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PHASING OF A NONSYNCHRONOUS ACCELERATOR SECTION
BY THE BEAM-INDUCTION METHOD

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1. INTRODUCTION

The so-called beam-induction method¹ has been proposed as one of the possible techniques to phase long linear electron accelerators. While this method appeared satisfactory from several points of view, there remained, however, a question as to its accuracy in the case of nonsynchronous operation, i.e., when accelerator temperature or frequency deviations cause the phase-velocity of the wave to differ from the velocity of light c of the electrons. The purpose of this report is to show that, even under conditions of nonsynchronism, this technique is as accurate as the "reactive beam-loading" method^{2,3}. Both methods would be acceptable in that they would yield a phase extremely close to the optimum. By optimum phase is meant one that yields maximum possible electron energy under the conditions of nonsynchronism of the accelerator section under consideration. The beam-induction technique has the advantage that it is more sensitive at low currents than the reactive beam-loading method.

¹All references will be found at the end of the text.

2. PHASING OF A SYNCHRONOUS SECTION

The principles of both beam-induction and reactive beam-loading techniques will be briefly reviewed.

In the beam-induction method, the particular klystron to be phased is first turned off or its pulse is displaced in time with respect to the beam pulse. One then observes the phase of the beam-induced wave at the downstream end of the corresponding accelerator section and compares it to a reference. For the klystron of a synchronous section to be correctly phased, this beam-induced signal must be 180° out of phase with the klystron phase. The klystron is then turned on again and phased until the angle of the resultant output signal (klystron plus beam waves) differs by 180° from the original beam signal.

In the reactive beam-loading method, the beam is first turned off and the phase of the klystron wave at the downstream end of the section is compared to a reference. If the klystron is phased correctly, turning on the beam, and thus adding a signal 180° out of phase, will modify the amplitude but not the angle of the output wave. Thus the klystron phase is adjusted until this condition is realized.

3. VECTOR REPRESENTATION OF THE NONSYNCHRONOUS CASE

It has been shown³ that the reactive beam-loading method can successfully be applied in the nonsynchronous case.

Assuming a difference δ between the electron and rf wave propagation constants throughout a section of length l

$$\beta_e - \beta_w = \delta \quad (1)$$

we recall, for example, that if the wave travels faster than the electron bunch ($\delta > 0$), the particles will gain the maximum possible amount of energy if they start at some angle θ_0 ahead of the crest and slip over and behind it as they travel down the accelerator section. The initial angle $(\theta_0)_{\text{optimum}}$, giving maximum possible energy transfer, was obtained mathematically (see Eq. (48), page 21³) and calculated for a particular example (see Table II(b), page 20³). While no known phasing method yields exactly this initial optimum θ_0 , it was shown that the

reactive beam-loading method nearly meets this requirement. As will be illustrated below in Fig. 1(h), by choosing an initial angle θ_0 [hereafter called $(\theta_0)_{\text{practical}}$] such that θ_ℓ , the corresponding angle at the output ($z = \ell$) remains unchanged when the electron wave is added, the decrease in energy is of the order of 0.5%* with respect to the optimum case (see Table II(c), page 20³).

The whole problem can be understood in terms of simple vector diagrams. A variety of possible cases is illustrated in Fig. 1 for both synchronism (cases a, b, and c) and nonsynchronism (cases d to h). \vec{E}_k , \vec{E}_e and \vec{E}_R are the klystron, electron and resultant waves respectively. Steady-state beam loading is assumed in all cases.

We see from Fig. 1(h) that the addition of the vector \vec{E}_e does not change the phase θ_ℓ of the resultant vector \vec{E}_R at $z = \ell$. Whether the beam is on or off, the angle of the output wave is $\theta_\ell = \theta_0 + \ell\delta$.

It should be noticed that, in the absence of the klystron signal [see Fig. 1(f)], the angle θ_e of the beam vector is always less than $\ell\delta$. This is an important result that can readily be interpreted physically as follows. Under nonsynchronism, the beam-induced wave as it appears at the end of a section ($z = \ell$) is made up of the sum of little wave packets induced all along the section. The wave packets induced near the input ($z = 0$) travel the same distance as the klystron wave under the same condition and are, therefore, shifted by the same angle. However, the wave packets generated further down the section travel a shorter distance and thus are shifted by a smaller angle. The resultant of all these packets is consequently shifted by a lesser angle than $\ell\delta$.

All the above vector additions are applicable because the beam-loading equations under consideration are linear, and superposition applies.

By examining Fig. 1 we now see that the final obtained angle θ_ℓ is the same whether the reactive beam-loading or the beam-induction method is used. Only the order in which the vectors \vec{E}_k , \vec{E}_e and \vec{E}_R are observed differs. In the beam-induction method, the phase of \vec{E}_e

*This particular example was worked out for the following set of parameters: $I\ell = 0.57$ nepers, $\ell = 10$ ft, $\delta = I = 0.187$ rad/m, $ri/E_0 = 0.26$. These conditions of nonsynchronism would be created by a frequency shift $|\Delta f| \sim 0.1$ Mc/sec or a temperature change $|\Delta T| \sim 2^\circ\text{C}$.

is equated to a reference signal in the absence of \vec{E}_k . The klystron is then turned on and its phase is adjusted so that \vec{E}_R is 180° out of phase with the reference and thus with \vec{E}_e ; hence, \vec{E}_k , \vec{E}_e and \vec{E}_R are collinear at the output.

The same result can be obtained mathematically by using the results of Reference 3. From Eq. (41)³, $I\ell$ being the attenuation per section,

$$\left[\tan \theta_e(\ell) \right]_{E_k=0} = \frac{e^{-I\ell} \left[(\delta/I) \cos \ell\delta + \sin \ell\delta \right] - (\delta/I)}{e^{-I\ell} \left[\cos \ell\delta - (\delta/I) \sin \ell\delta \right] - 1} \quad (2)$$

Notice that this beam phase angle is independent of the magnitude of the current.

The expression for $\tan(\theta_0 + \ell\delta)$ is given by Eq. (46)³. When $\theta_0 = (\theta_0)_{\text{practical}}$, given by Eq. (50)³,

$$\tan(\theta_0)_{\text{practical}} = \frac{(\delta/I) + e^{I\ell} \sin \ell\delta - (\delta/I)e^{I\ell} \cos \ell\delta}{1 - (\delta/I)e^{I\ell} \sin \ell\delta - e^{I\ell} \cos \ell\delta} \quad (3)$$

it is easy to show that

$$\tan(\theta_0 + \ell\delta) = \tan \theta_e(\ell) \quad (4)$$

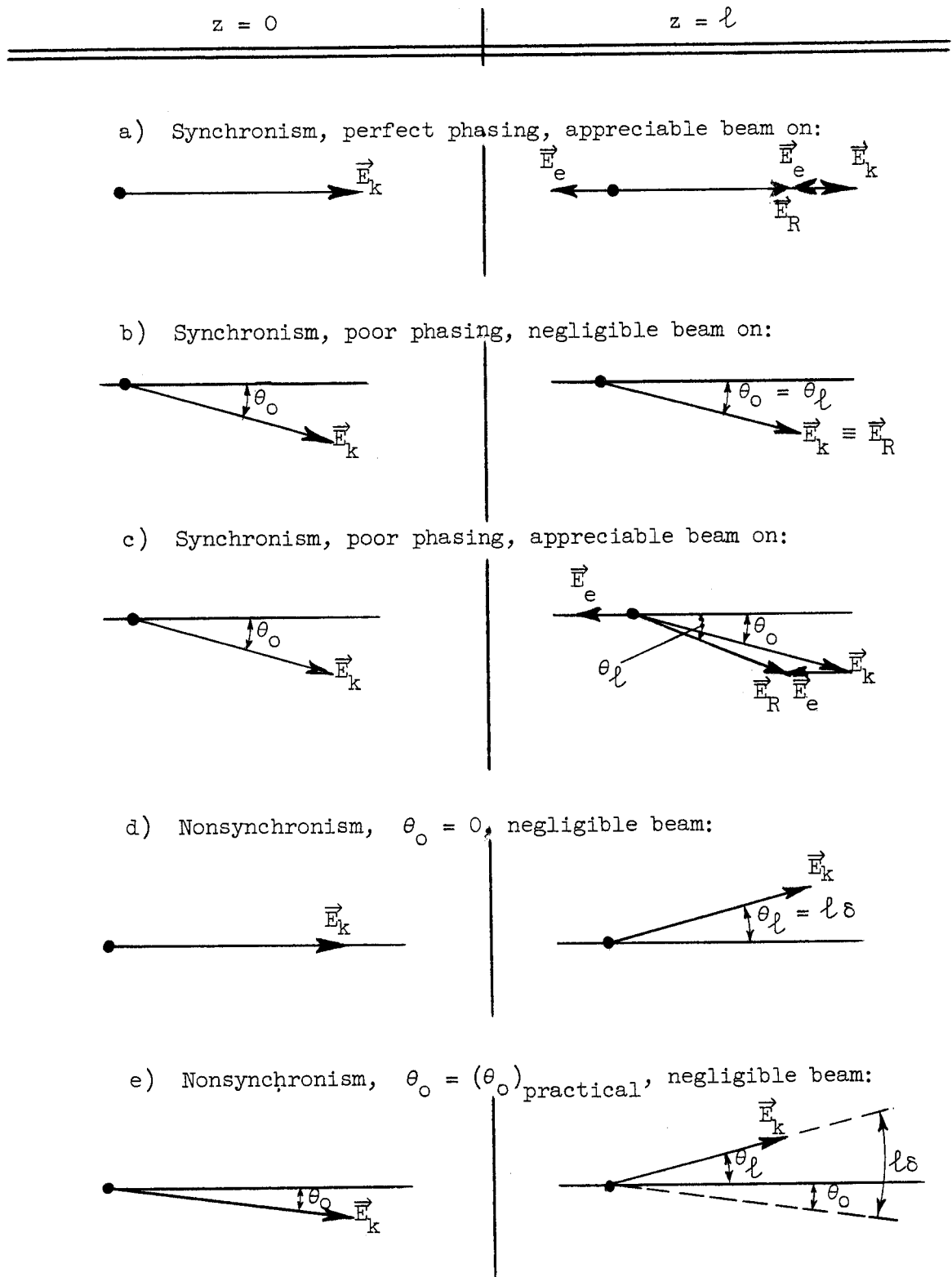
Quod erat demonstrandum.

4. CONCLUSION

The above vector diagrams and mathematical derivations show that applying either the beam-induction or the reactive beam-loading method yields the same initial klystron phase whether the section is synchronous or not. Thus, even under nonsynchronism, both methods give the same energy, which is very close to the maximum attainable energy. As mentioned earlier, at low beam currents the beam-induction method has the added advantage of being more sensitive.

REFERENCES

1. Status Report, M Report No. 260, Project M, Stanford University, Stanford, California, April 1961.
2. D. J. Goerz and R. B. Neal, "Phasing a Linear Accelerator from RF Phase Shift Due to Beam Loading Interaction," M Report No. 103, Project M, Stanford University, Stanford, California, December 1958.
3. G. A. Loew, "Non-Synchronous Beam Loading in Linear Electron Accelerators," M.L. Report No. 740, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, California, August 1960.



(Fig. 1 continued on page 7)

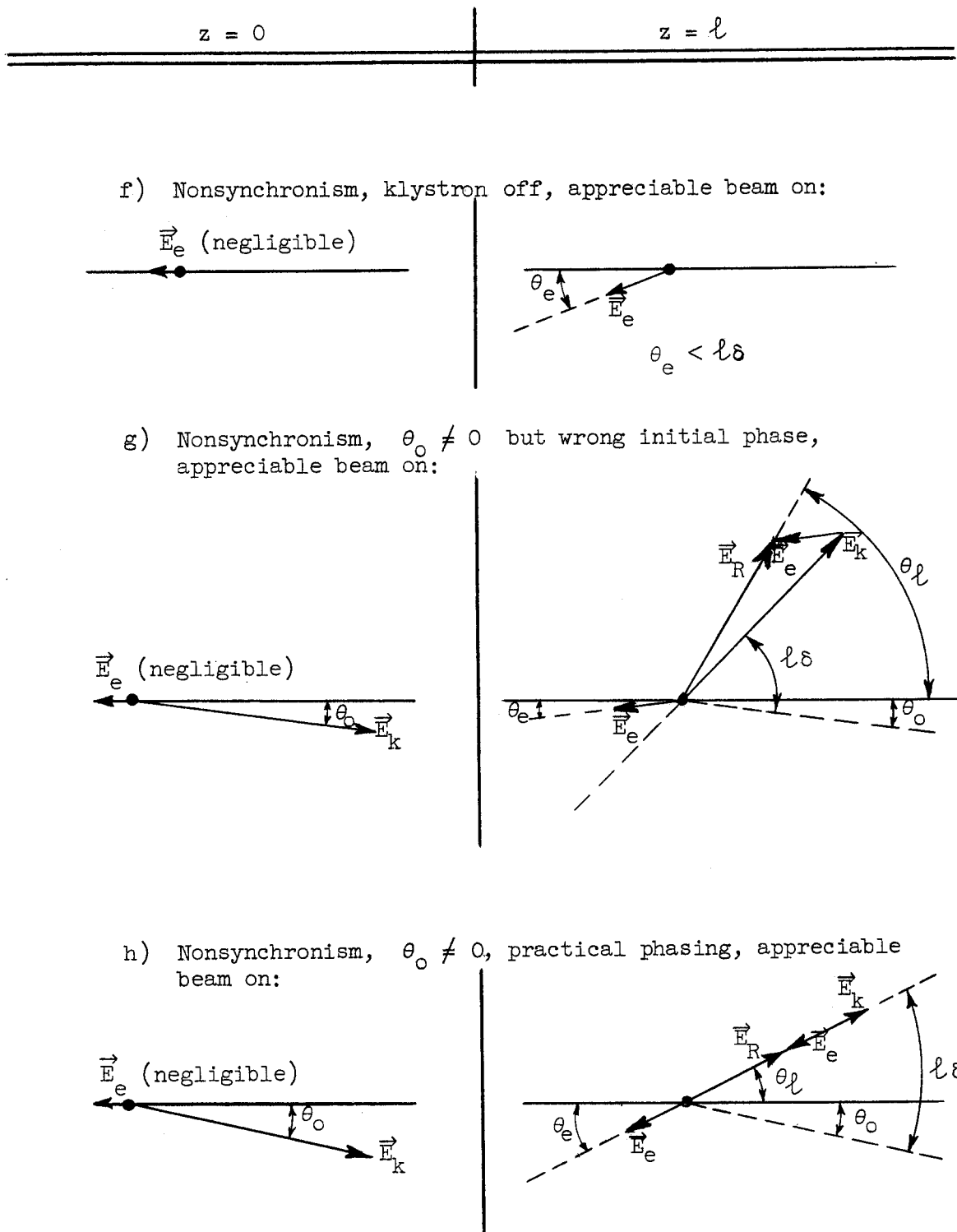


FIG. 1--Vector diagrams showing various cases of synchronous and non-synchronous phasing. (\vec{E}_k , \vec{E}_e and \vec{E}_R are the klystron, electron and resultant electric field vectors respectively.)