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WIDE-ANGLE PAIR PRODUCTION AT HIGH MOMENTUM TRANSFERS

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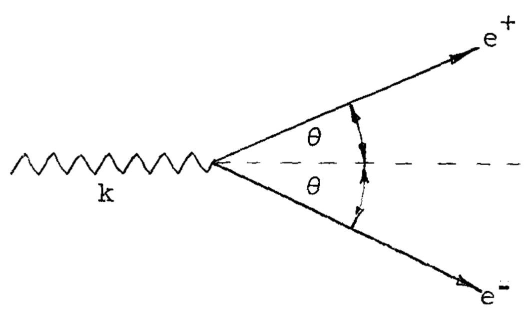
Several people have pointed out that wide-angle pair production on hydrogen probes the small-distance behavior of quantum electrodynamics.[1] Richter[2] has done such an experiment looking for single electrons produced at wide angles and has shown that to distances of the order of 0.9 fermi, quantum electrodynamics remains valid. It is the purpose of this note to investigate the possibility of extending this kind of measurement to large momentum transfers.

We consider an experiment in which the electron and positron, detected in coincidence, are both produced at an angle θ with respect to the incident photon:



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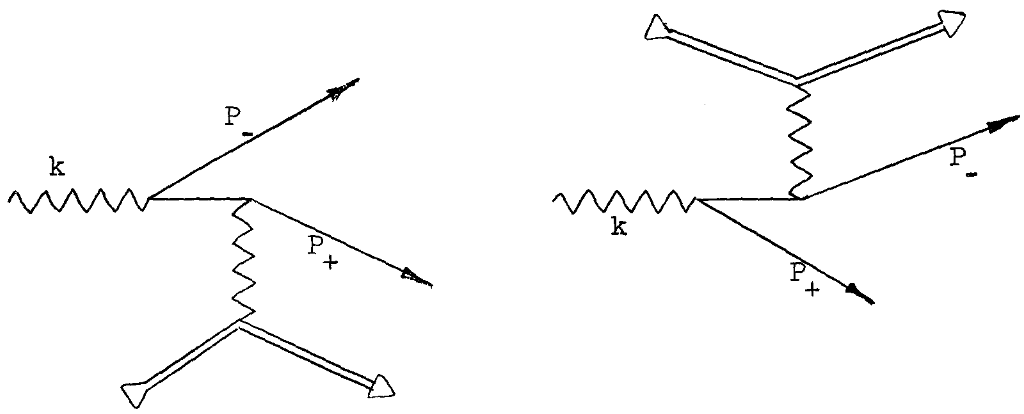
[1]T. D. Bjorken, S. D. Drell, and S. C. Frautschi, Phys. Rev. 112, 1409 (1958).

[2]B. Richter, Phys. Rev. Lett. 1, 114 (1958).

Bjorken, Drell and Frautschi[1] (hereafter referred to as BDF) have shown that this particular kinematics involves important theoretical simplification.

The diagrams of primary interest here, the Bethe-Heitler diagrams, are as follows:



Bethe-Heitler diagrams for wide-angle pairs.

The four-momentum transfer q_1^2 at the pair vertex, which will determine the sensitivity with which QED is being tested, is given by

$$(k - P_+)^2 = (k - P_-)^2 = - 2k \cdot P_+ = - 2k\, E_+ \,(1 - \cos\theta)$$

for the symmetric case chosen above. We assume that $P_+ = E_+$ and $P_- = E_-$ everywhere. BDG[1] have calculated the cross sections for these diagrams, including the effect of the nucleon form factor. It's important to note that this is possible; however, for our purposes the Bethe-Heitler result, which is much more transparent (at least to me), will suffice. The corrections to the B-H formula for the cases we consider are given by BDG[1] and can be taken into account later. The Bethe-Heitler result,

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[1]Bjorken, <u>et al., op. cit</u>.

for our special case (see first figure) can be written as

$$\frac{d\varphi}{d\Omega_+ d\Omega_- dE_+} = \frac{1}{137}\frac{e^4}{(2\pi)^2}\frac{1}{k}\frac{1}{q^2}\frac{\sin^2\theta}{(1 - \cos\theta)^2}$$

where k is the incident γ-ray energy, and q^2 is the square of the three-momentum transferred to the nucleon. In our case, this is also very close to the four-momentum transfer and determines how large an effect the nucleon form factors will have:

$$q^2 = k^2(1 - \cos\theta)^2$$

The B-H formula can be written

$$\frac{d\varphi}{d\Omega_+ d\Omega_- dE_+} = \frac{1}{137}\frac{r_o^2}{(2\pi)^2}\left(\frac{m}{k}\right)^2\frac{1}{k}\frac{\sin^2\theta}{(1 - \cos\theta)^4}$$

where $r_o = e^2/m$.

It's interesting to write this result in terms of the momentum transfer at the pair vertex:

$$q_1 = k\sqrt{1 - \cos\theta} \cong k\theta/2, \text{ for } \theta \ll 1$$

$$\sin^2\theta = \theta^2 = 2q_1^2/k^2$$

$$(1 - \cos\theta) \cong \theta^2/2 = q_1^2/k^2$$

Thus

$$\frac{d\varphi}{d\Omega_+ d\Omega_- dE_+} = \frac{1}{137}\frac{r_o^2}{(2\pi)^2}\left(\frac{m}{k}\right)^2\frac{1}{k}\frac{2q_1^2}{k^2}\frac{k^8}{q_1^8}$$

$$\frac{d\varphi}{d\Omega_+ d\Omega_- dE_+} = \frac{1}{137}\frac{r_o^2}{(2\pi)^2}\frac{m^2k^3}{q_1^6}$$

It appears that for fixed q_1 one gains by using higher k. This is in part illusory since, as k is increased, θ must be decreased for fixed q_1. For very small θ, the apertures must be very small to retain any resolution in q_1:

$$\frac{\Delta q_1}{q_1} \cong \sqrt{\left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta \theta}{\theta}\right)^2}$$

However, where resolution is not a problem, it is clear that the measurements should be made at as high a γ-ray energy as possible.

We consider now a specific experimental arrangement for which $q_1^2 = 10\ f^{-2}$. Such an experiment done to an accuracy of 10% is capable, in principle, of testing QED to distances of $\cong 10^{-14}$ cm.

We choose $\theta = 1/6$ radian (a much smaller angle gets us into the troubles with resolution indicated above). Since $q_1^2 = k^2\theta^2/2$ and we wish $q_1^2 = (10)(197)^2\ \text{Mev}^2$, we can determine k:

$$k = 5\ \text{Bev}.$$

Thus, the kinematics we are looking at are as follows:

$$\theta_+ = \theta_- = 1/6\ \text{radian}$$

$$P_+ = P_- = 2.5\ \text{Bev}$$

$$k = 5\ \text{Bev}$$

$$q_1^2 = 10\ f^{-2}$$

$$q^2 = k^2(1 - \cos\theta)^2 \cong \tfrac{1}{4}\ f^{-2}$$

We assume we have a two-magnet spectrometer to analyze the electron and positron, each with the characteristics $d\Omega = 5 \times 10^{-3}$ steradian and $\Delta p/p = 8 \times 10^{-2}$. The reasons for these choices are to allow a resolution in q_1 of about 20% and to minimize what may be a large background from π^0 pairs.*

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*By decreasing the energy interval seen by the spectrometer about 50%, one can make the detection of π^0 pairs kinematically impossible. However, the resolution suggested here will probably be sufficient.

Let N = expected counting rate/second

Q = number of equivalent quanta/second

$N_T = 10^{24}$ target atoms/cm^2 (10 in. liquid hydrogen target)

$$N = N_T Q \frac{dk}{k}\left(\frac{d\sigma}{d\Omega_+ d\Omega_- dE_+}\right) d\Omega_+ d\Omega_- dE_+$$

$$\frac{d\sigma}{d\Omega_+ d\Omega_- dE_+} = \frac{1}{137}\ \frac{(2.8 \times 10^{-13})^2}{(2\pi)^2}\ \frac{1}{10^8}\ \frac{1}{5 \times 10^3}\ \frac{3 \times 10^{-2}}{(1.5 \times 10^{-2})^4}$$

$$= 1.7 \times 10^{-35} \text{ cm}^2/\text{Mev-steradian}^2$$

Taking for the spectrometers $\Delta p/p = 8 \times 10^{-2}$,

$$\Delta E_+ = (8 \times 10^{-2})(2.5) \times 10^3 = 200 \text{ Mev}$$

$$N = 10^{24}\ Q\ \frac{400}{5000} \times (5 \times 10^{-3})^2\ (200)\ 1.7 \times 10^{-35}$$

$$= 6.7 \times 10^{-15}\ Q$$

The maximum tolerable value of Q will be determined by the chance rates. For this, we must make some estimate of the single rates in each detector. We assume that the detectors can certainly identify electrons. The two main sources of high-energy electrons will probably come from π^0 decay into Dalitz pairs and wide-angle pair production. We now estimate each of these contributions.

Dalitz Pairs

Let the cross section per steradian for production of a π^0 at 10^0 by a γ-ray of energy k be $d\sigma(k)/d\Omega$. Then the number of electrons arising from Dalitz pairs that will be detected will be given by

$$N_{el} = \frac{1}{80}\ N_T Q \left(\int_{k_{min}}^{k_{max}} \frac{d\sigma}{d\Omega}(k)\ P(k)\ \frac{dk}{k}\right) d\Omega$$

Where $P(k)$ is the probability that one of the electrons will have an energy appropriate to the detector. To estimate this, we make the pessimistic assumption that the energy E_{π^0} of the π^0 is equal to k. Then,

$P(k) = \Delta E/E_{\pi^0} = \Delta E/k$, where ΔE is the energy acceptance of the detector (200 Mev). The value of $(d\sigma/d\Omega)(k)$ is, of course, unknown. We guess at a cross section

$$\frac{d\sigma}{d\Omega} = 5 \times 10^{-30}\ \text{cm}^2/\text{ster}$$

independent of energy. Then

$$N_{el} = \frac{200}{80} \times 10^{24}\ Q\left(\int_{k_{min}}^{k_{max}} \frac{dk}{k^2}\right) 5 \times 10^{-30} \times 5 \times 10^{-3}$$

where $k_{min} = 2.5$ Bev and $k_{max} = 5$ Bev. Thus

$$N_{el} = \frac{200}{80} \times 10^{24} \times 5 \times 10^{-3}\ \frac{1}{5000}\ Q\ 5 \times 10^{-30}$$

$$\cong 1.2 \times 10^{-11}\ Q$$

Singles Rates from Large-Angle Pairs

Hough[3] gives the cross section for the production of an electron (or positron) at angle θ by a γ-ray of energy k as

$$\frac{d\varphi_-}{d\Omega_- dE_-} = \frac{1}{137}\frac{1}{4\pi}\, r_o^2 \left(\frac{m}{k}\right)^2 \frac{1}{k}\, S\left(k,\ \theta,\ \frac{E_-}{k}\right)$$

where S is a function of k, θ, and E_-/k too complicated to write down.

Then the number of electrons detected will be

$$N_\mu = N_T\, Q\left[\int_{k_{min}}^{k_{max}} \frac{d\varphi_-}{dE_- d\Omega_-}(k)\ S\left(k,\ \theta,\ \frac{E_-}{k}\right)\frac{dk}{k}\right] d\Omega dE$$

[3] P.V.C. Hough, Phys. Rev. 74, 80 (1948).

The integral is estimated numerically to be

$$21 \times 10^{-35}\ \text{cm}^2/\text{Mev-steradian}$$

$$N_{el} = 2 \times 10^{-11}\ Q$$

Thus the estimate for the total singles rate is

$$N_{el} = 3 \times 10^{-11}\ Q$$

We calculate the flux of γ-rays for which the accidental rate is equal to the true rate:

$$\frac{N_{Acc}}{N_T} = \frac{(2)\ (3)^2 \times 10^{-22}\ Q^2\ (\tau)\ (10^6)}{6.7 \times 10^{-15}\ Q\ (1.5)\ (360)} = 1$$

where τ is the resolving time, which we take to be $\tau = 10^{-9}$ seconds. We can then calculate that $Q = 2 \times 10^{12}/\text{sec}$.
This is only about 10% of the expected available beam from the monster. At this beam, the expected counting rate will be

$$N = 2 \times 10^{12} \times 6.7 \times 10^{-15} \times 60/\text{minute}$$

$$N = 0.8\ \text{counts/minute}$$

Obviously, these numbers are to be considered only as guides. I believe they indicate that the experiment is marginal and that if the Dalitz-pair rates should turn out to be wrong by a factor of 10, the experiment is probably not feasible, unless some technological improvement allows a resolving time $\tau \ll 10^{-9}$ seconds.

Detection Apparatus

The detection apparatus must have a very high discrimination against all particles other than electrons. It is easy to see that the number of π's entering the detector will be about 10^3 the number of electrons. We should like a discrimination of about 10^{-4} against the π's in the detector.

There are several possibilities for the detector, as follows.

(a) A total-absorption Cerenkov counter. This, at first sight, is simple and effective. However, the charge-exchange reactions in the counter ($\pi^- + p \rightarrow \pi^0 + n$; $\pi^+ + n \rightarrow \pi^0 + p$), with the subsequent development of the shower from the decay γ-rays, make it impossible to obtain the discrimination required. An estimate of the rejection of π's by such a counter is about 10:1 to 100:1 with the former figure more likely. I don't believe the situation can be much improved by any variations of this technique.

(b) A threshold Cerenkov counter. At 2.5 Bev/c, the velocity of a μ-meson is

$$\beta_\mu = 1 - \frac{1}{2\gamma^2} \qquad \gamma = 24$$

$$\beta_\mu = 0.999$$

The number of photons in the wavelength interval (3500 Å to 5000 Å) produced by one electron per cm of path length in a counter with $n = 1.001$ (n = index of refraction) is 0.7/cm. Thus a counter 3 meters long, with a collection efficiency for the light of 0.5 and a photo-cathode efficiency of 10% will give about 10 photo-electrons.

If one combines a total-absorption Cerenkov counter with a threshold Cerenkov counter, it isn't necessary to make the threshold counter insensitive to μ's, since μ's cannot be confused with electrons in the total absorption counter. For π's of 2.5 Bev/c momentum,

$$\beta_\pi = 1 - \frac{1}{2\gamma_\pi^2} = 0.9984$$

Thus a counter set to reject π's of 2.5 Bev/c ($n = 1.0016$) will have about 1 photon/cm in the appropriate wavelength interval. A 3-meter counter might be expected to yield about 15 photo-electrons, which should be sufficient.

There are obviously a number of other possibilities for detecting the electron. One might use focusing Cerenkov counters. The relativistic rise in ionization offers some possibilities for discriminating between electrons and π's. For instance, at 2.5 Bev/c, the ionization produced by electrons is about 50% greater than π's. Synchrotron radiation is another possiblity. For path lengths of the order of 10 cm in magnetic fields of the order of 10^5 gauss, the detection of electrons by their synchrotron radiation becomes reasonable. It might be mentioned in passing that for higher energies this technique becomes quite simple and is perhaps the only feasible method of distinguishing electrons from π's. I shall not analyze any of these other techniques in detail. It seems apparent that the problem of distinguishing 2.5 Bev electrons from π's is not insoluble.