SHOWER DEVELOPMENT AND HEATING IN THE ACCELERATING STRUCTURE OF A 50-BEV LINEAR ELECTRON ACCELÉRATOR

By

J. J. Muray

A.E.C. Contract AT(04-3)-363

Technical Report

M-276

September 1961

Project M
Stanford University
Stanford, California

TABLE OF CONTENTS

I. INTRODUCTION

The purpose of this report is to estimate the axial and radial temperature gradients in the accelerating structure of the Stanford two-mile accelerator if a fraction of the 60-μamp average beam-current at different energies (up to 50 Bev) strikes the structure, owing to mis-steering or malfunctioning.

E. L. Chu[1] has carried out an analysis of the surface heating of copper caused by the application of high-peak (low-average) and high-average (low-peak) rf power to the accelerator. This study indicates that the transient temperature rise under pulsed rf conditions is not significant and that only the average power need be considered. According to Chu's calculations, the temperature rise is 0.027°C per kw of peak rf power on 1 cm^2 of surface, which is negligible compared to the beam heating, as we shall see.

When a high-energy electron strikes one of the loading disks in the accelerating waveguide, it will start a series of high-energy interactions which not only heats the structure but also gives rise to penetrating radiation. The amount of this radiation and the necessary shielding for it has been calculated under normal operating conditions by DeStaebler.[2]

II. SHOWER FORMATION* IN THE ACCELERATOR STRUCTURE

When low-energy electrons (< 10 Mev) interact with matter, the energy is lost primarily by collision. Most of the lost energy is spent in exciting atoms or ejecting from atoms electrons at small energy, and is thus dissipated. When an electron has an energy considerably higher, of the order of 100 Mev or more, then the energy lost by radiation is predominant except for materials of low atomic number. The energy lost by radiation is fairly uniformly distributed among the secondary photons of all energies from zero up to the energy of the primary particle itself. The

---

[1]Status Report, M Report No. 246, Project M, Stanford University, Stanford, California, January 1961 p. 3.

[2]H. C. DeStaebler, Jr., "Review of Transverse Shielding Requirements for the Stanford Two-Mile Accelerator," M Report No. 262, Project M, Stanford University, Stanford, California, April 1961.
H. C. DeStaebler, Jr., "Radiation Levels Inside the Project M Accelerator Tunnel," M Report No. 263, Project M, Stanford University, Stanford, California, May 1961.

*See B.Rossi, High-Energy Particles, Prentice-Hall, Inc., New York (1952).

secondary gamma-rays may have sufficient energy to make the creation of electron-positron pairs, or materialization, probable. In this process the energy of the secondary gamma-ray is shared between the electron and positron. These new electrons radiate more photons, which again materialize into electron pairs or produce Compton electrons. Each step of this process of radiation and materialization yields an increase in the number of particles while decreasing the average energy of the individual particles. As the process goes on, more and more electrons fall into an energy range where radiation losses cannot compete with collision losses, until eventually the energy of the primary electron is completely dissipated in excitation and ionization of atoms. This formation of a large number of secondary particles from a single electron (or gamma-ray) is known as a cascade shower.

After the initial increase in the number of particles a point is reached where the average energy is too low to continue the multiplicative processes, and the number of particles gradually decreases as the energy of the individual particles is absorbed by matter. A general solution of the shower development problem would yield the probabilities of finding N_e and N_γ at any given depth in a shower and would determine the spatial and angular distributions of these particles. However, the theoretical problem in this form is hopelessly complex.

In the usual form, one can separate the longitudinal development and the radial development of a shower into two problems. The longitudinal shower development is discussed below; the radial shower development is discussed later in this report.

III. LONGITUDINAL SHOWER DEVELOPMENT

One of the most successful studies of a shower structure at energies from 50 to 50,000 Mev was the Monte Carlo calculation for air and aluminum absorbers performed by J. C. Butcher and H. Messel.[3] To apply this method to shower production, the fate of electrons and gamma-rays produced in the shower was determined by a digital computer. Each particle formed in the shower was followed until the gamma-ray energy fell below 5 Mev. It turns out that this method gives a result very similar to that of the usual shower theory[4] in Approximation A.

---

[3] J. C. Butcher and H. Messel, Nuclear Physics <u>20</u>,15 - 128 (1960).
[4] Rossi, <u>op. cit</u>.

The Monte Carlo calculation assumes that only four processes take place: (1) Bremsstrahlung, (2) pair production, (3) Compton effect, and (4) ionization loss. The distances are measured in terms of the depth t in radiation lengths, which is defined by

$$\frac{1}{t} = 4\alpha N r_o^2 \, Z(Z + 0.8)\rho \, \ell n(183 \, Z^{-1/3})/A$$

where the quantities N, α, r_o and ρ are, respectively, Avogadro's number, the fine-structure constant, the classical electron radius, and the density of the absorber.

Because this calculation was done for Al and air absorbers, for Cu we have to scale the energies. According to the suggestions of Butcher and Messel, all energies are scaled by the corresponding critical energies. Then

$$\frac{E_{Cu}}{E_{Al}} = \frac{\epsilon_{Cu}}{\epsilon_{Al}} = \frac{21.8}{48.8}$$

or

$$E_{Cu} = .45 \times E_{Al}$$

The result of this calculation can be summarized, after the proper scaling, to give the probability $P_n(t)$ of finding n electrons as a function of the radiation length t. Figures 1, 2 and 3 show the probability $P(t)$ of finding exactly n electrons against the depth t in units of radiation length in a copper absorber. The shower is initiated by an electron with primary energy $E_o = 50{,}000$ Mev, and each of the n electrons must have an energy greater than E.

From these probabilities one can calculate the mean number of particles in the energy interval $E_o - E$ as a function of the depth t in radiation lengths:

$$\bar{n}(E_o, E, t) = 1P_1(t) + 2P_2(t) + \dots + nP_n(t) = \sum_{n=o}^{\infty} nP_n(t)$$

In Figs. 4 through 8 the average number of electrons with energies greater than E is plotted against the depth t in radiation lengths,

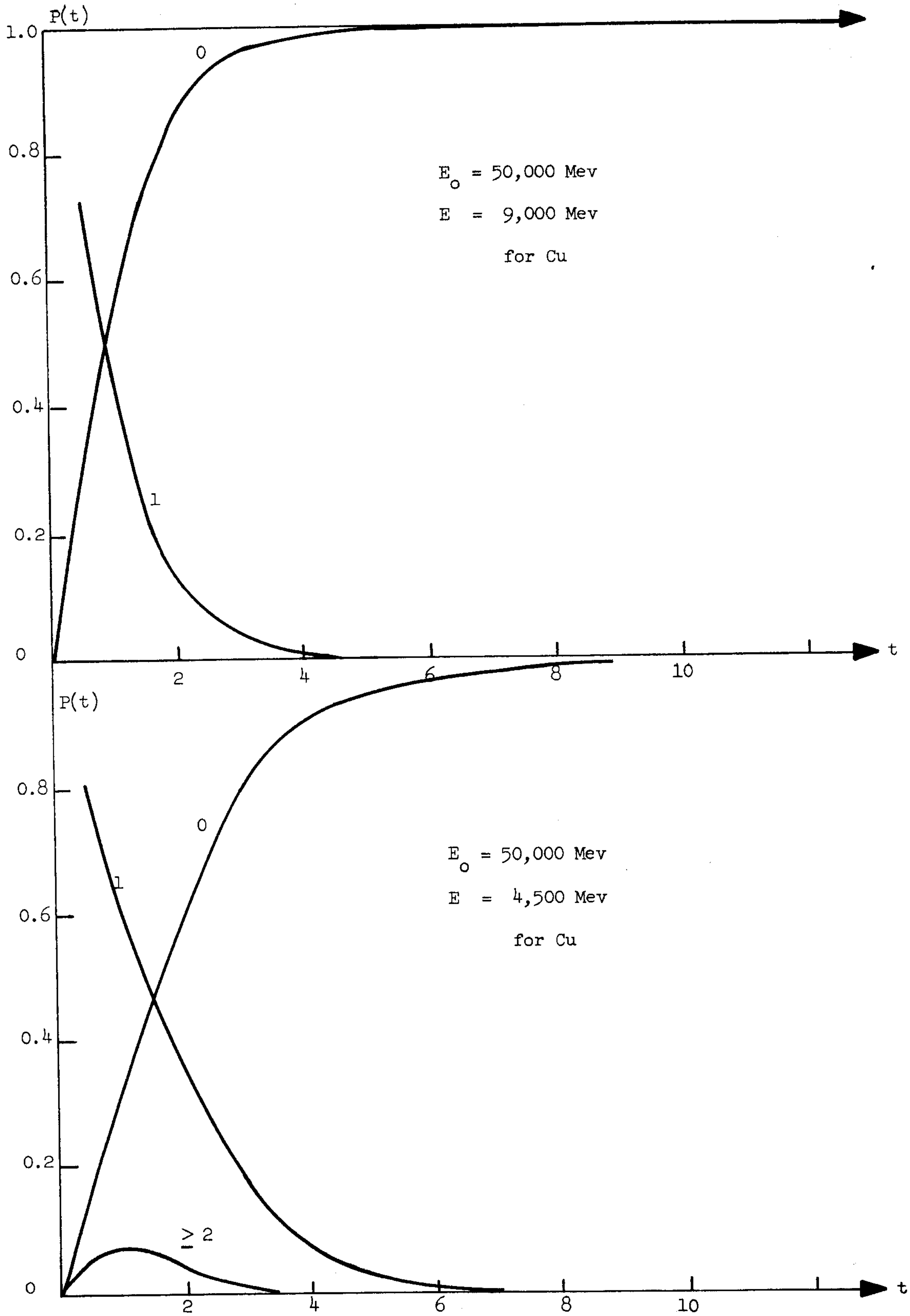
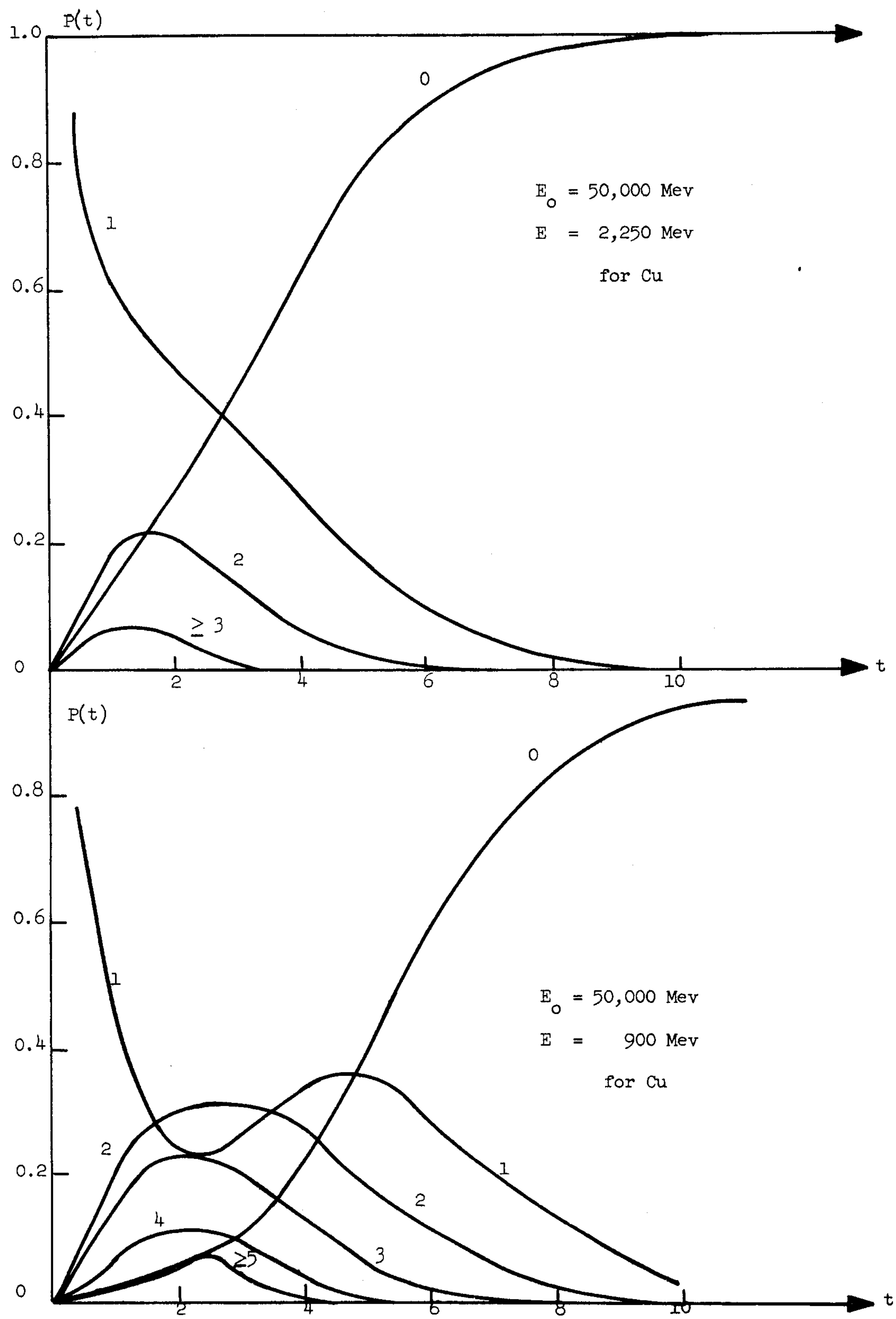
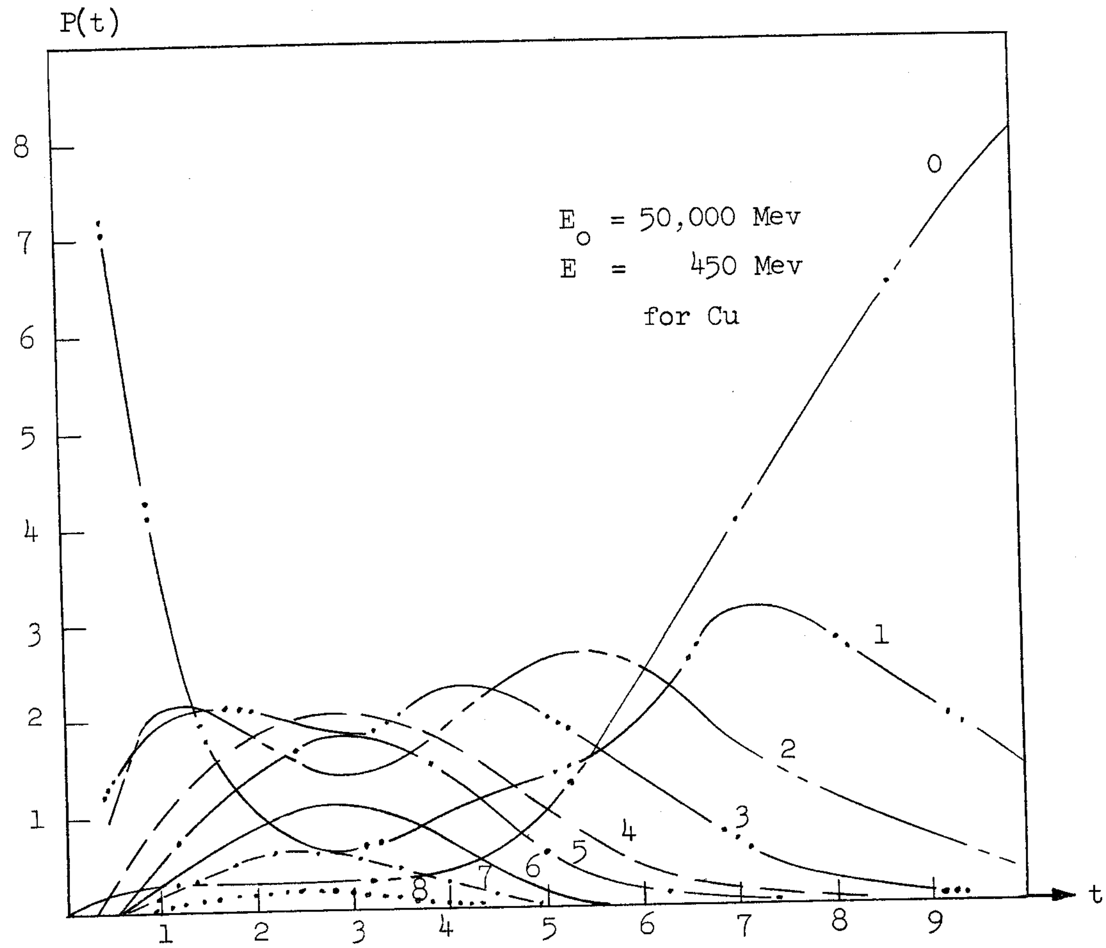

P(t)
1.0
0.8
0.6
0.4
0.2
0
0
1
E0 = 50,000 Mev
E = 9,000 Mev
for Cu
2
4
6
8
10
t
P(t)
0.8
0.6
0.4
0.2
0
0
1
≥ 2
E0 = 50,000 Mev
E = 4,500 Mev
for Cu
2
4
6
8
10
t

FIGURE 1



P(t)
1.0
0.8
0.6
0.4
0.2
0
0
1
2
≥ 3
E_o = 50,000 Mev
E = 2,250 Mev
for Cu
2
4
6
8
10
t
P(t)
0.8
0.6
0.4
0.2
0
0
1
2
3
4
≥5
1
2
E_o = 50,000 Mev
E = 900 Mev
for Cu
2
4
6
8
10
t

FIGURE 2


P(t)
8
7
6
5
4
3
2
1
E_o = 50,000 Mev
E = 450 Mev
for Cu
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
9
t

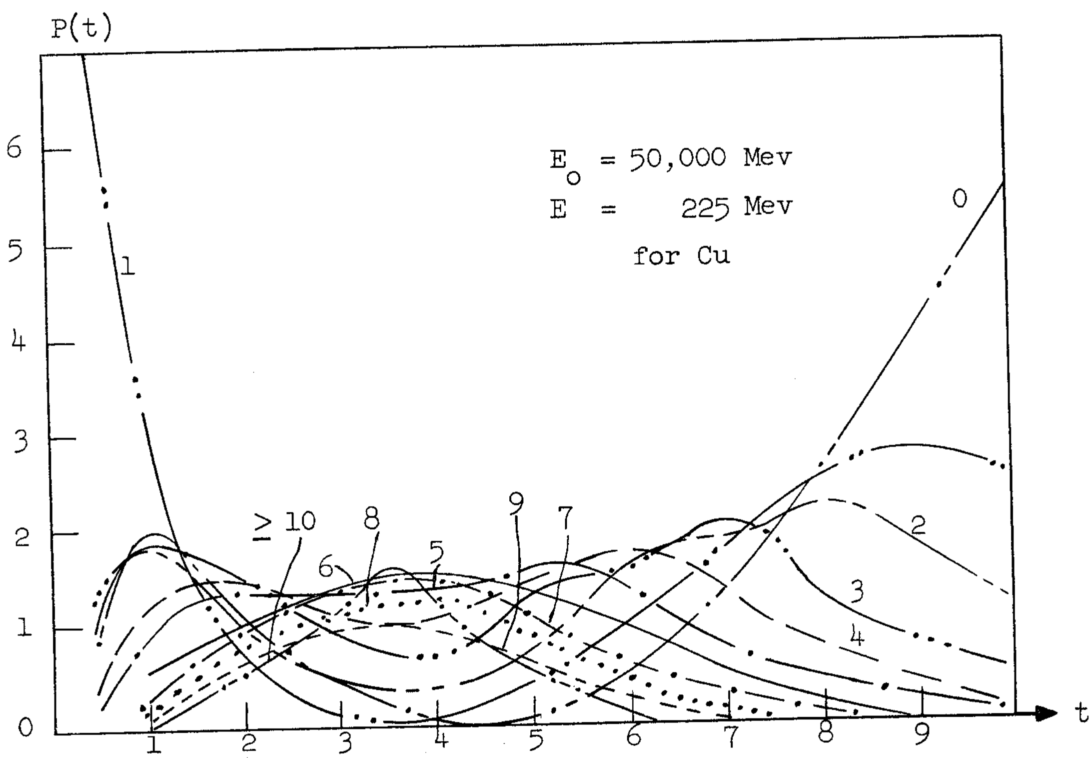

P(t)
6
5
4
3
2
1
0
E_o = 50,000 Mev
E = 225 Mev
for Cu
0
1
≥ 10
8
6
5
9
7
2
3
4
1
2
3
4
5
6
7
8
9
t

FIGURE 3

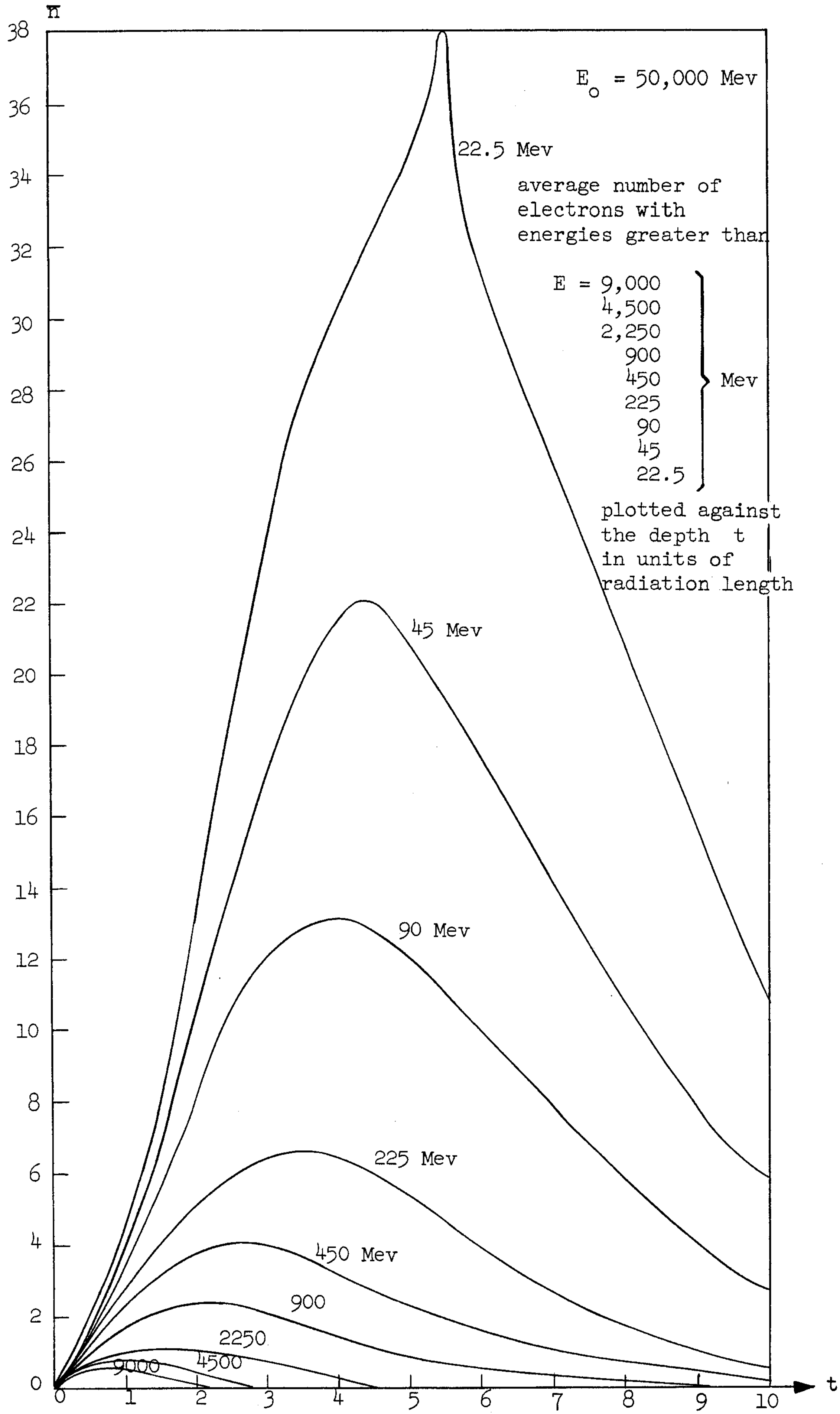

n̄
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
E_o = 50,000 Mev
22.5 Mev
average number of electrons with energies greater than
E = 9,000
4,500
2,250
900
450
225
90
45
22.5
Mev
plotted against the depth t in units of radiation length
45 Mev
90 Mev
225 Mev
450 Mev
900
2250
4500
9000
0
1
2
3
4
5
6
7
8
9
10
t

FIGURE 4

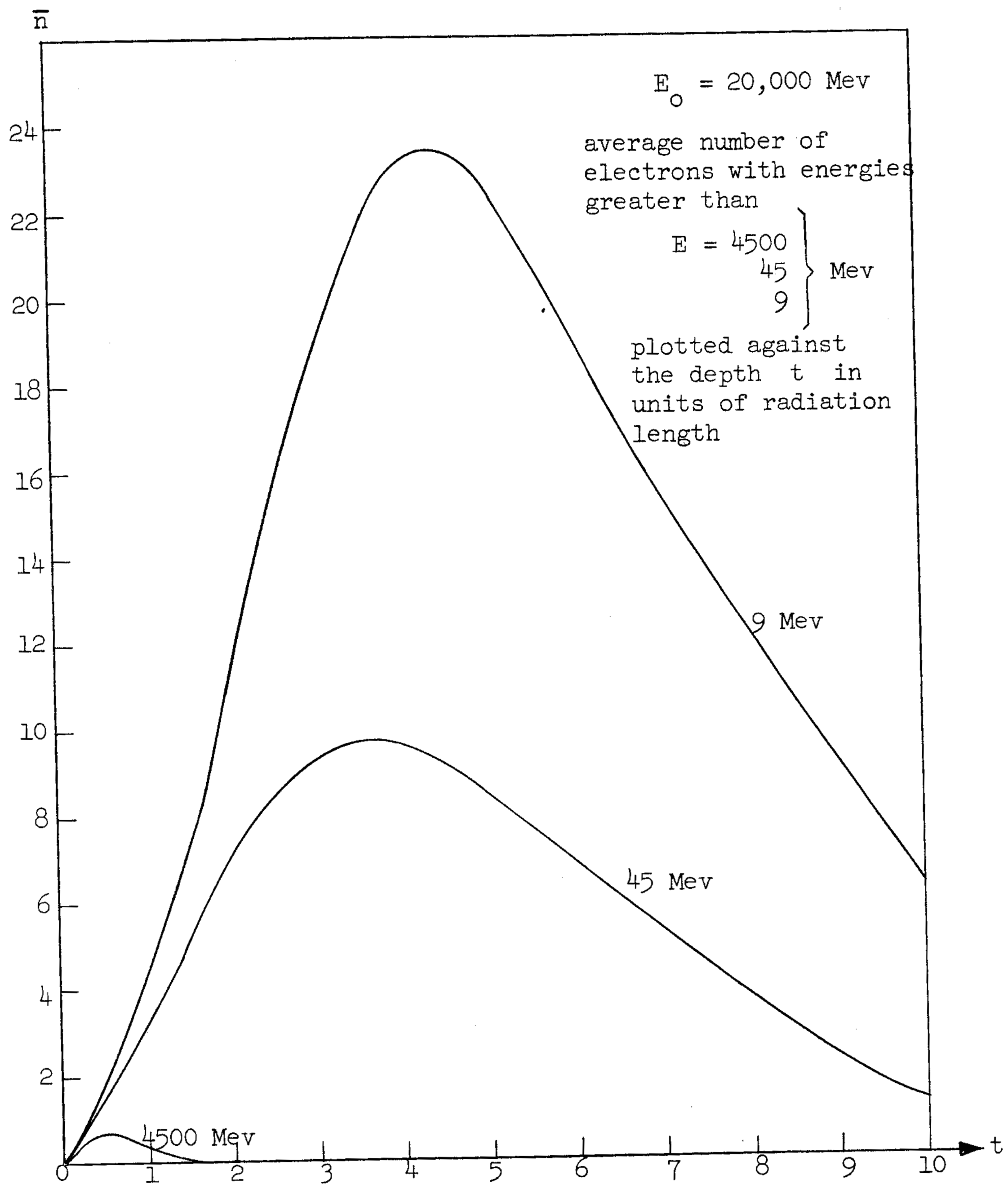
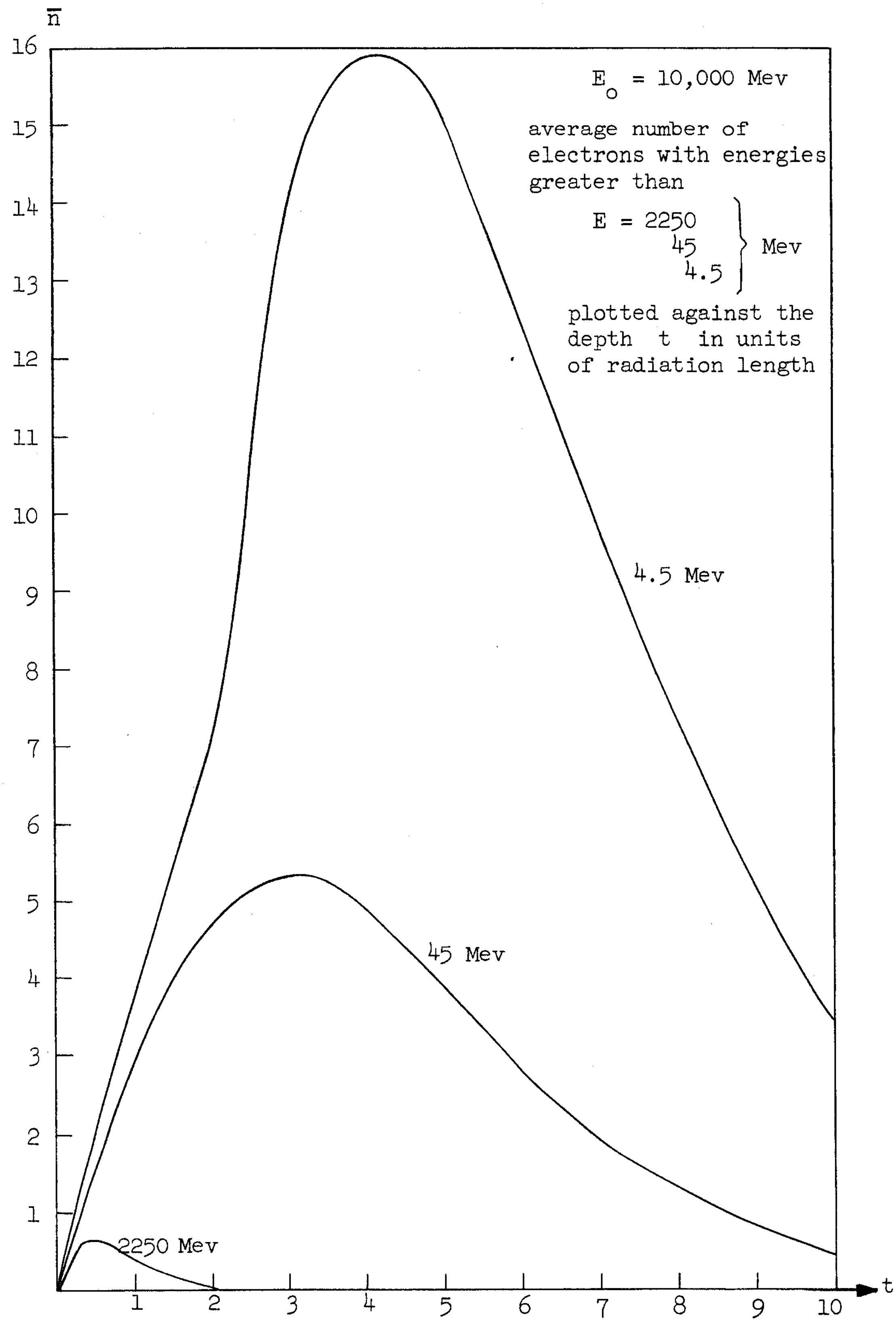
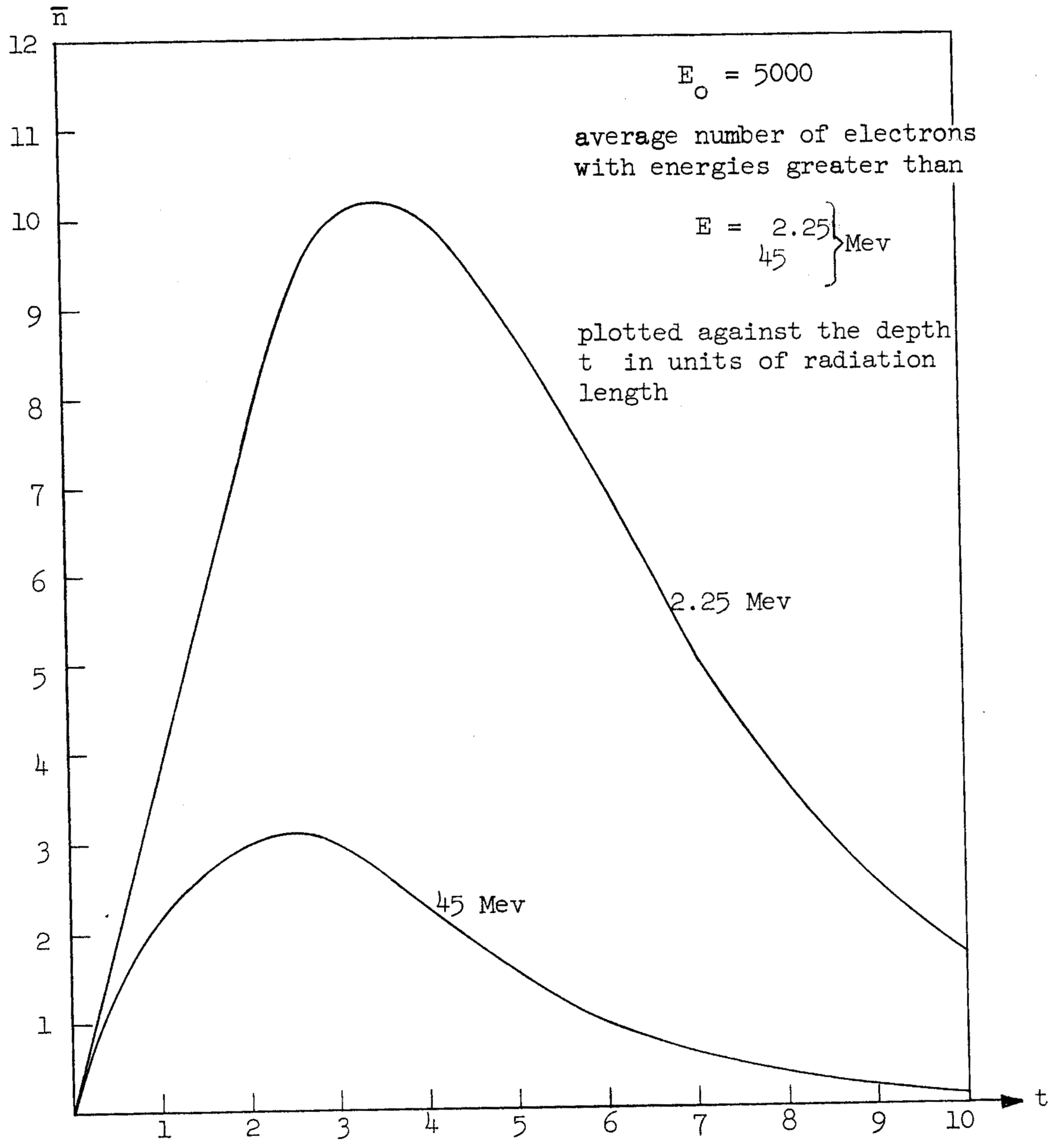

n̄
E_o = 20,000 Mev
average number of electrons with energies greater than
E = 4500, 45, 9 Mev
plotted against the depth t in units of radiation length
9 Mev
45 Mev
4500 Mev
t
0
1
2
3
4
5
6
7
8
9
10
2
4
6
8
10
12
14
16
18
20
22
24

FIGURE 5



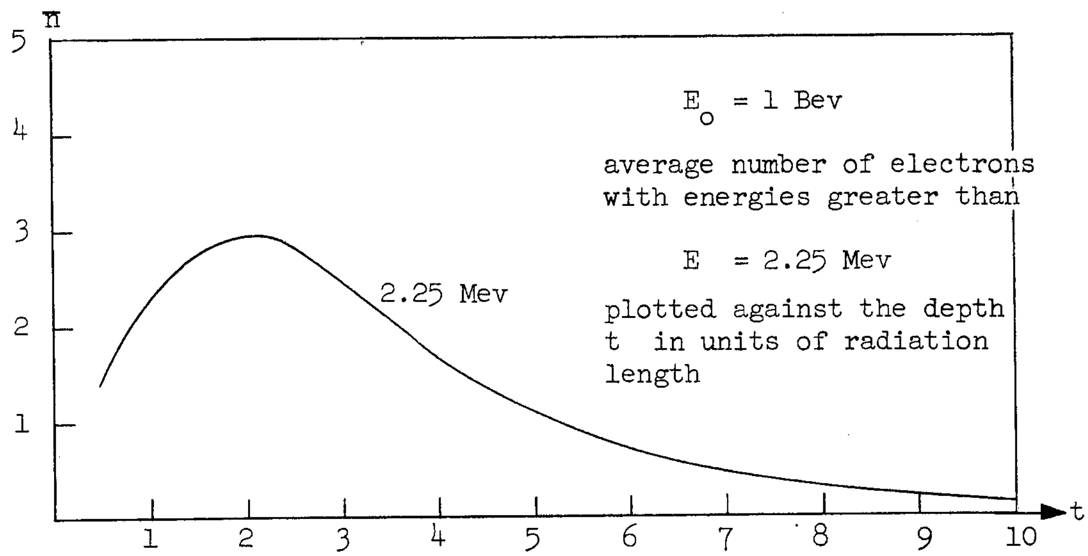
n̄
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
E_o = 10,000 Mev
average number of electrons with energies greater than
E = 2250, 45, 4.5 Mev
plotted against the depth t in units of radiation length
4.5 Mev
45 Mev
2250 Mev
1
2
3
4
5
6
7
8
9
10
t

FIGURE 6



$\bar{n}$
12
11
10
9
8
7
6
5
4
3
2
1
E_o = 5000
average number of electrons with energies greater than
E = 2.25 / 45 Mev
plotted against the depth t in units of radiation length
2.25 Mev
45 Mev
1
2
3
4
5
6
7
8
9
10
t

FIGURE 7



n̄
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
t
2.25 Mev
E
o
= 1 Bev
average number of electrons
with energies greater than
E = 2.25 Mev
plotted against the depth
t in units of radiation
length

FIGURE 8

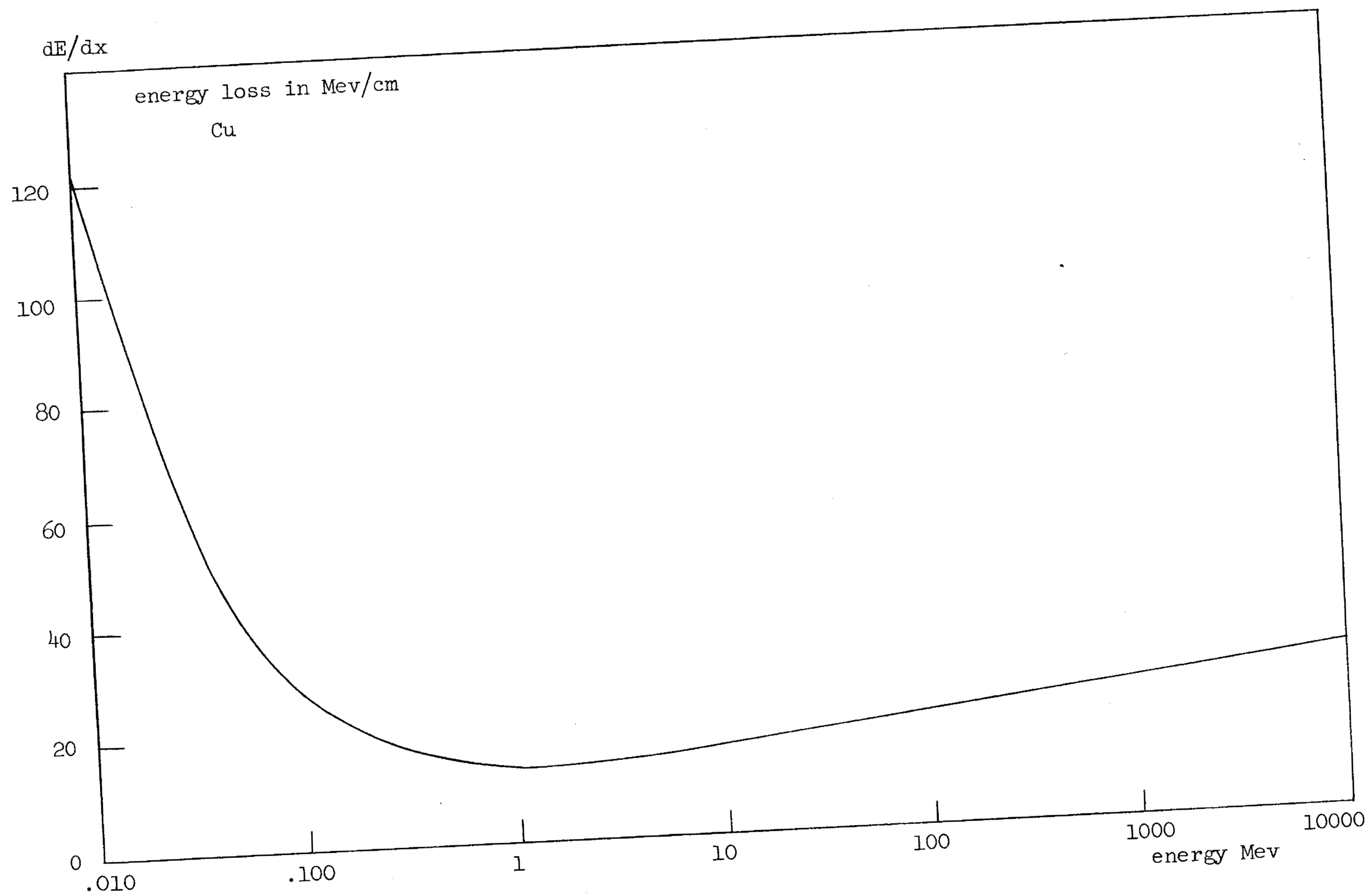


FIG. 9--Electron energy loss vs energy

for 5 cases of electron-initiated showers with primary energies $E_o = 50,000$; 20,000; 10,000; 5,000; and 1000 Mev.

Since the heating in the accelerator structure actually measures the amount of energy absorbed by the structure one would expect that the beam power deposited in the loading disks can be expressed as

$$W(E_o, E, t) = \sum_{E}^{E_o} W_E(E, E + \Delta E, t) = \sum_{E}^{E_o} \bar{n}(E, E + \Delta E) \frac{dE}{dx}(\bar{E})$$

where $\frac{dE}{dx}(\bar{E})$ is the energy loss per unit length per electron with energy $\bar{E}$. Figure 9 shows the energy loss of an electron as a function of its energy.

The geometry of the actual waveguide structure is shown in Fig. 10. Because the radiation length in Cu is 13 gr/cm^2 with density of 8.9 gr/cm^3 this corresponds to a distance of

$$\frac{t}{\rho} = 1.46 \text{ cm}$$

or

$$\frac{1.46}{.575} = 2.58 \text{ disks}$$

The power deposited per radiation length in the energy interval E, $E + dE$ per incoming electron can be written as

$$\bar{n}(E, E + \Delta E, t) \frac{dE}{dx}(\bar{E}) \times 1.46$$

Thus the dissipated power (in watts) per disk (for the actual numbers, using Figs. 4-9) is

$$\frac{W_E}{D} = \bar{n}(E, E + dE, t) \frac{dE}{dx}(\bar{E}) \times \frac{1.46}{2.58} \times 1.6 \times 10^{-13}$$

This quantity integrated over the energy range $E_o - E$ is plotted in Fig. 11 as the depth in radiation length per electron with different incident energy E_o.

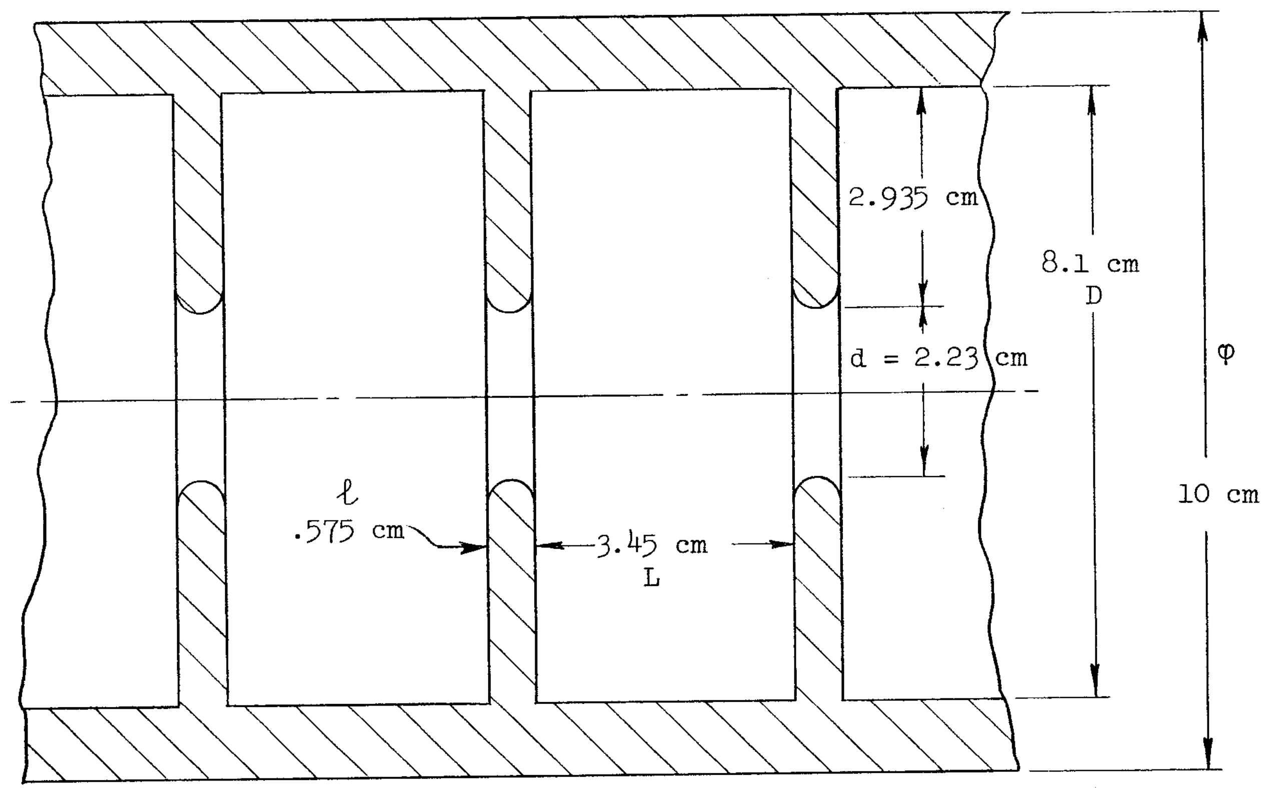


FIGURE 10

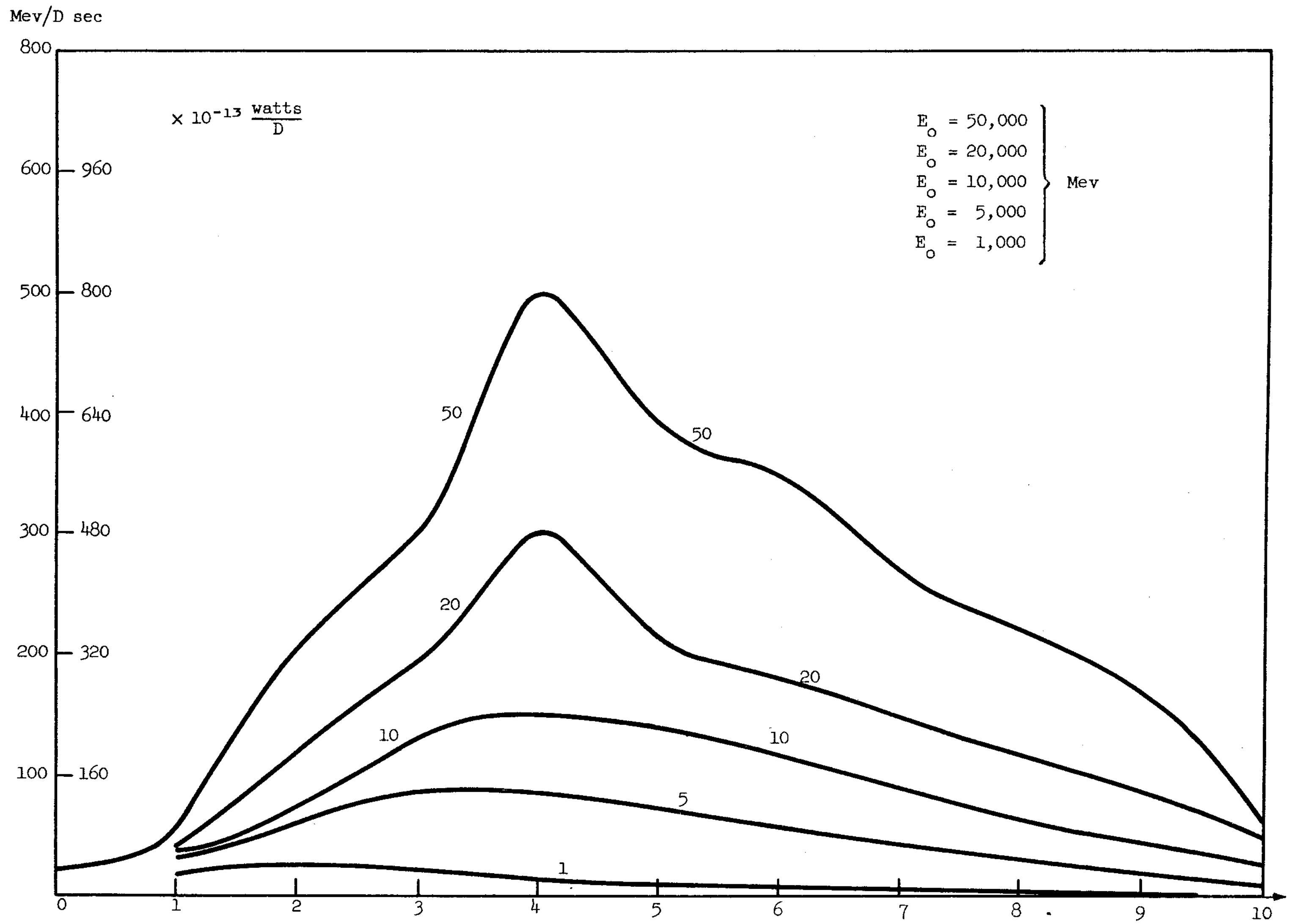


FIG. 11--Dissipated power per disk per electron vs radiation length.

Because of the shower development in the structure, the maximum heating will occur where

$$\frac{d}{dt}\left[\sum_{E}^{E_o} \bar{n}(E,\ E + \Delta E,\ t)\ \frac{dE(\bar{E},\ t)}{dx}\right] = 0$$

Since the location of the maximum of the number of electrons moves farther away from the striking point with increasing energy, one would expect that the distance between the striking point and the hottest disk would also increase. This is shown in Fig. 12.

IV. HEAT CONDUCTION

In the heat-conduction calculations it is supposed that the beam strikes the inside surface of the disk with equal electron density independent of the azimuth angle. This approximation underestimates the heat-source density and the heat conduction of the surface. In calculating the temperature difference using this model one has to solve the heat-conduction equation with cylindrical boundary conditions. When the temperature of the outer surface of the waveguide is T_1, the temperature difference between the inside and outside surfaces of the waveguide disk can be calculated from the solution of this heat conduction problem which is, using the proper units,

$$\Delta T = T_1 - T_2 = 0.088\ \frac{N \log_{10}(r_2/r_1)}{\lambda \ell}$$

or, with the dimensions of the waveguide structure,

$$\Delta T = 1.14 \times 10^{-1}\ nN_o$$

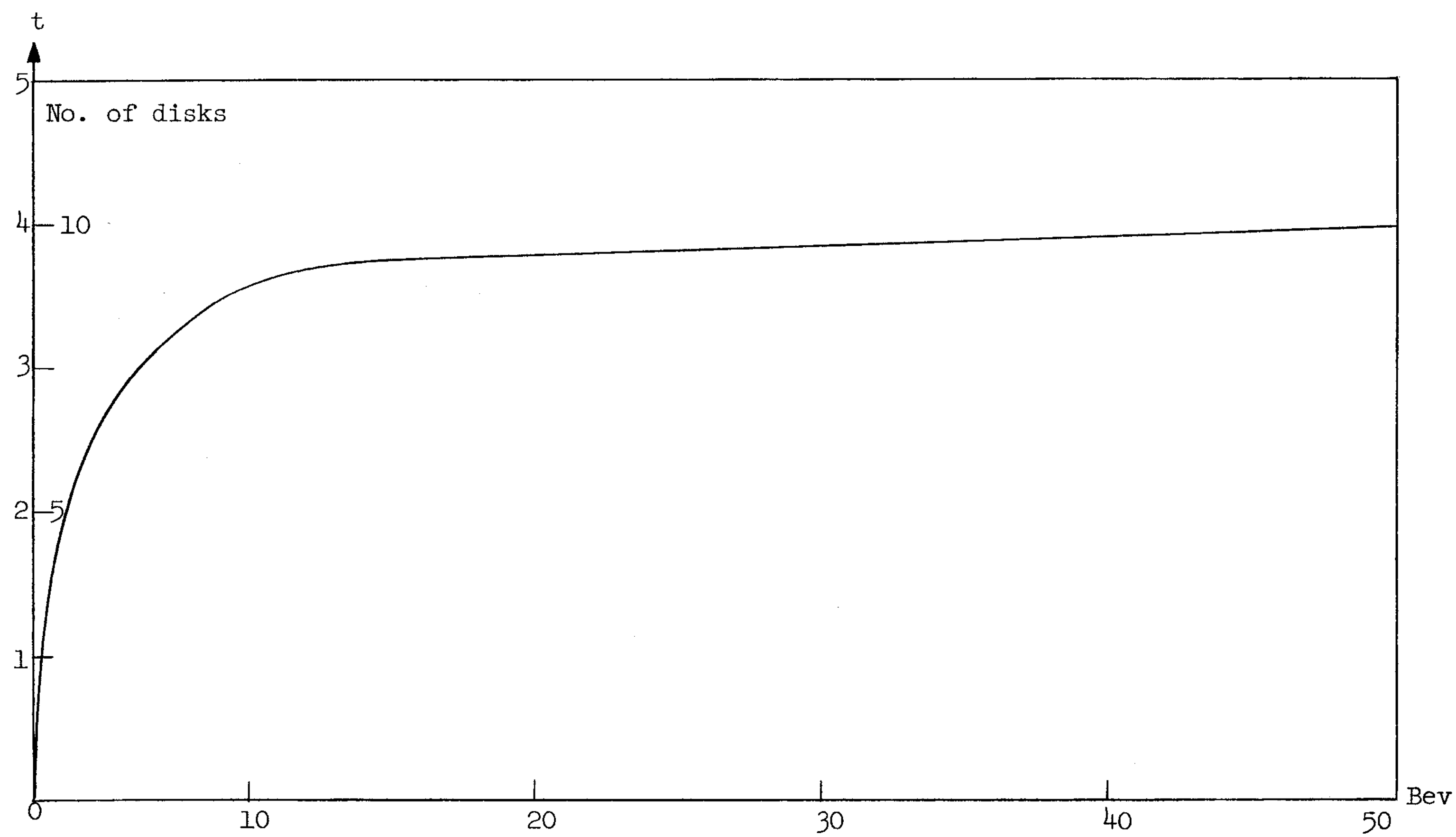


FIG. 12--The distance between the hitting point and the hottest disk vs energy.

where

N is the total dissipated power (in watts)

r_2 and r_1 are the outside and inside diameters of the iris

ℓ is the thickness of the disk (in centimeters)

$\lambda = .94$ is the heat-conduction coefficient (in units of $cal/cm^{o}C/sec$)

$N_o = (W/D)$ is the dissipated power per disk per electron

n is the number of electrons per second which strike the inside surface of the disk.

Now one can calculate the maximum number of electrons n which can hit the structure causing a 100-degree temperature difference between the outside and inside of the disk. To do this one can use the following simple equation:

$$n = \frac{\Delta T}{N_o \times 1.14 \times 10^{-1}} = \frac{10^3}{N_o \times 1.14}$$

If one uses for N_o the value which is shown in Fig. 11, n is calculable. The design value for n is 3.6×10^{14} when the total beam strikes the target. Figure 13 shows the maximum number of electrons versus the energy (or length if there is a linear energy increase) which cause a 100-degree temperature rise in the iris when the beam strikes the waveguide structure.

The curve in Fig. 13 shows that if the total beam were to hit the structure with 50-Bev energy, the temperature rise would be 3200 degrees. At 1 Bev a 53-μamp average beam can hit the structure and cause only a 100-degree temperature rise. This result could be checked experimentally with the present Mark III accelerator by using the 1-Bev beam with 1-μamp average current. Using a water-cooled waveguide section, one would expect to measure approximately a 1.9-degree temperature rise using thermistors with an accuracy of 0.1 degree in a simple bridge circuit.

If the beam strikes the structure asymetrically, i.e., at only one point on the inside surface, the temperature difference at other points on the inside surface can be approximated as follows. Since

$$N = nS = n2\pi\ell r_1$$

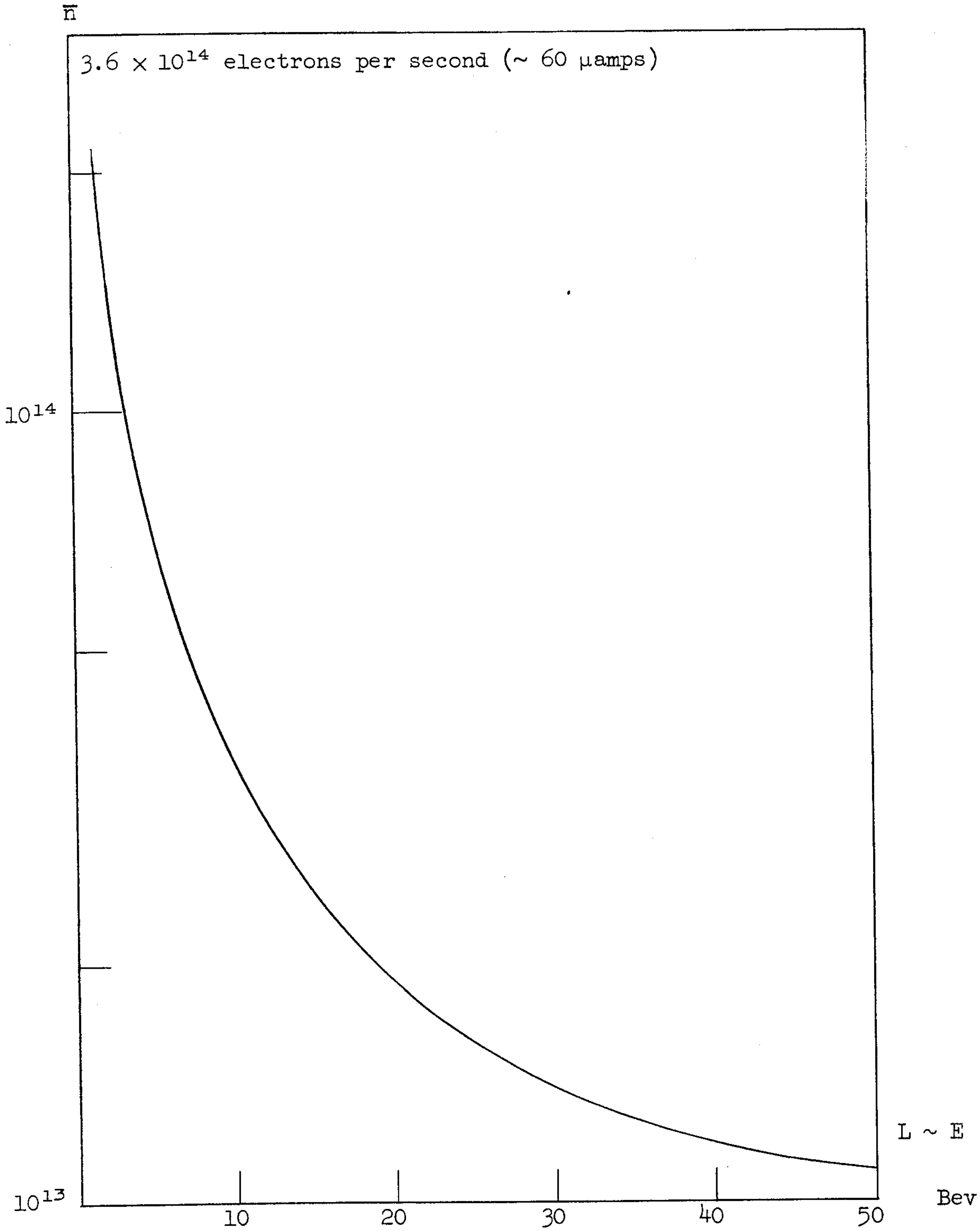


FIG. 13--The maximum number of electrons vs energy which cause a 100-degree temperature rise in the iris when the beam is hitting the waveguide structure.

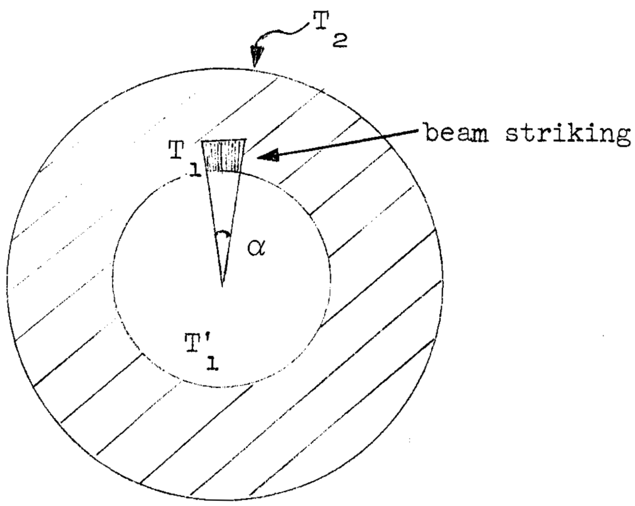
where n (in watts/cm²) is the power density, and S is the inside surface, one can write

$$T_1 - T_2 = \Delta T = 0.088 \frac{n\, 2\pi r_1 \ell \log (r_2/r)}{\lambda \ell} = 0.55 \frac{n r_1 \log (r_2/r_1)}{\lambda}$$

and at the opposite side of the striking point the temperature difference is

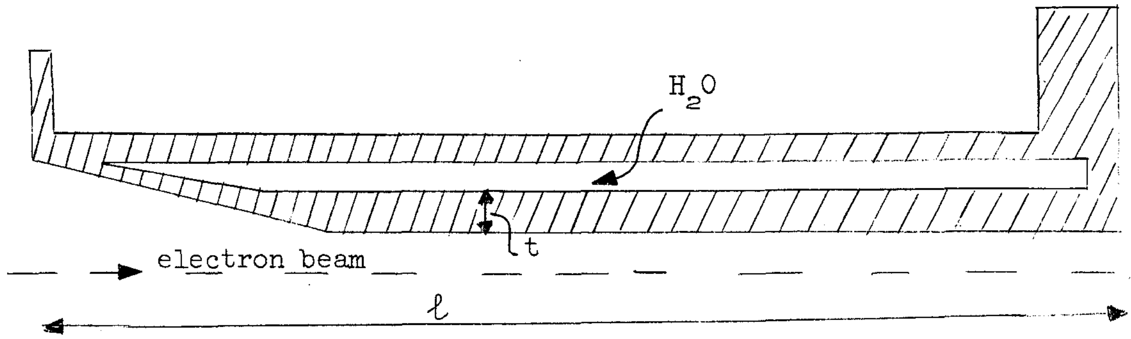
$$T_1' - T_2 = \Delta T_1 = 0.088 \frac{n \times \alpha r_1 \ell \log (r_2/r)}{\lambda \ell}$$

where the symbols are defined as shown in the following sketch.



Since the temperature difference exists along the inside surface, the iris will deform because of the thermal stresses, which will cause a shift in the resonant frequency in this cavity. The effect of nonuniform heating must be investigated further.

It is worthwhile to note that by using a water-cooled collimater between the waveguide sections the temperature gradient in the structure could be reduced. Using a water-cooled tapered collimator, as sketched below, the inside surface temperature could be reduced by a factor of 10, but the effect of Coulomb scattering on the beam-energy spectrum in such a collimator is far from clear



The length ℓ and thickness t of this collimator depend on its location in the accelerator because the maximum of the electron-number distribution shifts with the beam energy (see Fig. 12).

V. RADIATION COOLING

In addition to the heat conduction, heat radiation also contributes to the cooling of the hot surfaces. The radiated power from a surface area F at temperature T can be expressed in the following form

$$N = S \times F$$

where F is in cm², N is in watts/cm², and

$$S = \sigma e_g T^4 = 5.67 \times 10^{-4}\, e_g \left(\frac{T}{100}\right)^4 \quad \text{(in units of watts/cm}^2\text{)}$$

where e_g is the emission constant, and T is in degrees Kelvin.

Figure 14 shows S, as the surface temperature, for copper. From this curve it is evident that the radiation cooling, on the order of 100 watts, is negligible compared to the conduction cooling.

VI. SURFACE COOLING

In the previous sections it was supposed that the outside surface is held at constant temperature T_1, which is close to room temperature under normal conditions. However, in the case of the beam hitting the structure, the total beam energy is dissipated in a distance L, which is the length

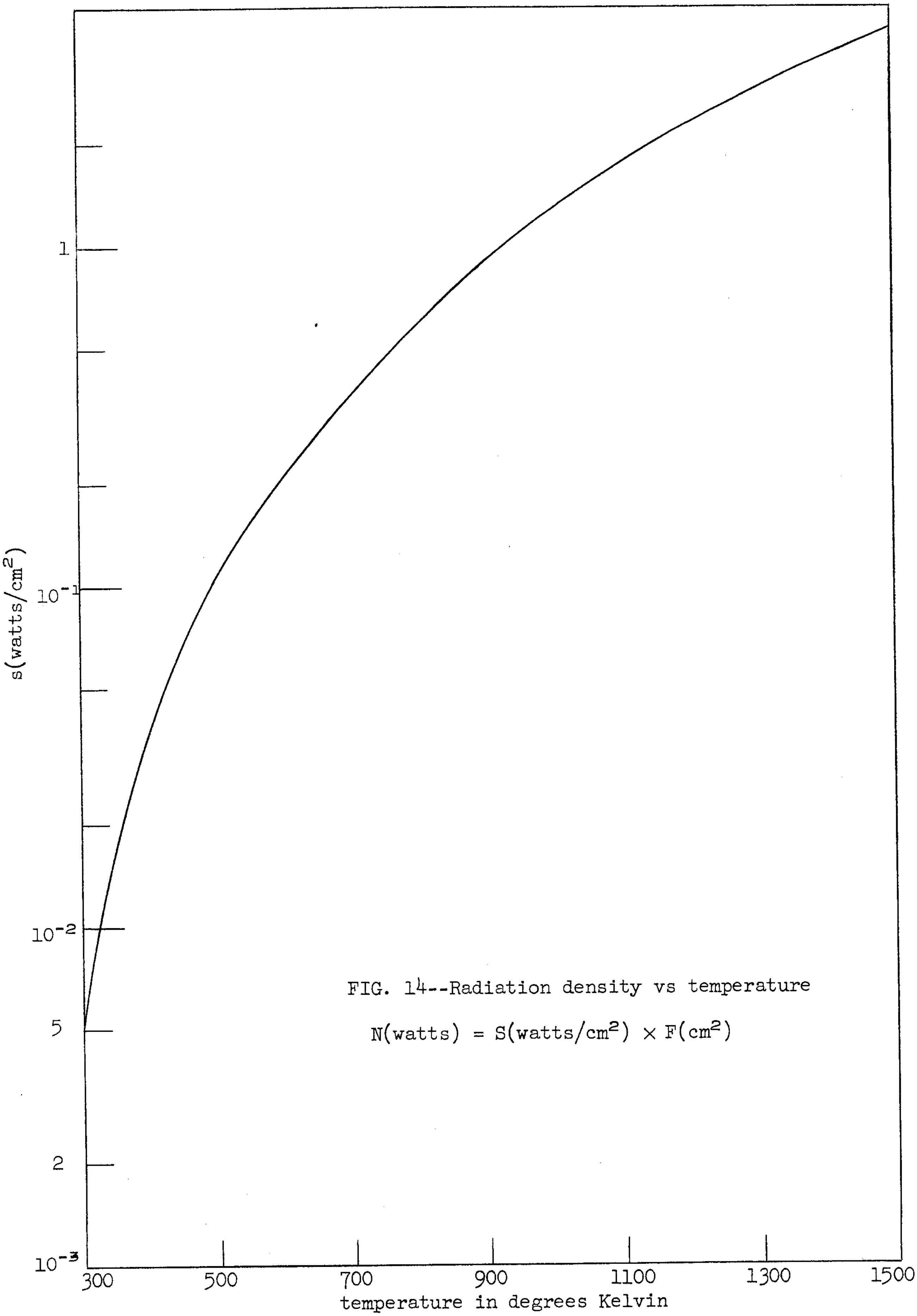


FIG. 14--Radiation density vs temperature

N(watts) = S(watts/cm²) × F(cm²)

of the shower. Because L is a function of the energy in the high-energy case, the dissipating surface will be longer, which helps the situation. It is interesting to calculate the average surface-power density which has to be removed by the cooling system when the total beam power is dissipated in the structure.

The quantity

$$\frac{W_c}{cm^2} = \frac{EI}{L(E)\pi\Phi}$$

where Φ is the outside waveguide diameter, is shown in Fig. 15, where the length L is taken from Fig. 11. Figure 15 clearly shows that the power density that has to be removed is a factor of 10 higher than the conventional maximum with water-cooling systems (which is 500 w/cm^2).

VII. RADIAL DEVELOPMENT OF THE ELECTRON-INDUCED SHOWER

Up to this point we have supposed that the shower develops only in the direction of the incident particle. Such an approximation is particularly good for high energy and low atomic number, but one also has to take into account those effects which lead to a radial development. These are:

1. Multiple Coulomb scattering of electrons by nuclei and electrons.
2. Compton scattering of gamma-rays by electrons.
3. The angular separation in pair production from the parent gamma-ray.
4. The angular separation of the Bremsstrahlung gamma-ray from the direction of the incident electron.

Of these effects, the multiple Coulomb scattering contributes most to the scattering, and the other processes may be neglected in a first approximation of the radial development of a shower.

The multiple-scattering angle can be calculated from the following formula:

$$\Theta = 10^{-3} \left(\frac{21}{E}\right) t^{\frac{1}{2}}$$

where E is measured in Bev.

Figure 16 shows Θ versus the energy of the beam for Coulomb scattering and for the processes of (2), (3) and (4) where the production angle

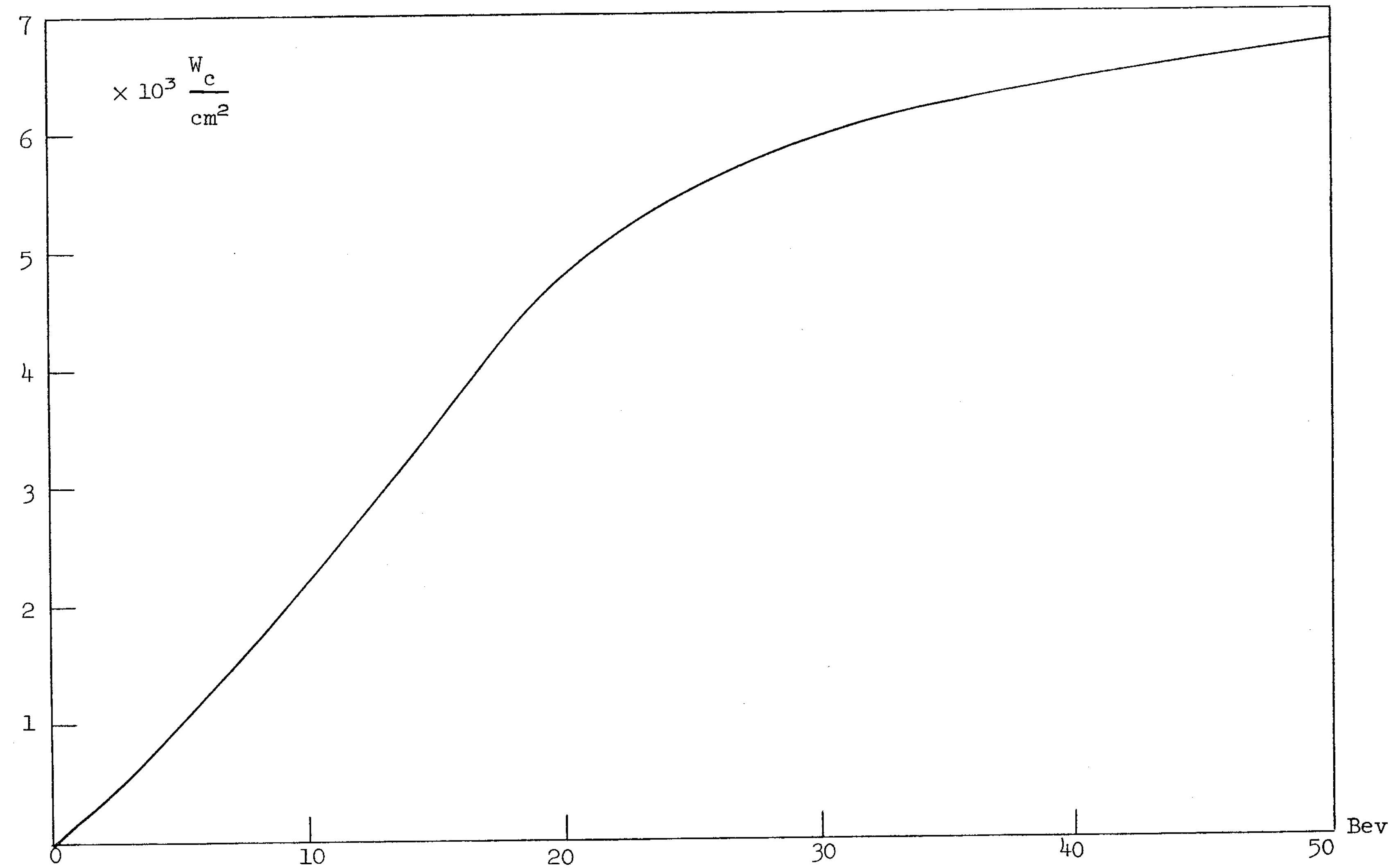


FIG. 15--The average surface power density vs energy when the total beam power is dissipated in the waveguide structure.

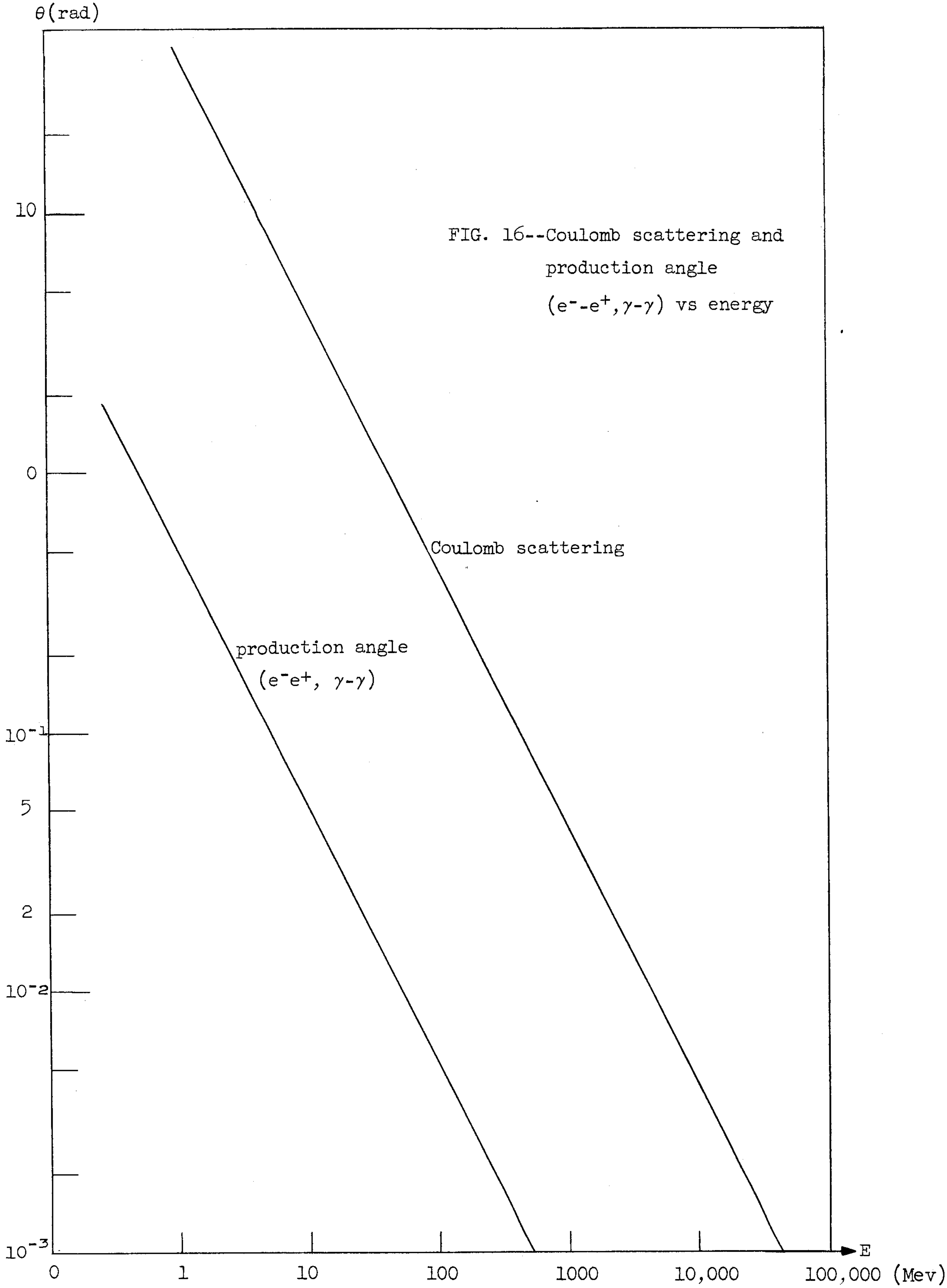


FIG. 16--Coulomb scattering and production angle ($e^- - e^+, \gamma - \gamma$) vs energy

is proportional to $(mc^2/E) \ln (2E/mc^2) \sim (mc^2/E)$. At a distance of 35 cm (10 disks) the area occupied by the beam is shown in Fig. 17, using only the Coulomb-scattering angle.

From Fig. 17 it is evident that the beam starts to scatter out from the waveguide below 100 Mev where the area occupied by the beam is larger than the cross section of the waveguide (314 cm^2). These particles contribute to the radiation background in the tunnel and to the activation of the air.[5]

VIII. CONCLUSIONS

The shower development in the waveguide structure smears out the dissipated power and, with this, the heat source along the guide into a distance of 20 cavity lengths. Even in this case the temperature rise in the disk would be about 3200° if a total beam of 60 μamps at 50 Bev were to strike the structure. The temperature gradient can be decreased by using collimators between the waveguide sections, and protection afforded by a fast interlock system. The radiation cooling is negligible compared to the conduction cooling. Below 100 Mev some of the striking particles scatter out of the waveguide and contribute to background radiation and activation of the air.

Under normal operating conditions the temperature rise caused by beam heating probably would not exceed 10 degrees.

---

[5] H. C. DeStaebler, Jr., Project M Source Book, Section VI (internal document), Project M, Stanford University, Stanford, California, August 1961.

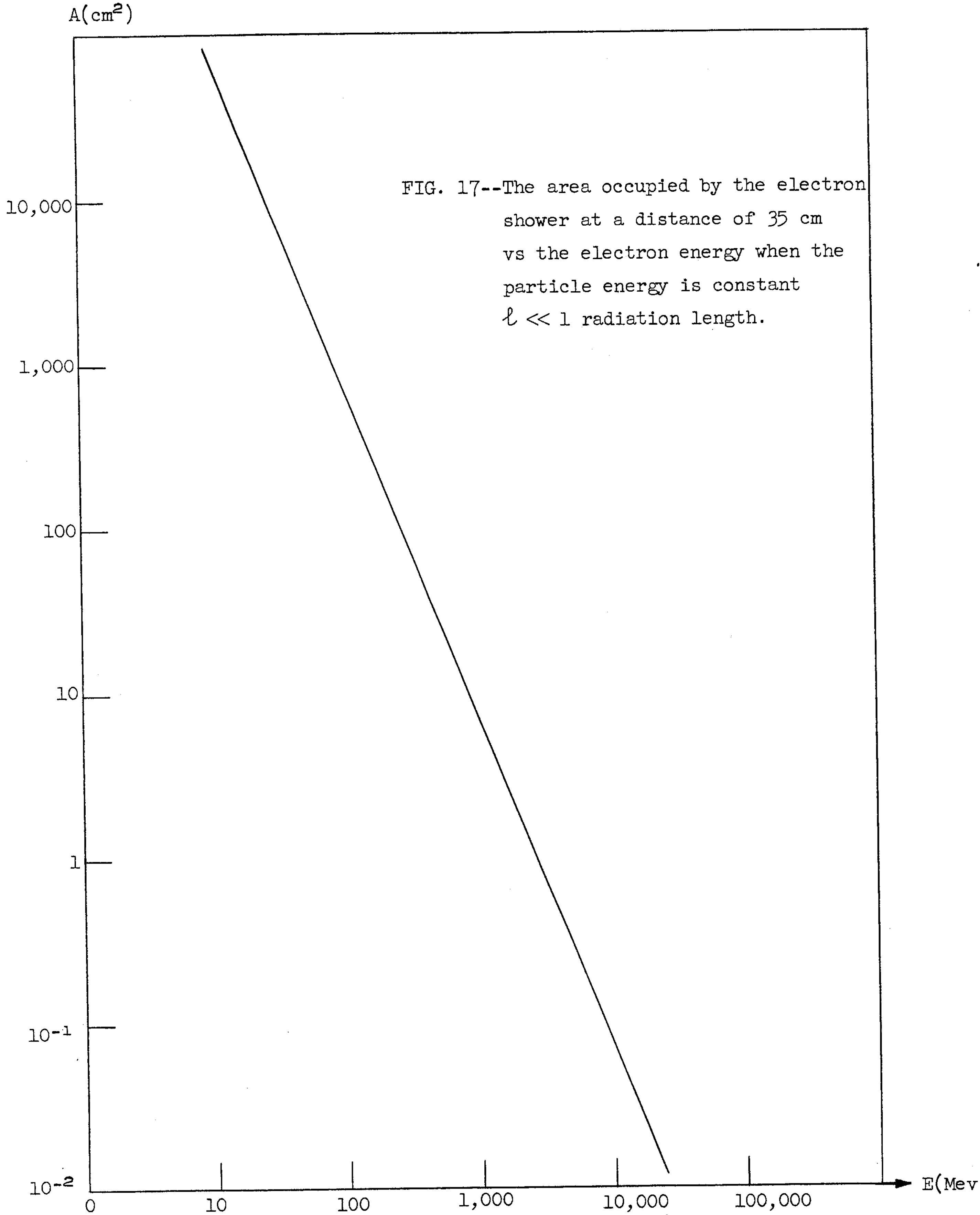


FIG. 17--The area occupied by the electron shower at a distance of 35 cm vs the electron energy when the particle energy is constant $\ell \ll 1$ radiation length.