1}{k}\sin^{-1}\left(\frac{1}{kf_1}\right) \tag{10}$$

$$\ell_2 = \frac{1}{k}\sinh^{-1}\left(\frac{1}{kf_2}\right) \tag{11}$$

where

$$k = \sqrt{\frac{1}{B\rho}\frac{\partial B}{\partial x}}$$

The total length of the system measured along the central trajectory is

$$L_t = \ell_1 + 2t + 2d \tag{12}$$

The minimum momentum resolution that can be obtained from this system, which is a zero-width slit, is given by the ratio of the (1,2) to the

(1,3) element of the horizontal transformation matrix, all times the initial divergence θ_o.

$$\left(\frac{\Delta p}{p}\right)_{min} = \frac{[\rho(1 - \cos\alpha) + D\sin\alpha]\theta_o}{\left(\cos\alpha - D\frac{\sin\alpha}{\rho}\right)\left[t + L\left(1 - t/f_2\right)\right] + \left(1 - t/f_2\right)\left(\rho\sin\alpha + D\cos\alpha\right)} \tag{13}$$

The focal length of the quadrupole lens Q is determined by setting the (2,3) element of the matrix to zero. This makes θ_s independent of $\Delta p/p$ and ensures that the central ray of any momentum deviation crosses the symmetry axis perpendicularly. The vanishing of the (2,3) matrix element requires

$$-\frac{1}{2f}[\rho(1 - \cos\alpha) + D\sin\alpha] + \sin\alpha = 0$$

from which we calculate

$$f = \frac{1}{2}\left(D + \rho\tan\frac{\alpha}{2}\right) \tag{14}$$

The combined focal length of the quadrupole pair f_1 and f_2 in the horizontal plane is F_x. Associated with this focal length there is an object distance p and an image distance q, each measured from the appropriate principal plane. The distance $d = L + \rho\alpha + D$ is a very close approximation to q. We may calculate an excellent approximation to p by considering the following matrix transformation

$$\begin{pmatrix} x \\ \theta \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 - \frac{t}{f_1} & t \\ -\frac{1}{F_x} & 1 - \frac{t}{f_2} \end{pmatrix} \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_o \\ \theta_o \end{pmatrix}$$

We require a focus at a distance P to the left of the center of f_1 as well as at the symmetry plane. For both x and x_0 to vanish we must have the (1,2) matrix element vanish also. This gives

$$P\left(1 - \frac{t}{f_1}\right) + t + d\left(- \frac{P}{F_x} + 1 - \frac{t}{f_2}\right) = 0$$

Solving for P we have

$$P = \frac{-\left(t + d - td/f_2\right)}{1 - t/f_1 - d/F_x} \qquad (15)$$

This completes the necessary analysis. Once the following input parameters are specified--electron energy E, θ_0, α, B, dB/dx, t and L--it is possible to solve for the values of f_1 and f_2 which provide double focusing at the symmetry plane from Eqs. (7) and (9). In addition one can calculate the best obtainable energy resolution for a given initial divergence from Eq. (13). The length of the quadrupoles is calculated from Eqs. (10) and (11) while the total system length is given by Eq. (12). The focal length of the quadrupole at the symmetry plane is given by Eq. (14).

As a first step in the evaluation of this system the minimum energy resolution was calculated as a function of drift length D for a representative choice of system parameters. The result is shown in Fig. 7. The important conclusion to be drawn from this graph is that the resolution does not improve dramatically for drift lengths in excess of, say, 75 meters. If we set a lower limit of 0.1% in $\Delta p/p$, then the range of drift lengths that are of interest lie between 35-75 meters.

The dependence of energy resolution on the beam energy is depicted in Fig. 8. One readily concludes that the energy dependence is weak. The really important factor in resolution is the initial beam divergence θ_0. Eq. (13) shows that $\Delta p/p$ is directly proportional to θ_0.

Figure 9 shows the half length of the system as a function of energy for two different drift lengths. This graph shows that with a modification in length, namely an increase in length of the bending magnet, the

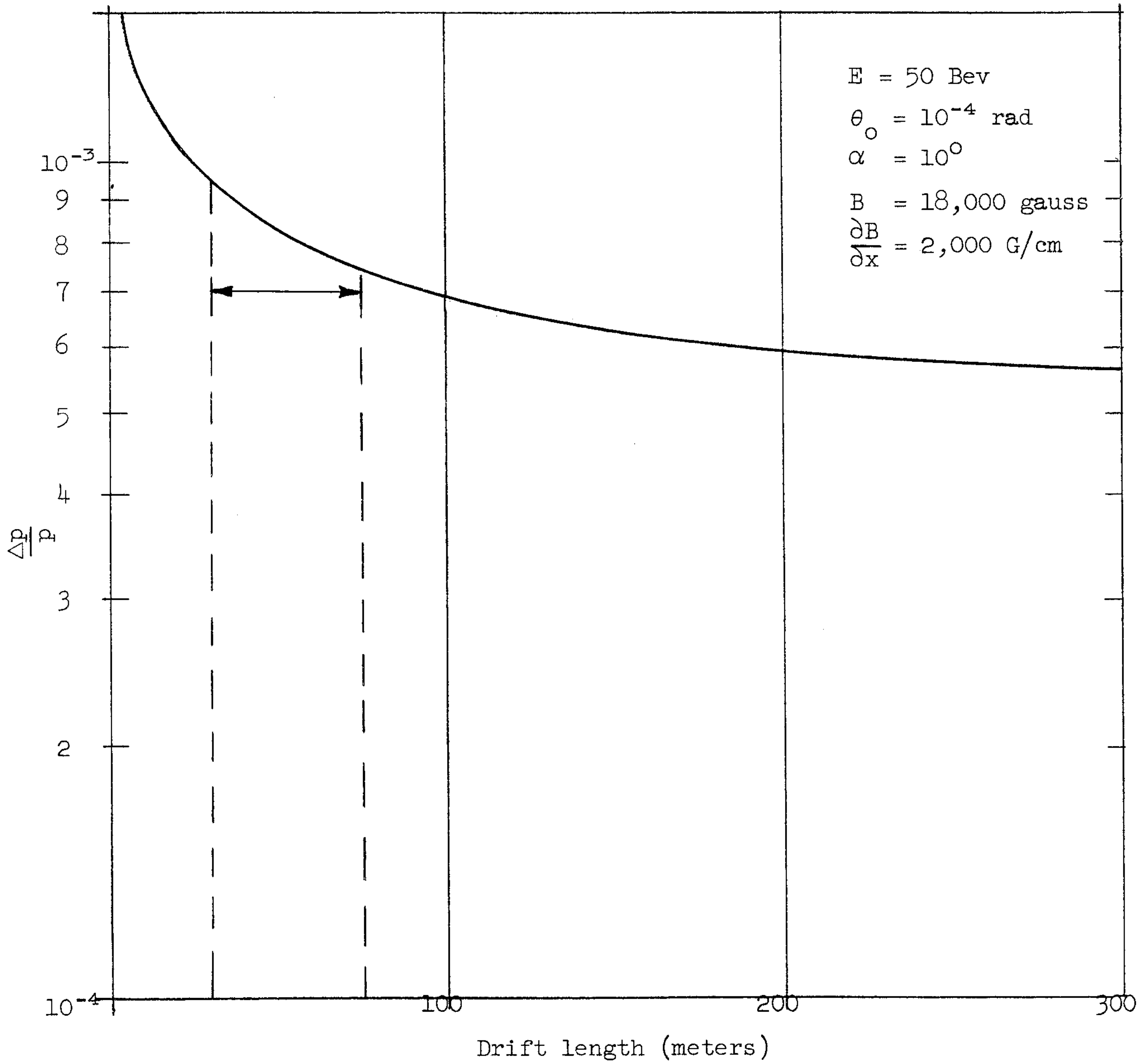

FIG. 7--$\Delta p/p$ as function of drift length.

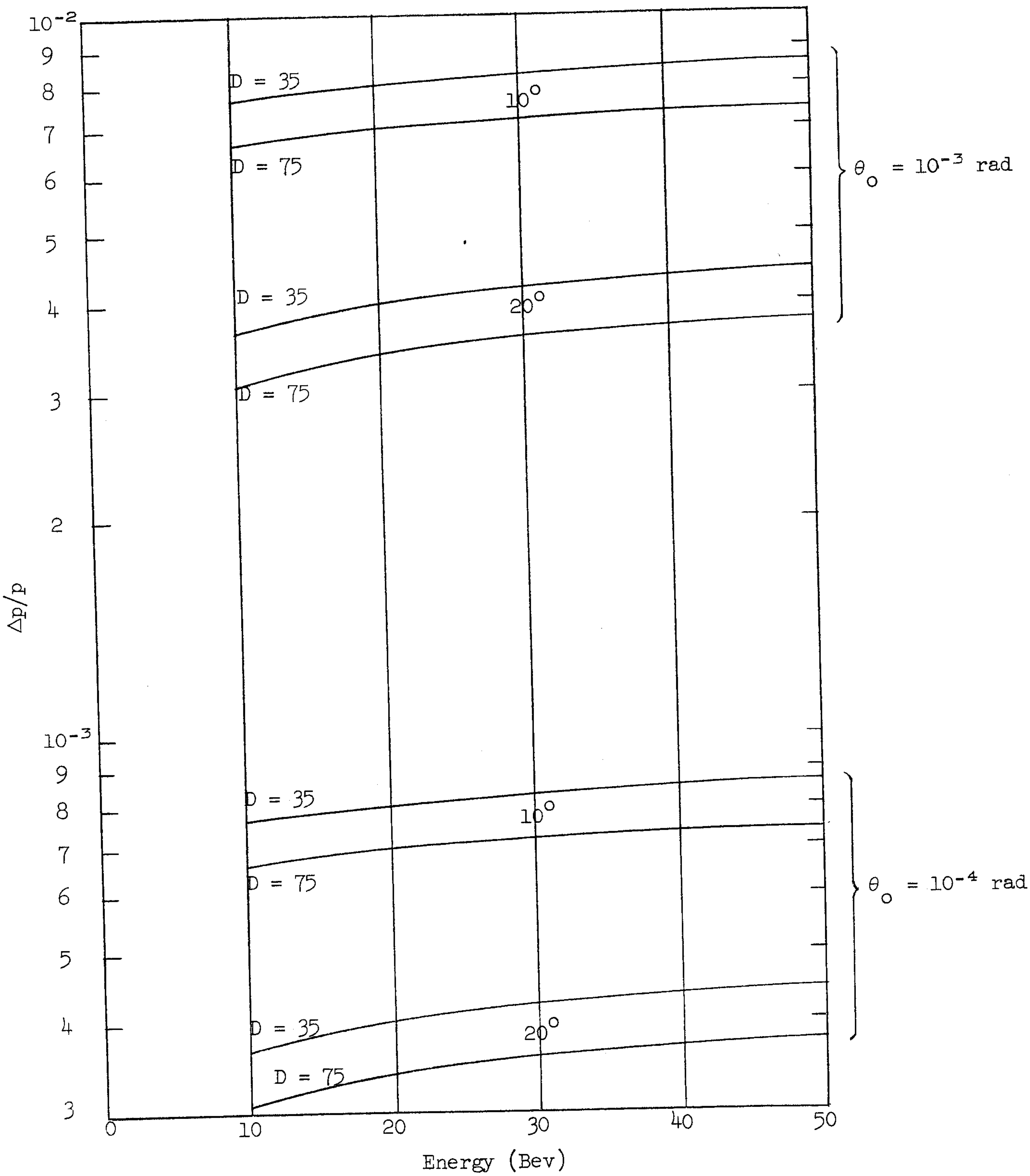

FIG. 8--$\Delta p/p$ as a function of the beam energy for fixed drift lengths (D = 35, 75m) and fixed beam divergence, θ_o.

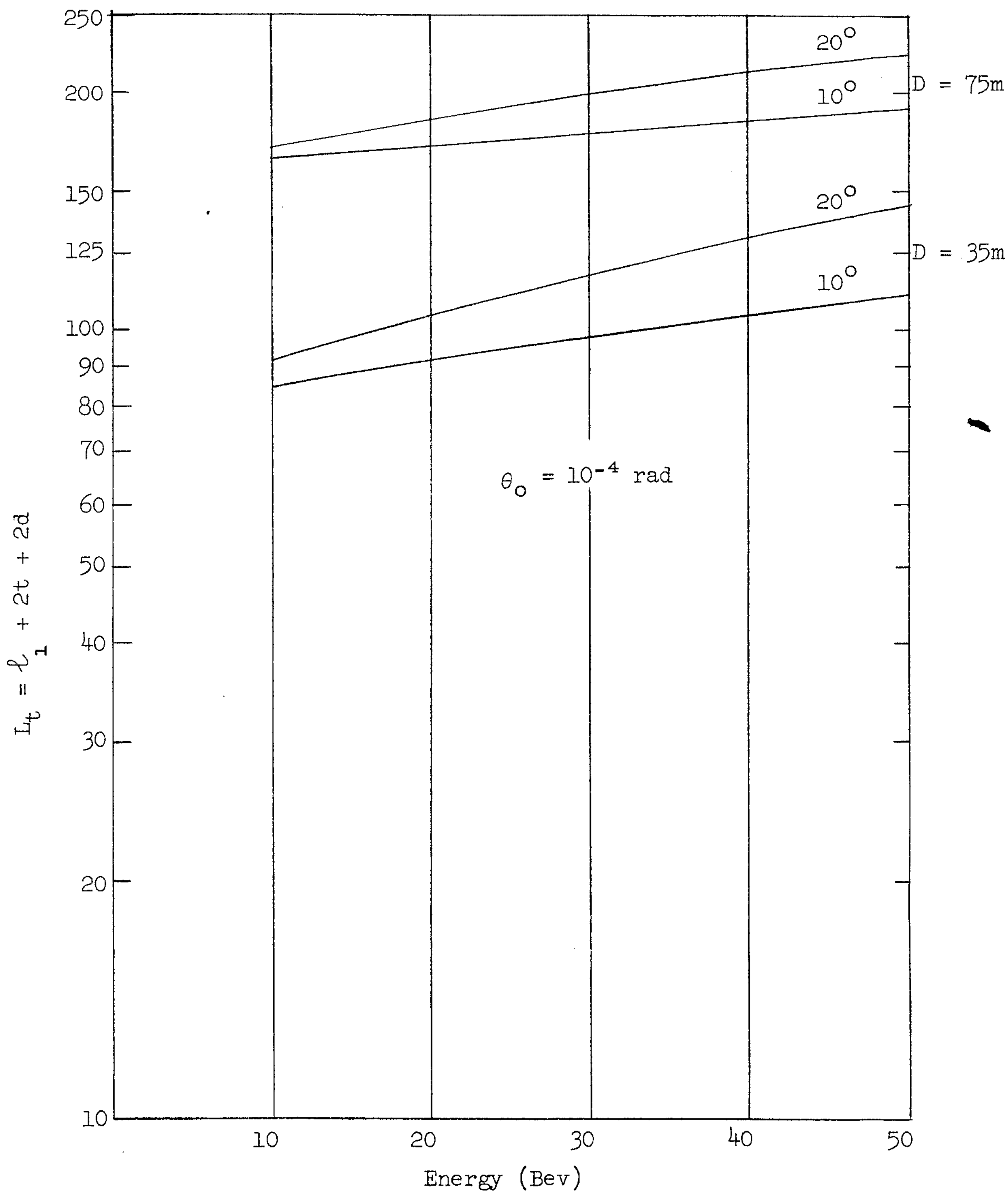

FIG. 9--The total length of the system as a function of the energy for two drift lengths and two angles.

system will work for a wide range of energies. This fits in with the philosophy of building the bending magnets in 10-foot sections and adding more sections as the beam energy increases.

The length of the quadrupole magnets in the system increases with energy for a fixed gradient in the field. This fact is shown in Fig. 10. This presentation was convenient from the point of view of calculations. In practice the length will be fixed by the highest achievable gradient and the highest electron energy; then the gradient, not the length, will be reduced as the energy is decreased.

Finally, Fig. 11 gives the object distance as a function of energy for two drift lengths. These curves were calculated from Eq. (15).

IV. CONCLUSIONS

This report has presented some detailed calculations relating to two possible deflection systems for the beam switchyard. On the basis of the first-order theory used in the analysis, both systems have the necessary optical properties, including satisfactory resolution (provided $\theta_o \lesssim 10^{-4}$ radians). However the KLB system is preferable to the tri-wedge system when costs are considered.

From Figs. 8-11 we can devise a possible switchyard for the primary beam. As an example of the use of these graphs we assume the following system parameters: $E = 20$ Bev, $\theta_o = 10^{-4}$ radians, $\alpha = 20^o$, $B = 18{,}000$ gauss, $\partial B/\partial x = 2{,}000$ gauss/cm, $t = 2$ meters, $L = 2$ meters and $D = 35$ meters. The minimum resolution obtained from Fig. 8 is $\Delta p/p)_{min} = 0.40 \times 10^{-3}$. The total length of the system Eq. (12) from Fig. 9 is 104 meters. The lengths of the quadrupoles in the doublet are, from Fig. 10: $\ell_1 = 0.30$ meters and $\ell_2 = 0.315$ meters. For the given gradient this corresponds to focal lengths of $f_1 = 11.3$ meters and $f_2 = -10^5$ meters from Eqs. (10) and (11). The focal length of the quadrupole at the symmetry plane is calculated from Eq. (14) to be 20.8 meters. This corresponds to an effective magnet length of 0.159 meters.

The same quantities for $D = 75$ meters are: $\Delta p/p)_{min} = 0.34 \times 10^{-3}$, $L_t = 184$ meters, $\ell_1 = 0.195$ meters, $\ell_2 = 0.202$ meters, $f_1 = 17.1$ meters, $f_2 = 15.7$ meters, $f = 40.8$ meters and $\ell = 0.081$ meters.

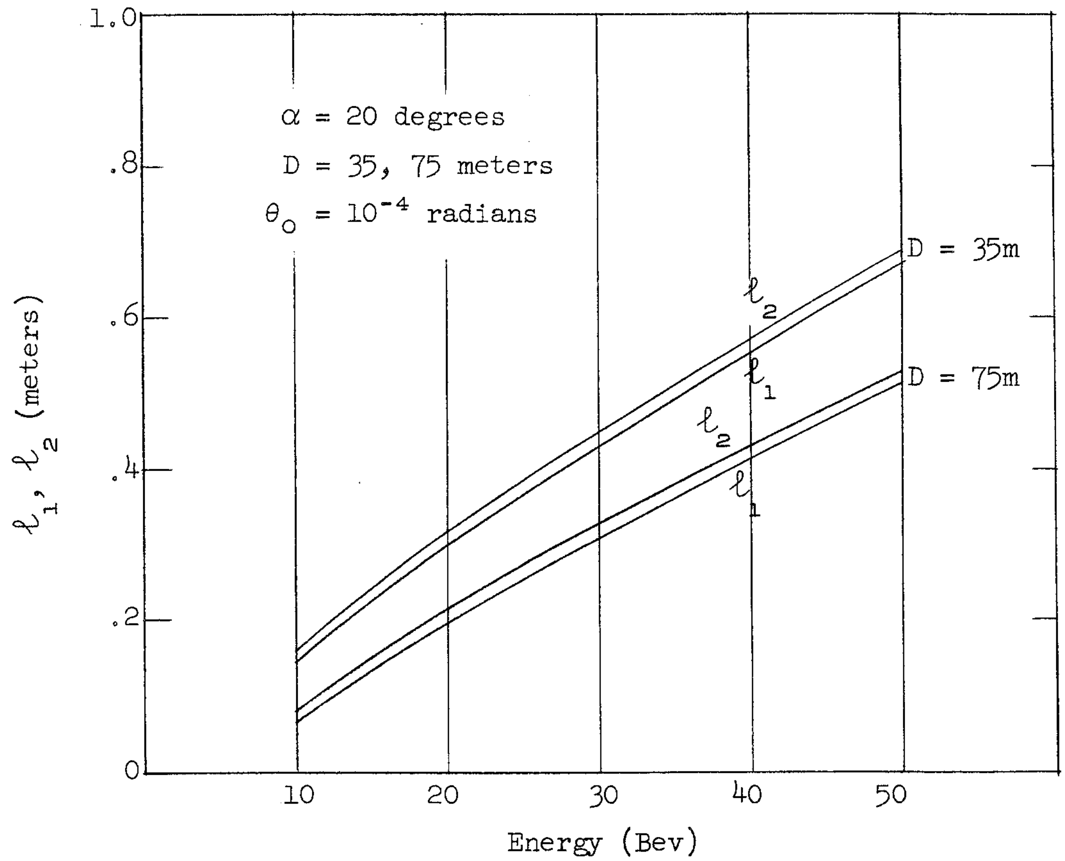

FIG. 10--Length of quadrupoles in doublet vs energy.

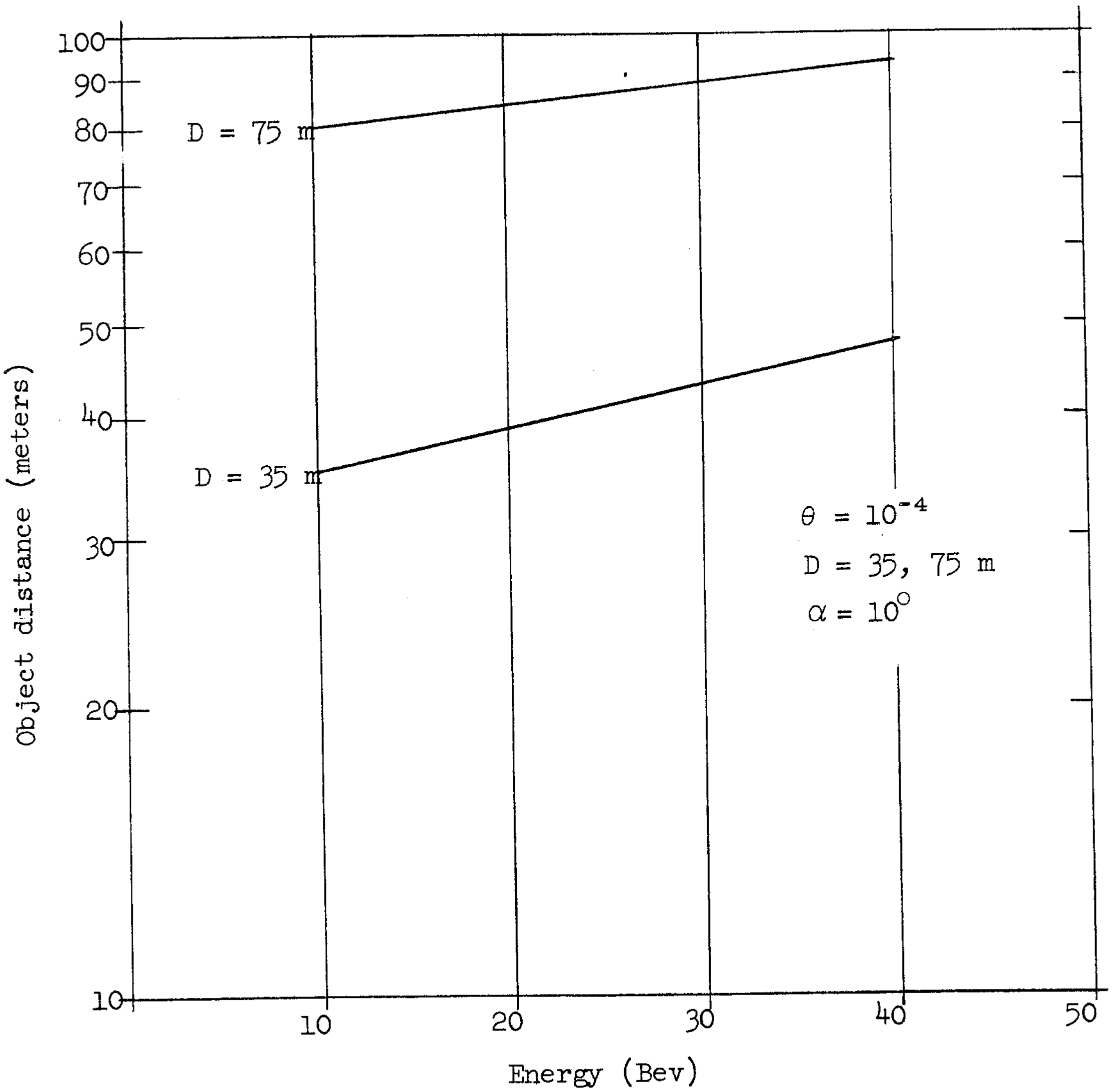

FIG. 11--The object distance p vs the energy.