Project M
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PHOTON EXPERIMENTS WITH M
(A slightly optimistic survey)

By

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We consider some of the obvious photon experiments possible with M to see what sort of problems can be expected when these experiments are extended to the multi-Bev region. We limit ourselves to reactions in which the final state contains two particles. Final states containing three or more essentially uncorrelated particles offer particular problems to M because of its short duty cycle, and we do not consider them here.

π-Meson Photoproduction from Hydrogen

$$\gamma + p \rightarrow \pi^0 + p \tag{1}$$

$$\gamma + p \rightarrow \pi^+ + n \tag{2}$$

Both of these reactions can be extended to high energy with little difficulty.[*] For reaction (1), we can use the well-tested technique of looking for coincidences between the proton and one γ-ray from the π^0 decay. This technique becomes easier as the energy increases. The γ-ray, detected in a total-absorption Čerenkov counter, becomes easier to detect, and its energy determination becomes more accurate as the π^0 energy increases. For instance, for a 5-Bev π^0, a Cerenkov counter with an aperture of $\pm 1.60^\circ$ captures both of the decay γ-rays. One can expect the counter to have an energy resolution of about 5% full-width or better. Since the

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[*]Dogmatic statements such as this arise from a desire for brevity. Caveat Lector.

γ-ray pulses will be among the largest, the background rates in the Cerenkov counter should be small. It's probably necessary, certainly desirable, to magnetically analyze the proton. With an analyzer of 1% momentum resolution and an angular aperture of 1°, the γ-ray energy can be determined to a few percent without using any information obtained from the Cerenkov counter. The protons can be distinguished from lighter particles with a gas Cerenkov counter up to energies of 10 to 20 Bev, although because of the coincidences it may not be necessary to identify the proton with great certainty. One obvious observation: detecting the recoil proton alone makes the separation of single pion production from double pion production very difficult and is probably unfeasible at energies greater than about 5 Bev.

Reaction (2) [$\gamma + p \rightarrow \pi^{+} + n$] is a bit more difficult, but not much. Again, to be sure that we are dealing with single pion production, it will be necessary to observe the pion and neutron in coincidence. The neutron can be detected with an efficiency of 10% without much trouble, which should be comfortably adequate. There is perhaps some question as to the single rate in the neutron counter, which might be a scintillation counter in which the neutron makes a recoil proton. We assume the neutron counter can be adequately shielded from stray radiation, and the charged particles from the target can be swept out with a rather modest magnet. The arrangement might look something like Fig. 1.*

If the cross sections remain as high as 10^{-31} cm^2/sr (at 1 Bev they are about 30 times that) one can have counting rates of about 1 count/sec with very modest beam ($\approx 5 \times 10^{11}$ Q/sec). I believe we can conclude with confidence that extension of reactions (1) and (2) to the multi-Bev region will present no difficult problems.

Strange-Particle Photoproduction

$$\gamma + p \rightarrow K^{+} + \Lambda^{0} \qquad (3)$$

$$\gamma + p \rightarrow K^{+} + \Sigma^{0} \qquad (4)$$

*Some method will have to be devised for calibrating the neutron counter.

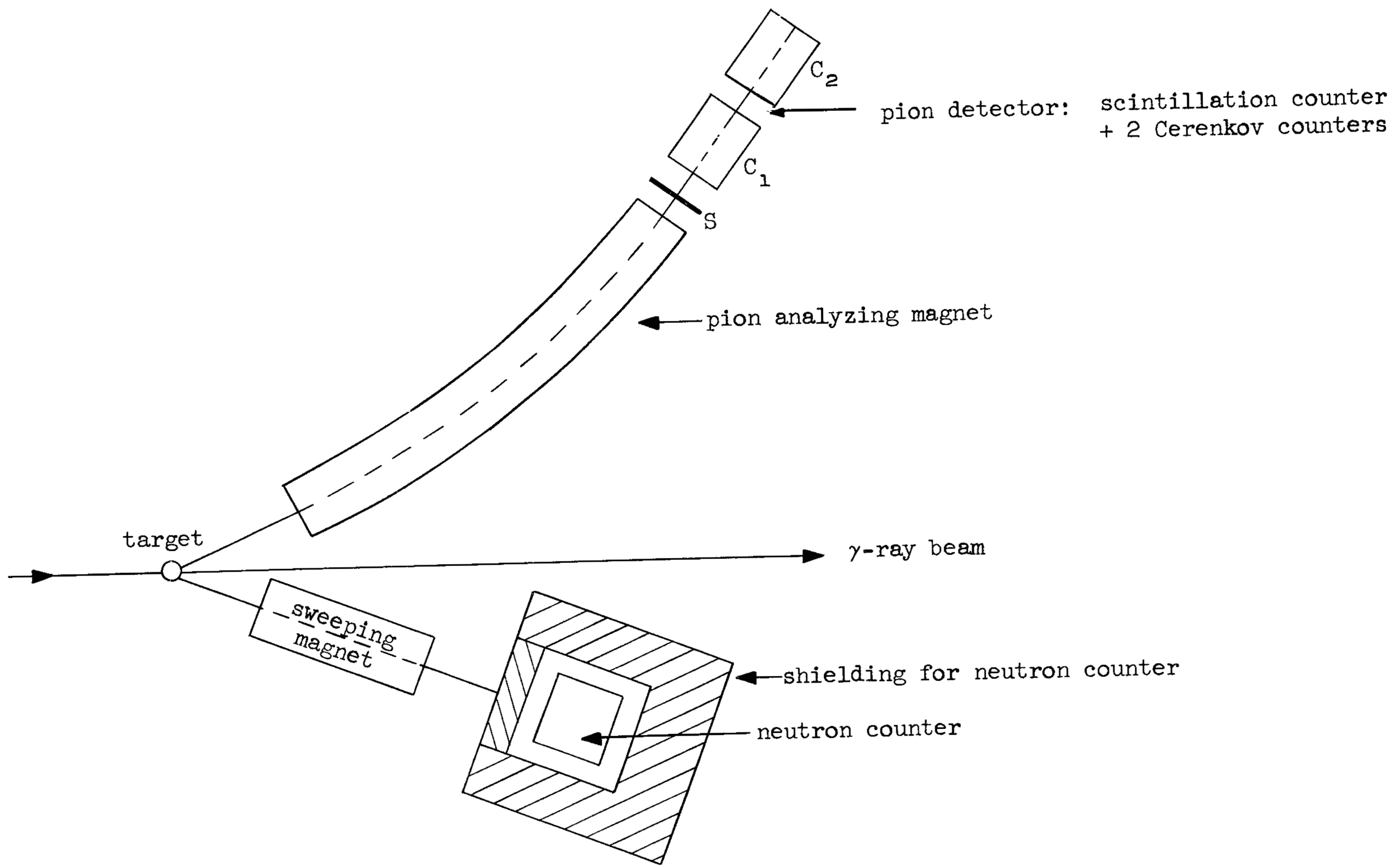

FIG. 1--Possible experimental arrangement for detection of $\gamma + p \rightarrow n + \pi^+$.

There are other strange-particle reactions, but these two seem to be the simplest. We first consider the feasibility of extending to higher energies techniques that have proven successful at about 1 Bev.* The method consists of measuring the yield of K-mesons of a given momentum at a fixed laboratory angle as the peak energy of the bremsstrahlung beam is varied. The K-mesons so measured may arise from either reactions (3) or (4) or from reactions in which additional particles are produced, e.g., $\gamma + p \rightarrow K^+ + \Lambda^0 + \pi^0$. By carefully controlling k_{max} (the peak energy of the photon beam) and with sufficiently accurate determination of the K-meson momentum, we can distinguish between the K's from the various processes. Schematically, we might expect the yield curves to look like Fig. 2.

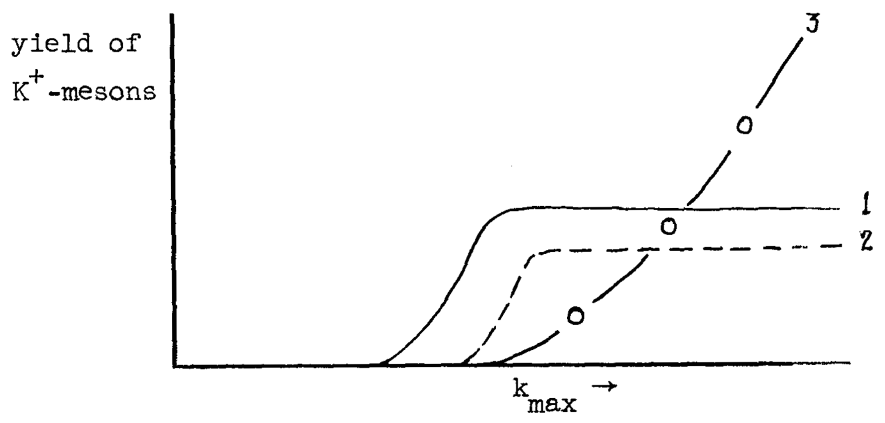

FIG. 2--Yield of K^+-mesons as function of k_{max} for fixed lab angle and momentum of the K-meson. Curve 1 is the yield from $\gamma + p \rightarrow K^+ + \Lambda^0$. Curve 2 is the yield from $\gamma + p + K^+ + \Sigma^0$. Curve 3 is the yield from $\gamma + p \rightarrow K^+ + \Lambda^0 + (\pi's)$.

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*B. D. McDaniel, A. Silverman, R. R. Wilson, and G. Cortellessa, "Photoproduction of K-mesons," Phys. Rev. Lett. 1, 109 (1958).

The resultant total yield from the three processes in Fig. 2 is then something like Fig. 3.

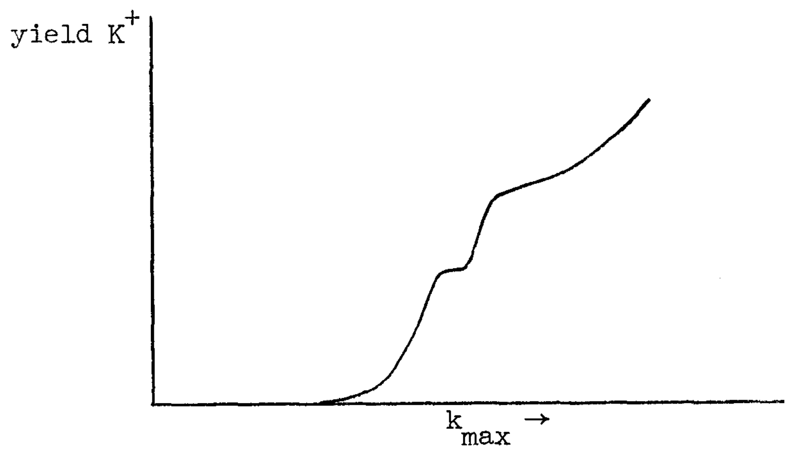

FIG. 3--Resultant yield curve of K^+-mesons as a function of k_{max}.

The yields from reactions (3) and (4) can be separately extracted from such a yield curve.

We imagine the K's to be magnetically analyzed. What must be the spectrometer resolution to make this separation possible? We can determine this from the kinematics. We define the following quantities:

k = γ-ray energy.

$\vec{P}_{K\Lambda}$ = momentum of K-meson produced at lab angle θ by k in reaction (3).

$\vec{P}_{K\Sigma}$ = momentum of K-meson at same lab angle produced by k in reaction (4).

k' = γ-ray energy necessary to produce a K-meson of momentum $P_{K\Lambda}$ at angle θ in reaction (3).

We also define

$$\Delta P_K = P_{K\Lambda} - P_{K\Sigma}$$

$$\Delta k = k' - k$$

Figures 4 through 7 show some of the relevant kinematics. Figure 4 plots K-momentum vs $\theta_{c.m.}$ and θ_L vs $\theta_{c.m.}$ for various k's. Figure 5 plots $\Delta P_K/P_K$ vs $\theta_{c.m.}$ and tells what the momentum resolution of the analyzer must be to separate reactions (3) and (4). Figure 6 plots $\Delta k/k$ vs $\theta_{c.m.}$ and shows how carefully the peak energy must be controlled to separate reactions (3) and (4). Figure 7 plots $(1/k)\,(dk/d\theta)$ vs θ_L and tells us the maximum angular aperture allowable for the spectrometer if we wish to separate reactions (3) and (4).

From these graphs we can see that at 3 Bev we require a resolution in energy of $\approx 3\%$, a momentum resolution of $\approx 3\%$, and an angular resolution $\approx 1.5^\circ$, in order to separate the K-meson yields from the two processes. Table I shows the $\Delta P/P$, $\Delta k/k$, and $\Delta\theta$ required to separate the two processes at various energies.

k(Bev)	$\Delta P_K/P_K$	$\Delta k/k$	$\Delta\theta$
3	3.5%	3.5%	1.3°
5	1.9%	1.9%	0.65°
10	0.9%	0.9%	0.25°

TABLE I--Required resolution in momentum, γ-ray energy, and angle to separate K-meson yields from reactions (3) and (4) at all center-of-mass angles.

Thus a spectrometer with $\Delta P/P = 10^{-2}$ and $\Delta\theta = 1^\circ$ will work reasonably well up to 5 Bev (although somewhat marginally at 5 Bev). At 5 Bev the definition in energy required, $\Delta k/k \approx 2\%$, is not a severe limitation since the thin-target bremsstrahlung spectrum is reasonably well up on the plateau for $k/k_{max} = 0.98$ ($k_{max} = 5$ Bev).

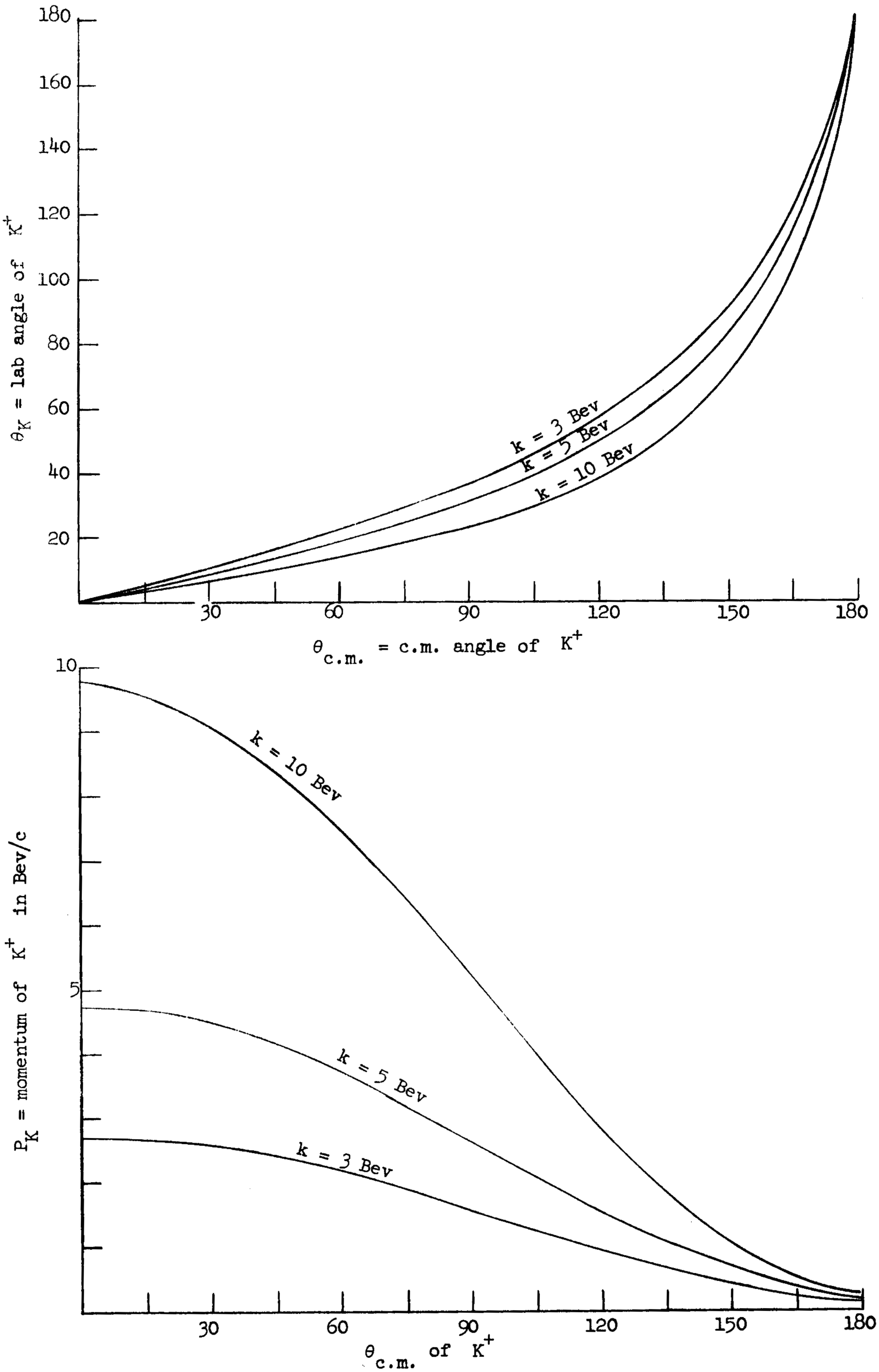

FIG. 4--Kinematics for reaction $\gamma + p \rightarrow K^+ + \Lambda^o$.

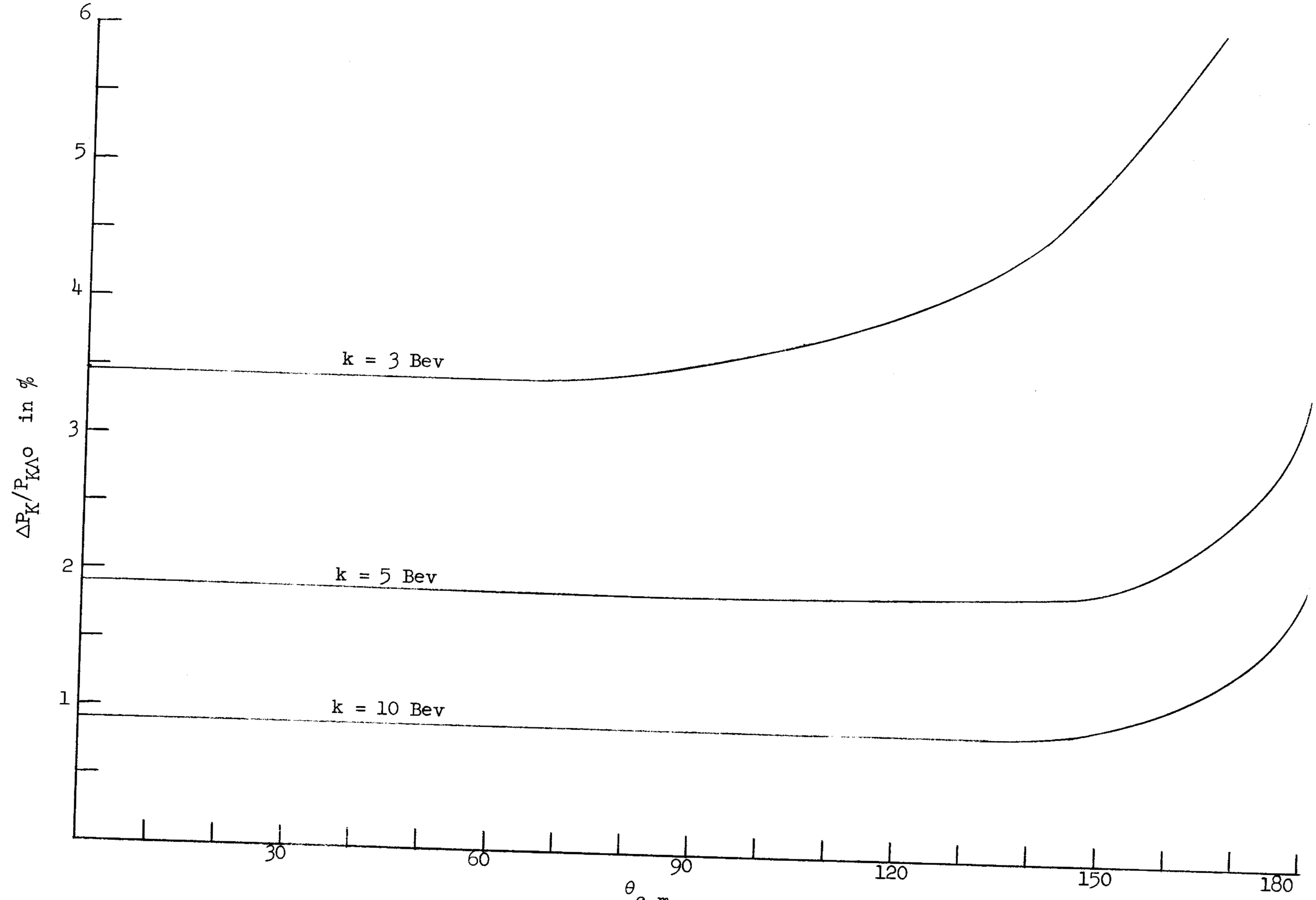

FIG. 5--$\Delta P_K = P_{K\Lambda^o} - P_{K\Sigma^o}$ (see text for definition of symbols).

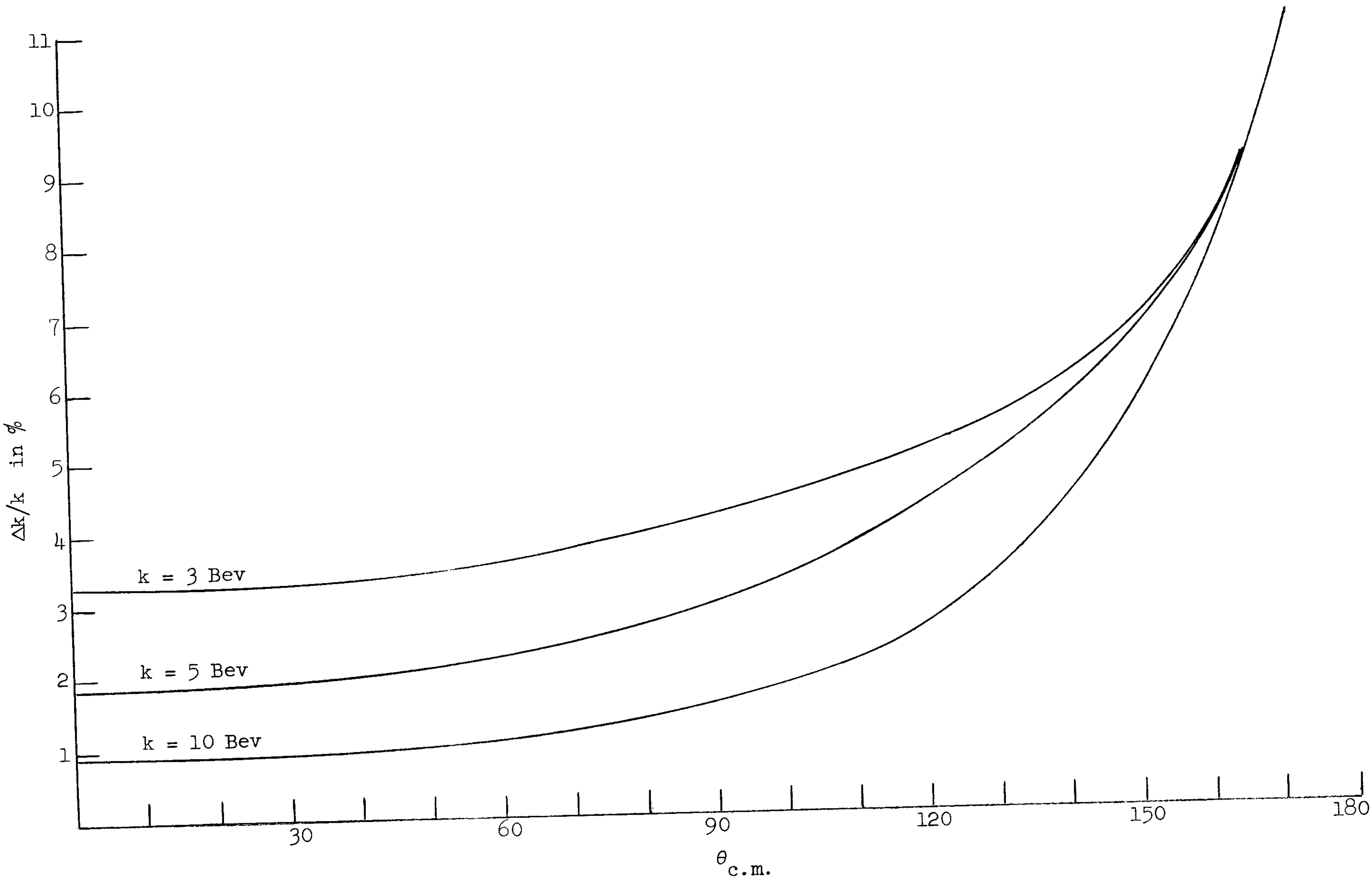

FIG. 6-- $\frac{\Delta k}{k} = \frac{k' - k}{k}$ (see text for definition of symbols).

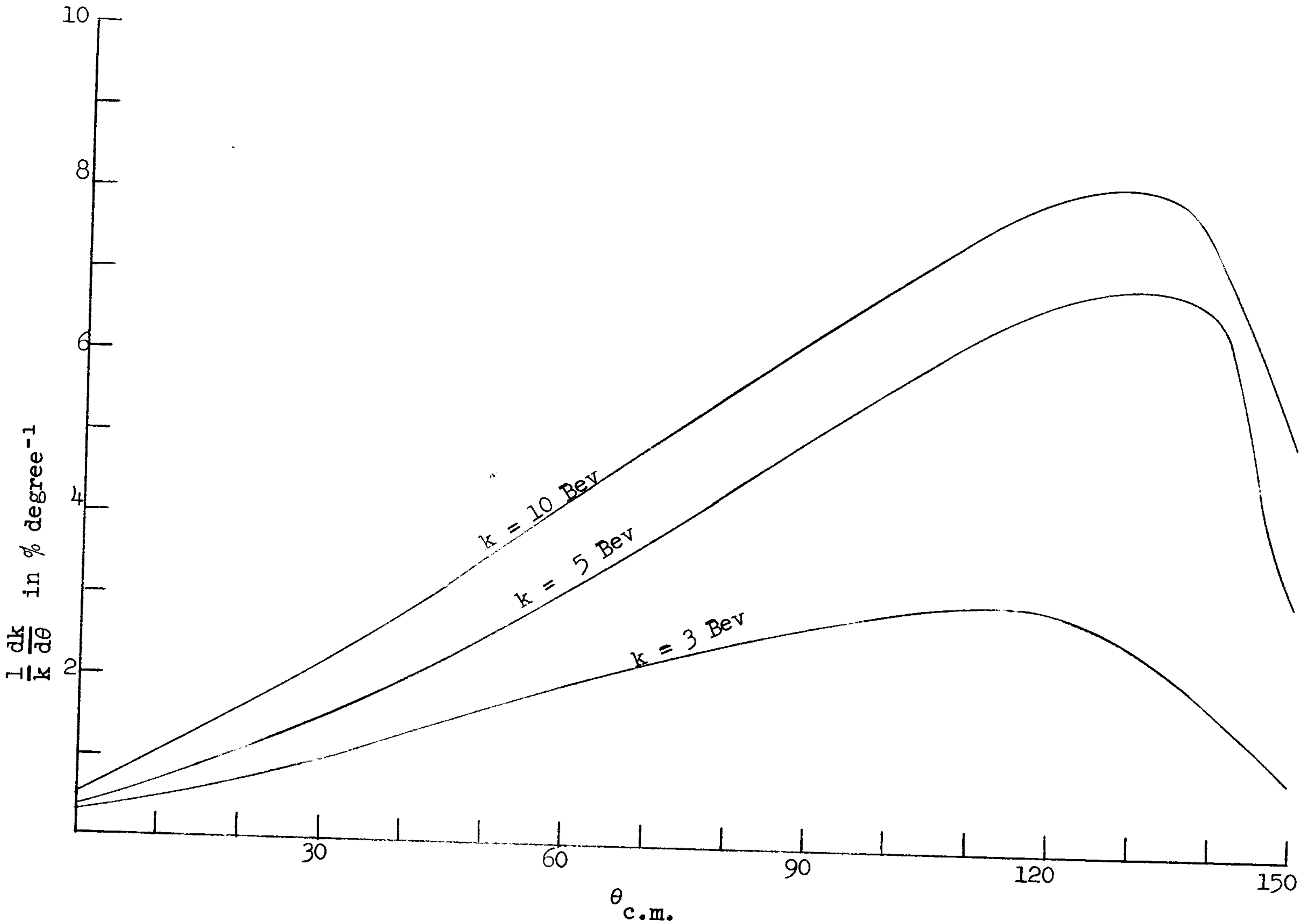

FIG. 7--$\gamma + p \rightarrow K^+ + \Lambda^o$ Rate of change of k with θ.
k = γ-ray energy θ = lab angle of K^+

At 10 Bev the spectrometer characteristics, particularly the angular resolution required, become more stringent. In addition, one has to work on the rapidly varying part of the photon distribution. It appears that up to approximately 5 Bev this technique is feasible but that it deteriorates at higher energies. Before we look into the question of possible techniques at higher energies, we discuss the detection apparatus at $k \leq 5$ Bev.

For low momenta K's produced at large c.m. angles, the detection of the K's offers no particular problem. Such K's have been detected by various techniques, and I will not discuss them. For K-momenta in the region of 5 Bev/c it appears that threshold Cerenkov counters can be used. Consider a detection system consisting of a scintillation counter S (which may not be necessary), a gas Cerenkov counter C_1, which detects K's but not protons, and another gas Cerenkov counter C_2, which detects π's but not K's (see Fig. 8). A K-meson is then identified by a coincidence between S and C_1, in anticoincidence with C_2.

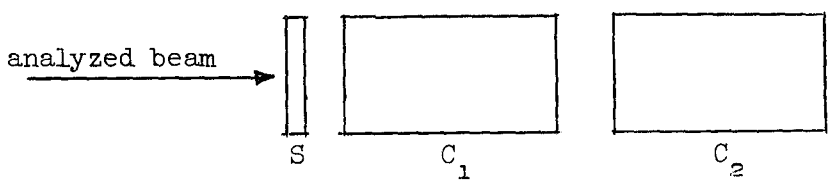

FIG. 8

Consider the characteristics of C_1 and C_2 for $P = 5$ Bev/c:

$\beta_p = 0.982$ = velocity of a proton at 5 Bev/c

$\beta_K = 0.995$ = velocity of a K-meson at 5 Bev/c

$\beta_\pi = 1.000$ = velocity of a π-meson at 5 Bev/c

Let N_1 = index of refraction of C_1 and N_2 = index of refraction of C_2. Then we require that

$$N_1 \beta_p = 1 \qquad\qquad N_1 = 1.018$$

$$N_2 \beta_K = 1 \qquad\qquad N_2 = 1.005$$

Following Mozley,* we write

$$\theta^2 = \frac{2}{\beta}(\beta + \eta - 1)$$

where $\eta = N - 1$. The number of photons per unit path length in the wave-length interval from 3500 to 5000 A^o is

$$\frac{dk}{d\ell} \approx 390 \sin^2 \theta \approx 390\ \theta^2 = 390\ \frac{2}{\beta}(\beta + \eta - 1)$$

For C_1,

$$\frac{dk}{d\ell} \approx (390)\ (2)\ (.995 + .018 - 1) \approx 10 \text{ photons/cm}$$

For C_2

$$\frac{dk}{d\ell} \approx (390)\ (2)\ (.005) \approx 3.9 \text{ photons/cm}$$

We make ℓ_1 = 100 cm. Assuming a light collection efficiency of $\frac{1}{2}$ and a photoefficiency of 10%, we get

N_1 = 50 photoelectrons produced by a K-meson in C_1

N_2 = 18 photoelectrons produced by a π-meson in C_2.

These numbers seem perfectly adequate. We can make an estimate of the efficiency required of C_2 by guessing at the number of π's per K produced by the γ-ray beam. There is, of course, no data to use. However, at 1 Bev the ratio is known to be $N_\pi/N_K \approx 10^3$.** At the higher energies of the CERN and Brookhaven machines (with protons incident on nuclei) the ratio $N_\pi/N_K \approx 10$.† We guess at a ratio $N_\pi/N_K \approx 100$. Thus the efficiency

*R. F. Mozley, Project M Source Book (Internal Document), Sec. V, "Research Area Design," September 1960.

**McDaniel, op. cit.

†W. F. Baker, et al., "Particle Production by 10-30 Bev Protons Incident on Al and Be," Phys. Rev. Lett. 1, 101 (1961).

of C_2 for rejecting π's should be $\approx 99.9\%$, although one could probably tolerate an efficiency of 99%. This does not seem a difficult problem.

The detection system as outlined is very primitive. There are a number of ways in which it may be improved. For instance, π-meson pulse heights in C_1 will be approximately 1.8 times the pulse height of the K's. This fact can be used to help eliminate π's. The Cerenkov angle for 5 Bev/c K's is $\theta_K = 9.22^\circ$, and for π's, $\theta_\pi = 10.87^\circ$. With an angular spread of 1° in the analyzed beam, this difference can be useful.

We conclude that reactions (3) and (4) can be measured up to k = 5 Bev by presently available techniques. The magnetic analyzing system should probably consist of two analyzers, one for P_K up to about 1 Bev/c, corresponding to large c.m. angles; and another for $P_K > 1$ Bev/c. We estimate the counting rates for a beam of 5×10^{11} Q/sec (about 1% of maximum beam obtainable). Assume a cross section $d\sigma/d\Omega = 10^{-31}$ cm^2/sr, which is approximately equal to that at 1 Bev. Then

$$N = N_T Q \frac{P_K}{k} \frac{dk}{dP_K} \frac{dP_K}{P_K} d\Omega_L \left(\frac{d\sigma}{d\Omega}\right)_{c.m.} f$$

where N = counts/sec, N_T = number of target atoms/cm^2, k = photon energy, and f = fraction of K's which do not decay in 10 m flight path.

We now select the following parameters:

$$N_T = 6 \times 10^{23}/\text{cm}^2 \text{ (6 inch long } H_2 \text{ target)}$$

$$Q = 5 \times 10^{11} \text{ equivalent quanta/sec}$$

$$\Delta P_K/P_K = 10^{-2} \qquad k = 5 \text{ Bev}$$

$$d\Omega_L = 2 \times 10^{-3} \qquad \theta_{c.m.} = 45^\circ$$

With these parameters, $N = (6 \times 10^{23})(5 \times 10^{11})(10.1)(10^{-2})(2 \times 10^{-3})$ $(10^{-31})(.7)$

$$N \approx 4.2 \text{ counts/sec}$$

The same cross section gives a rate of about 0.5 counts/sec at $\theta_{c.m.} = 135^\circ$ (again $\Delta P_K/P_K = 10^{-2}$ and $d\Omega_L = 2 \times 10^{-3}$ sr). Thus, even with these very modest assumptions about the beam strength and solid angle of the spectrometer, the counting rates one can expect are very good. Two orders of magnitude decrease in these rates would not be prohibitive.

Photon Energies Above 5 Bev

As noted above, the spectrometer requirements and the necessity for working within 1% of the upper end of the photon spectrum make the previous technique undesirable at energies substantially above 5 Bev.* If we can detect the Λ^o in coincidence and distinguish those produced directly from those arising from the Σ^o decay ($\Sigma^o \rightarrow \Lambda^o + \gamma$), then these requirements could be greatly relaxed. We shall see below that it seems feasible to detect the Λ^o's with an efficiency of $\approx 10\%$ and to make the necessary identification.

Let D = mean distance a Λ^o goes before decay, τ = proper lifetime of Λ^o, P_{Λ^o} = its momentum, and M_{Λ^o} = its mass. Then

$$D = \frac{\tau \ P_{\Lambda^o}}{M_{\Lambda^o}}$$

Thus for $P_{\Lambda^o} = 5$ Bev/c, D is 38 cm. This is a convenient distance for observing the Λ^o in a spark chamber.

For definiteness, we imagine a measurement at $k = 10$ Bev and $\theta_{c.m.} = 90^o$. The K^+-meson spectrometer characteristics are taken to be $\Delta P/P = 10^{-2}$ and $\Delta\theta = 1^o$. Then

$$\theta_{\Lambda^o} = 21.3^o \qquad P_{\Lambda^o} = 5.64 \text{ Bev/c}$$

$$\theta_{\Sigma^o} = 20.9^o \qquad P_{\Sigma^o} = 5.67 \text{ Bev/c}$$

If θ_{K^+} is known to 1^o, then θ_{Λ^o} and θ_{Σ^o} are known to approximately the same accuracy (perhaps a factor of two better). We place a 50-cm-deep spark chamber at 1 meter from the target. The experimental arrangement is shown in Fig. 9. The probability of the Λ^o decaying in the active volume of the spark chamber is $\approx 10\%$. The Σ^o decays essentially immediately in the target. The angles are such that the Λ^o observed in the spark chamber

*It is perhaps worth noting that the difficulty is not in detecting the K's. The counter system described in the previous section would be adequate for this purpose. Thus, the sum of reactions (3) and (4) could probably be measured. It is hard to predict how useful this would be.

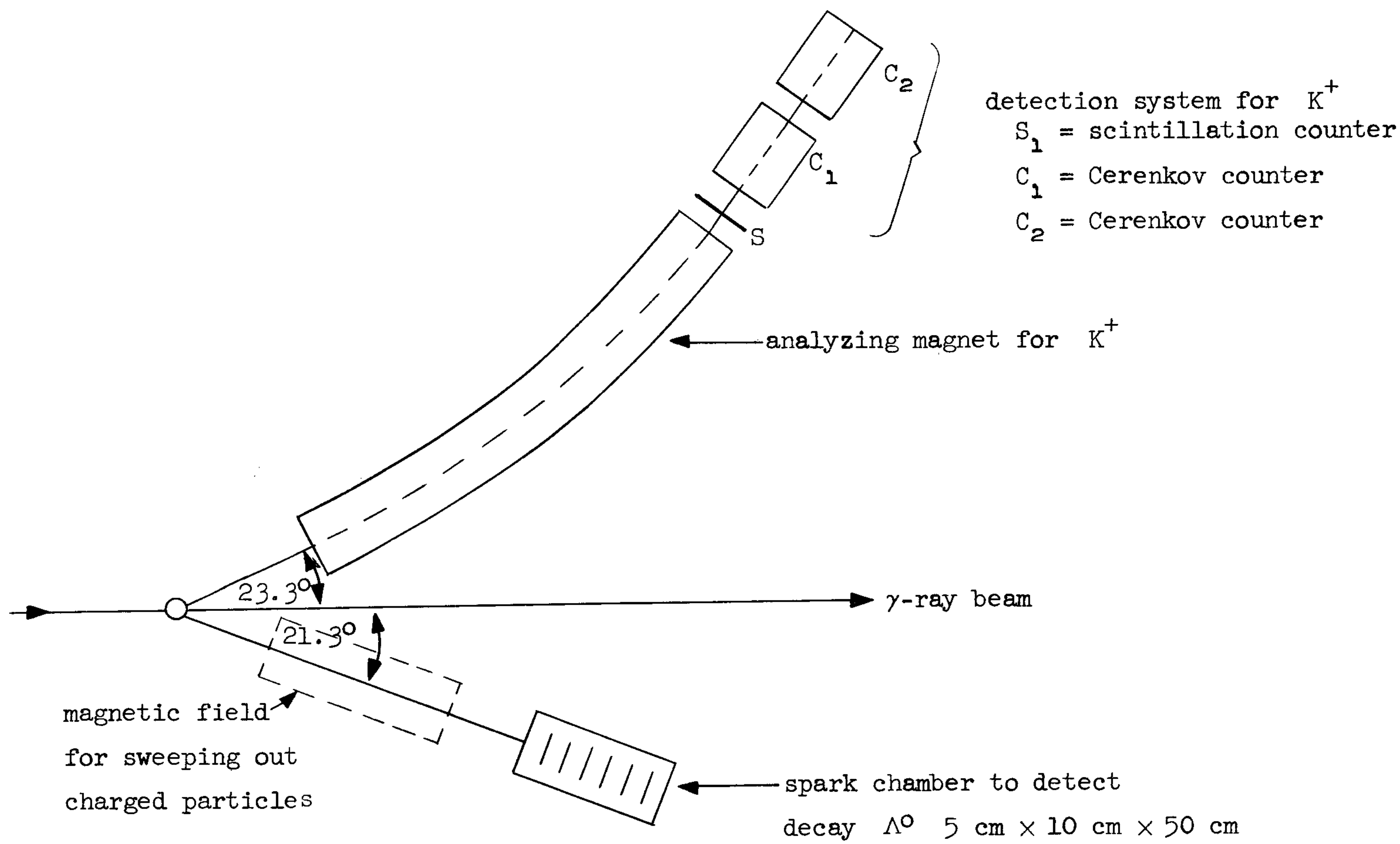

FIG. 9--Possible arrangement for detecting the $\Lambda^o + K^+$ in coincidence for the reactions $\gamma + p \rightarrow K^+ + \Lambda^o$; $\gamma + p \rightarrow K^+ + \Sigma^o$. Photon energy = 10 Bev. Angles correspond to $\theta_{c.m.} = 90^o$.

could have come from either reaction (3) or reaction (4). However, the momentum distribution of the decay protons is different in the two cases, and this fact can be used to distinguish them, as follows.

Assume monoenergetic Λ^o's and Σ^o's, and let β_{Λ^o} be the velocity of the Λ^o. The laboratory energy of the decay proton is then given by

$$E_p = \gamma_{\Lambda^o} (\overline{E}_p + \beta_{\Lambda^o} \overline{P}_p \cos \overline{\theta})$$

where $\overline{E}_p$ = energy of proton in rest frame of the Λ^o, $\overline{P}_p$ = momentum of proton in rest frame of the Λ^o, $\overline{\theta}$ = angle of decay proton in rest frame of the Λ^o, and $\gamma_{\Lambda^o} = 1/(1 - \beta^2_{\Lambda^o})^{\frac{1}{2}}$. Thus for a fixed $\overline{\theta}$, corresponding to a fixed lab angle between the π^+ and the proton, E_p is unique.

We now consider the Σ^o decay:

$$\Sigma^o \rightarrow \Lambda^o + \gamma$$

We define E_{Σ^o} = lab energy of the Σ^o

β_{Σ^o} = velocity of Σ^o

k = lab energy of the decay γ

E_{Λ^o} = lab energy of the decay Λ^o

$\overline{k}$ = energy of decay γ in rest frame of Σ^o

$\overline{E}_{\Lambda^o}$ = energy of decay Λ^o in rest frame of Σ^o

$\overline{\theta}$ = angle of decay γ in rest frame of Σ^o

The energy distribution of the decay γ's is uniform between the limits

$$\gamma_{\Sigma^o} \overline{k} (1 + \beta_{\Sigma^o}) > k > \gamma_{\Sigma^o} \overline{k} (1 - \beta_{\Sigma^o})$$

The energy distribution of the Λ^o is uniform in the interval

$$E_{\Sigma^o} - \gamma_{\Sigma^o} \overline{k} (1 + \beta_{\Sigma^o}) < E_{\Lambda^o} < E_{\Sigma^o} \overline{\theta} \gamma_{\Sigma^o} \overline{k} (1 - \beta_{\Sigma^o})$$

Thus

$$2 \frac{(E_o)_{max} - (E_o)_{min}}{(E_o)_{max} + (E_o)_{min}} = \frac{\Delta E_{\Lambda^o}}{\langle E_{\Lambda^o} \rangle_{av}} = \frac{2 \overline{k} \beta_{\Sigma^o}}{\langle E_{\Lambda^o} \rangle_{av}} \gamma_{\Sigma^o} \approx \frac{2 \overline{k} \beta_{\Sigma^o}}{M_{\Lambda^o}}$$

as $\beta_{\Sigma^0} \rightarrow 1$. Thus

$$\frac{\Delta E_p}{E_p} = \frac{\Delta E_{\Lambda^0}}{E_{\Lambda^0}}$$

for fixed $\overline{\theta}$ and $\beta_{\Lambda^0} \approx 1$.

Therefore, starting with a monenergetic, relativistic Σ^0, the protons resulting from the two decays in cascade $\Sigma^0 \rightarrow \Lambda^0 + \gamma$, $\Lambda^0 \rightarrow p + \pi^-$ will have a uniform energy distribution spread over about 13% for a fixed opening angle between the proton and the pion, whereas the proton resulting from the decay of a monoenergetic Λ^0 will have a unique energy. This statement is reasonably accurate for all Σ^0 with momenta greater than about 1 Bev/c. The energy distribution of the decay protons thus offers a means of distinguishing the Λ^0's directly produced from those arising from Σ^0 decay. Figure 10 shows the calculated distribution of the protons for the conditions described earlier ($k = 10$ Bev, $\Delta P_K/P_K = 10^{-2}$, $\Delta\theta = 1^\circ$, and $\theta_{c.m.} = 90^\circ$). This technique, in principle, will go to arbitrarily high energies.

What beam intensity can we tolerate with the arrangement shown in Fig. 9 before saturating the spark chamber? We assume that the spark chamber cannot resolve tracks within a single beam pulse. We make the following optimistic assumptions: (a) all particles observed in the spark chamber originate in the target, and (b) the magnet between target and chamber gets rid of all charged particles. The chamber background will then consist primarily of γ-rays and neutrons coming from the target, the γ-rays from the decay of π^0's, and the neutrons from charged meson production.

The number of γ-rays, N_γ, which enter the spark chamber per pulse is approximately

$$N_\gamma = 2N_T\, Q \left(\int \frac{d\sigma}{d\Omega}(k) \frac{dk}{k} \right) \frac{d\Omega_{c.m.}}{d\Omega_L}(k)\, d\Omega_L$$

where N_T = number of target atoms, $(d\sigma/d\Omega)(k)$ = differential cross section for production of a π^0 at energy k, $d\Omega_L$ = solid angle subtended by chamber, and Q = equivalent quanta through the target per pulse. We take $N_T = 6 \times 10^{23}$ atom/cm^2.

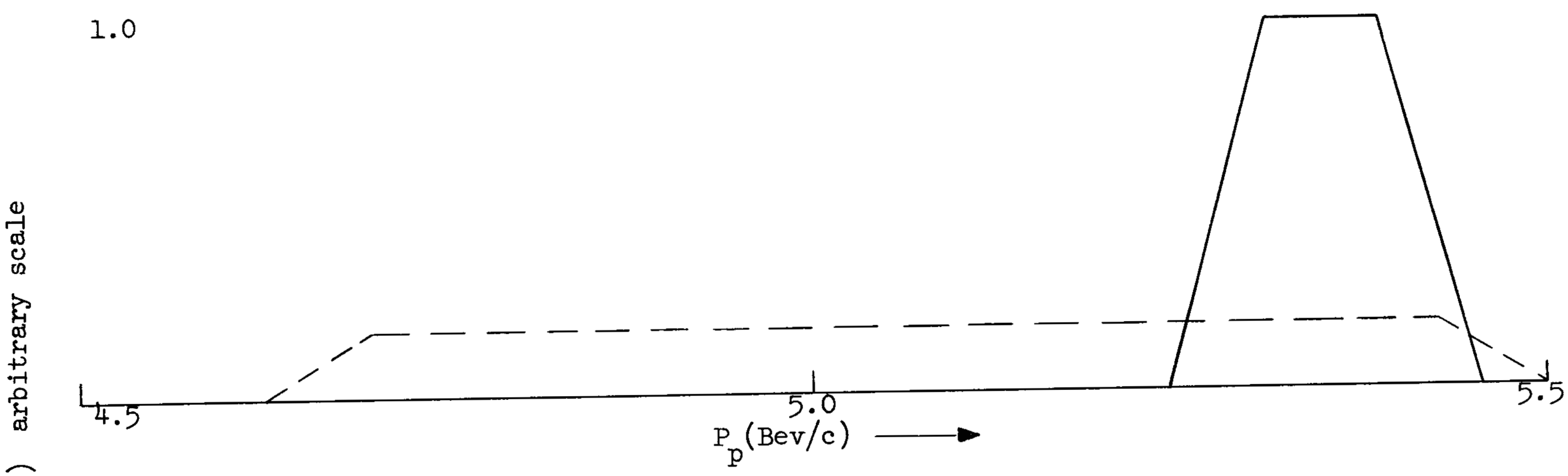

FIG. 10--Distribution in momentum of decay protons from $\Lambda^o \rightarrow p + \pi^-$ for case considered in text ($k = 10$ Bev, $\theta_{c.m.} = 90^o$, $\Delta P_K/P_K = 0.01$, $\Delta\theta = 1^o$)

Solid curve from $\gamma + p \rightarrow K^+ + \Lambda^o$

Dashed curve from $\gamma + p \rightarrow K^+ + \Sigma^o$

Curves normalized to same area

This case calculated for proton decay angle = 0^o. For any other decay angle, distribution similar but shifted to lower momenta.

Then

$$\frac{d\sigma}{d\Omega} = 10^{-29} \text{ for } 0.2 \leq k \leq 1.0 \text{ Bev}$$

or

$$\frac{d\sigma}{d\Omega} = 3 \times 10^{-30} \text{ for } 1.0 \leq k \leq 10 \text{ Bev}$$

Let the chamber have a cross section of 10 cm × 5 cm, and take $d\Omega_{c.m.}/d\Omega_L\ (k) = 5$ for all energies. Then

$$N_\gamma = (2)\ (6 \times 10^{23})\ Q\ (5)\ \frac{5 \times 10^{-3}}{1.56} \left[10^{-29}\ \ln 5 + 3 \times 10^{30}\ \ln 10 \right]$$

or

$$N_\gamma \approx 3 \times 10^{7}\ Q$$

and a similar number of neutrons.

We assume the chamber material has a radiation thickness of 1/50 radiation length. The number of electron pairs produced in the chamber is

$$N_{pairs} = 6 \times 10^{9}\ Q$$

Assuming 10 tracks/pulse is tolerable, we can stand approximately $Q = 10^9$ equivalent quanta/pulse. This beam intensity would yield counting rates of $K - \Lambda^0$ coincidences of 2 to 20/min for the experiment described above for differential cross sections of 10^{-31} cm^2/sr, the larger rate being for small c.m. angles and the smaller rate for the larger c.m. angles. There seems to be nothing in these estimates that establishes the unfeasibility of this technique.

Conclusion

For those inclined to optimism, it appears feasible to investigate reactions (3) and (4) by detecting the K^+ and Λ^0 in coincidence up to energies of 10 Bev and probably considerably higher. It should be noted that being forced to detect the Λ^0 is a disguised blessing since it provides a measurement of the Λ^0 polarization, which is a very useful bit of information.