MUON YIELDS FROM PION DECAY AND ELECTROMAGNETIC PAIR PRODUCTION

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I. INTRODUCTION

The problem of the shielding design for muons produced in π-μ decay and in electromagnetic pair production is quite serious, because the muons are stopped only by ionization energy loss instead of nuclear interaction. In this report the muon flux is calculated for two different shielding geometries just outside of the shielding wall.

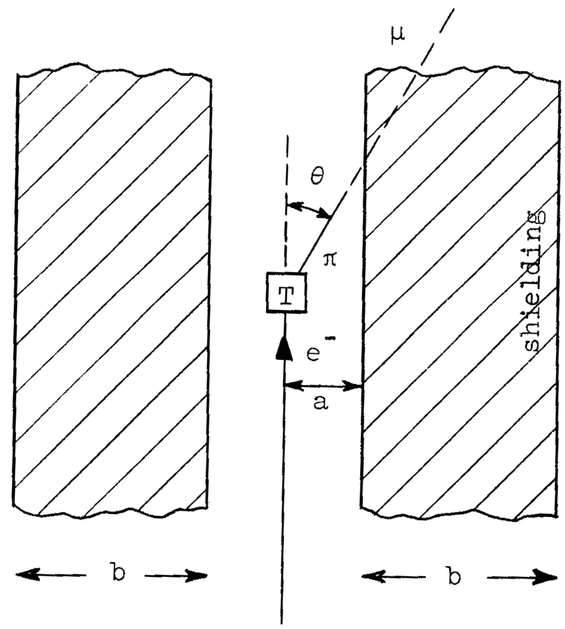
The pions are produced in single or multiple photoproduction events, while the photons arise from electron-photon showers in the target. These π-mesons either undergo nuclear interactions or decay to muons; the first process is much more probable in matter, but the latter process remains to constitute a shielding problem.

II. PION DECAY YIELDS

The pion yields are calculated by K. G. Dedrick* based on the following model. The electron beam produces a soft shower on striking a target. Some of the photons in the shower are then absorbed by nucleons in the target nuclei. The photon absorption cross sections have been taken as $\sigma = 1 \times 10^{-28}$ cm^2 for all photon energies; this is probably too large because at 300 Mev the resonance cross section is 2×10^{-28} cm^2. The number of pions which are boiled off from the nucleons is calculated according to a statistical model. Dedrick's final results are expressed as the number of pions produced per unit pion energy per unit solid angle in the laboratory when one electron (45 Bev) is incident on the target.

First, let us consider a cylindrical shielding geometry, using the electron beam as the axis of symmetry; b is the shielding thickness, a is the inside tunnel radius, and θ is the direction of the outcoming pion relative to the beam axis. The geometry can be pictured as shown in the following sketch. If the pion yield vs the pion energy at a given angle is known in the laboratory system as

*K. G. Dedrick, "Calculation of Pion Photoproduction from Electron Accelerators According to the Statistical Model," M Report No. 228, Stanford University, October 1960; and K. G. Dedrick, "More Calculations of Photopion Yields," M Report No. 229, Stanford University, October 1960.



cylindrical shielding geometry

$$\frac{1}{X_0}\frac{d^2Y}{dE_\mu d\Omega} = N(\theta,E)$$

the muon yield can be calculated from the π-decay in flight between the target and the inside surface of the shielding. Then for muons

$$N(\theta_\mu,E_\mu) = N_\pi(\theta_\pi,E_\pi) \times \text{fractional decay in flight} \quad .$$

We suppose that $\theta_\pi = \theta_\mu$, the multiple scattering is negligible for the muons, and $E_\mu = E_\pi$.

If the traveling time is $T = (1/c)(a/\sin\theta)$ to the inside wall of the shielding, and the pion lifetime is

$$t = \frac{t_0}{\sqrt{1-\beta^2}} = \frac{t_0 E_\pi}{m_\pi c}$$

in the laboratory system, then the muon yield can be expressed as

$$N_\mu = N_\pi \frac{T}{t} = N_\pi \frac{(1/c)\cdot(a/\sin\theta)}{(t_0/m_\pi c^2)\cdot E_\pi} = N_\pi \cdot \frac{3.47 \times 10^{-2}}{\sin\theta \times E_\pi(\text{Bev})}$$

using $a = 2$ meters.

Figure 1 gives the muon yield for one incident electron of energy $E_0 = 45$ Bev. The results in Fig. 1 when multiplied by the radiation length $X_0(\text{gram/cm}^2)$ give the number of muons per steradian per Mev of muon energy.

If one wants to know the muon flux outside of the shielding, one has to take into account the shielding thickness difference in the different directions (θ). This effective shielding thickness is plotted in Fig. 2, using 1 Bev energy loss for muons in 3 meters of earth shielding material. In Fig. 2 the minimum energy (cut-off energy) with which the muon can cross the shielding is plotted vs the angle. Using these cut-off energy values one can see, for example, that no muon below 28.5 Bev can penetrate at $\theta = 6$ degrees. The shadowed area in Fig. 1 shows the different directions of the energy regions in which the muon can contribute to the flux outside of the shielding.

In calculating the flux outside of the shielding, one has to integrate the yield as a function of the energy because

$$\phi_\mu = \frac{N_e}{X_0} \int_{E_{min}} \int_\Omega \frac{d^2Y}{dE\cdot d\Omega}\, dE d\Omega = \frac{1}{X_0} \int_{E_{min}(\theta)} \frac{d^2Y}{dE\cdot d\Omega} \cdot dE \times N_e \times \frac{A}{r^2}$$

at θ direction, where N_e is the number of electrons in the beam ($N_e = 4 \times 10^{14}$);

$r = (a + b)/\sin\theta$;

$X_0 = 52$ gr/cm^2 for carbon;

$A = 10^{-4}$ m^2; and

$E_{min}(\theta)$ is the minimum energy required to penetrate the shield at angle θ (Fig. 2).

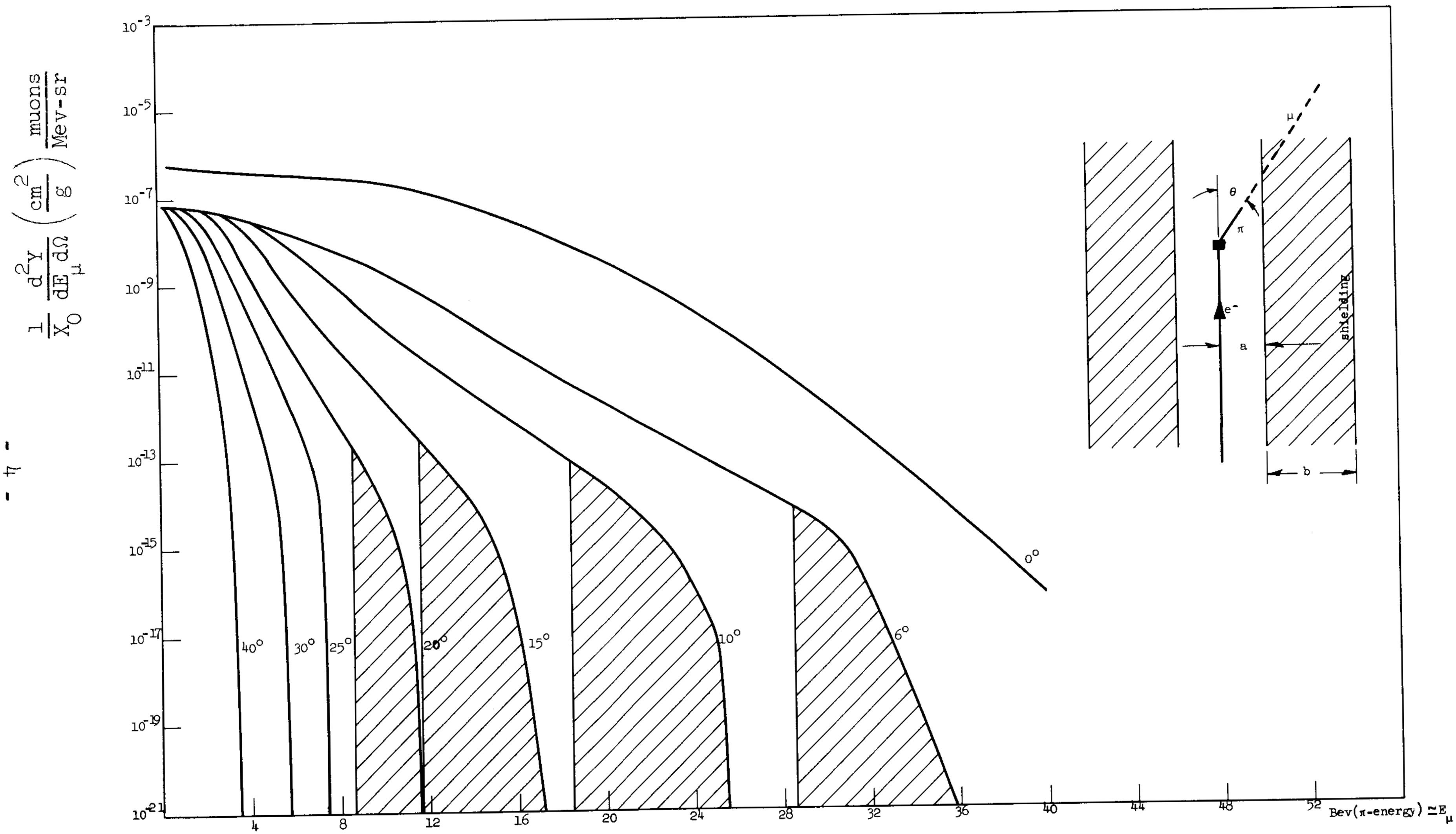


FIG. 1--Muon yield at E_0 = 45 Bev vs muon laboratory energy in Bev.

$$N_\mu = N_\pi \frac{am\pi c^2}{c \sin\theta\, t_\pi E_\pi} = N_\pi \frac{3.47 \times 10^{-2}}{\sin\theta \times E_\pi}$$

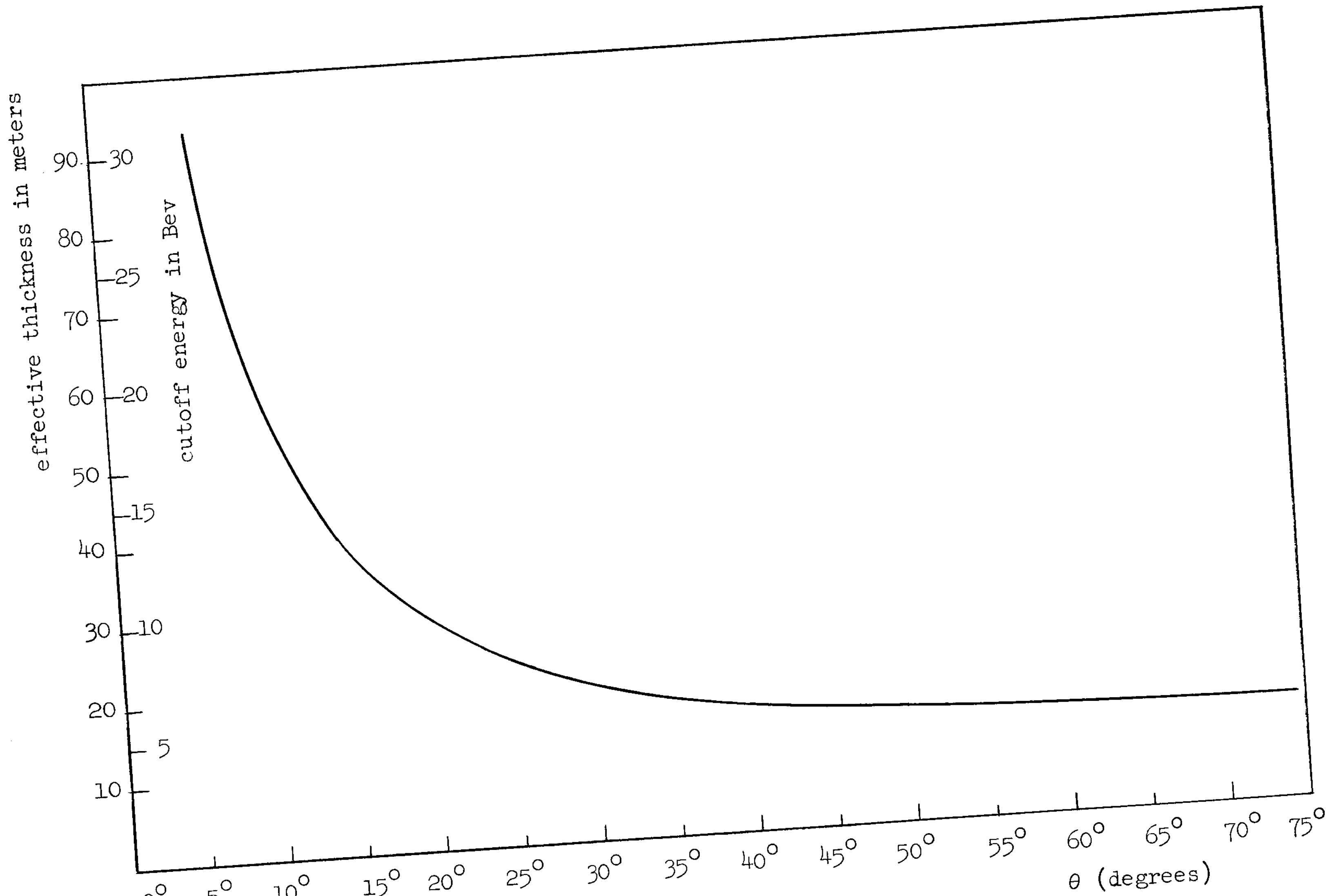
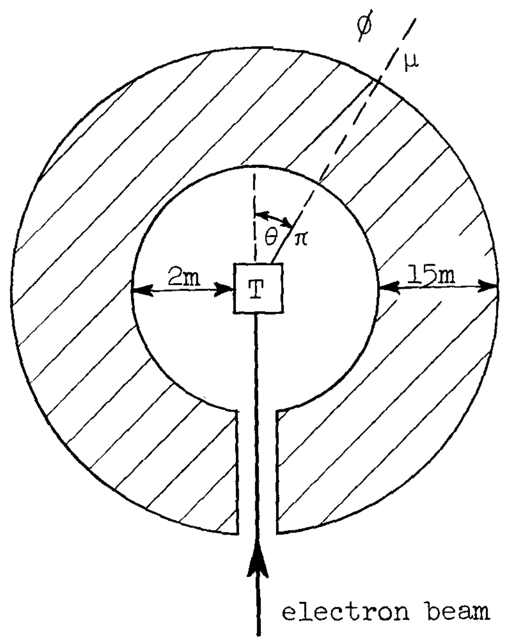


FIG. 2--The effective thickness of the shielding vs θ .
$b = 12$ m

Using a linear instead of logarithmic yield diagram, one can integrate for different angles. The result is given in Fig. 3. One can see that even at the maximum (around 15^{o}) the flux is 0.341 particle/cm^2, which can be compared with the "radiation-worker" tolerance of 7 minimum-ionization muons per cm^2/sec (30 mr per 40 hours).

Let us consider now another shielding geometry which may well be important in the case of the target design in the end station. The shielding geometry now is spherical around the target and has a thickness b = 15 meters, and a = 2 meters, as shown in the following sketch.



spherical shielding geometry

Again the muon yield is

$$N_\mu = N_\pi \frac{T}{t} = N_\pi \times \frac{a}{ct_0} \frac{m_\pi c^2}{E_\pi} = N_\mu \times \frac{3.47 \times 10^{-2}}{E_\mu (\text{Bev})}$$

which is shown in Fig. 4. Integrating the shadowed area for a carbon target, one gets the flux of muons vs angle shown in Fig. 5.

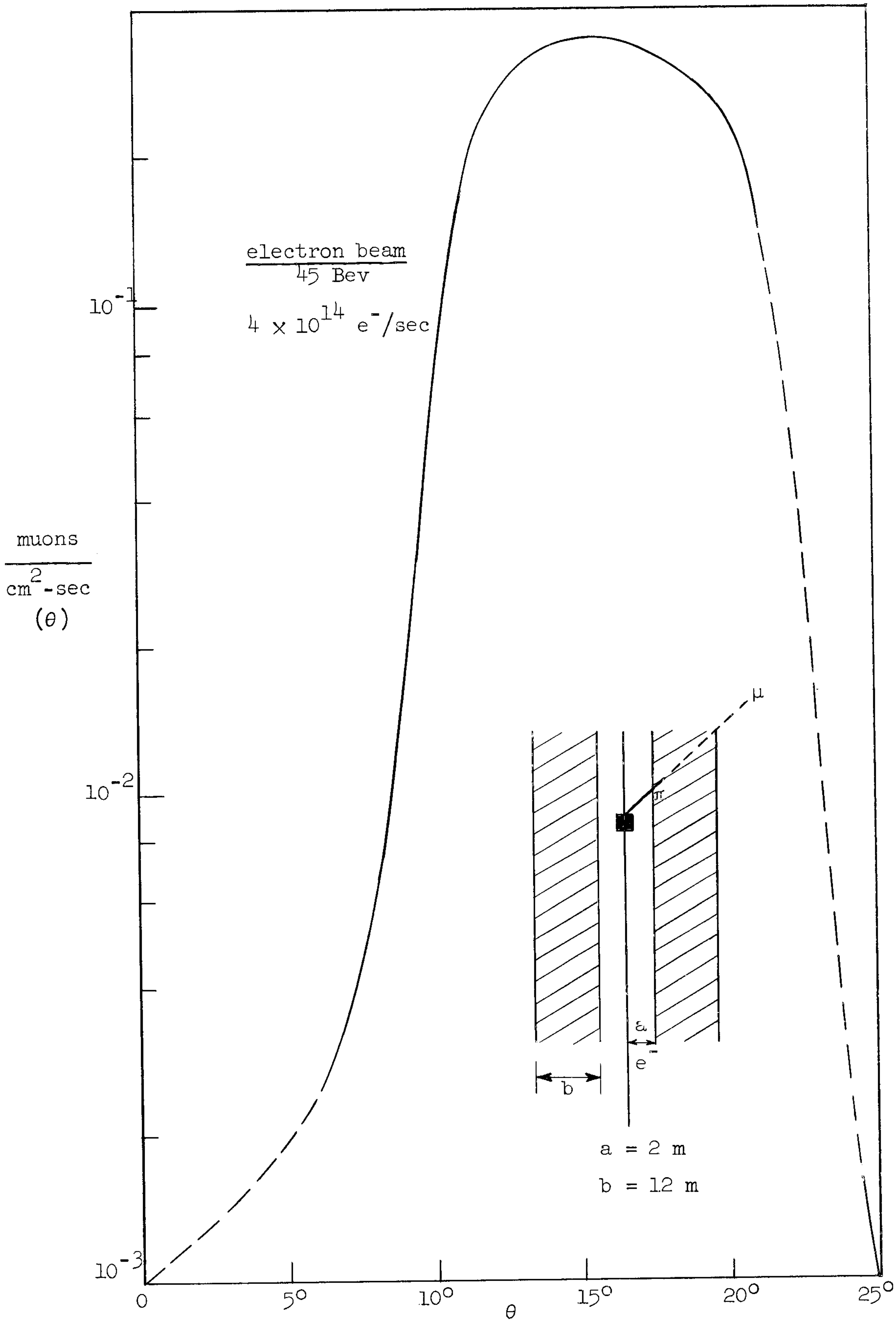


FIG. 3--Muon flux vs angle.

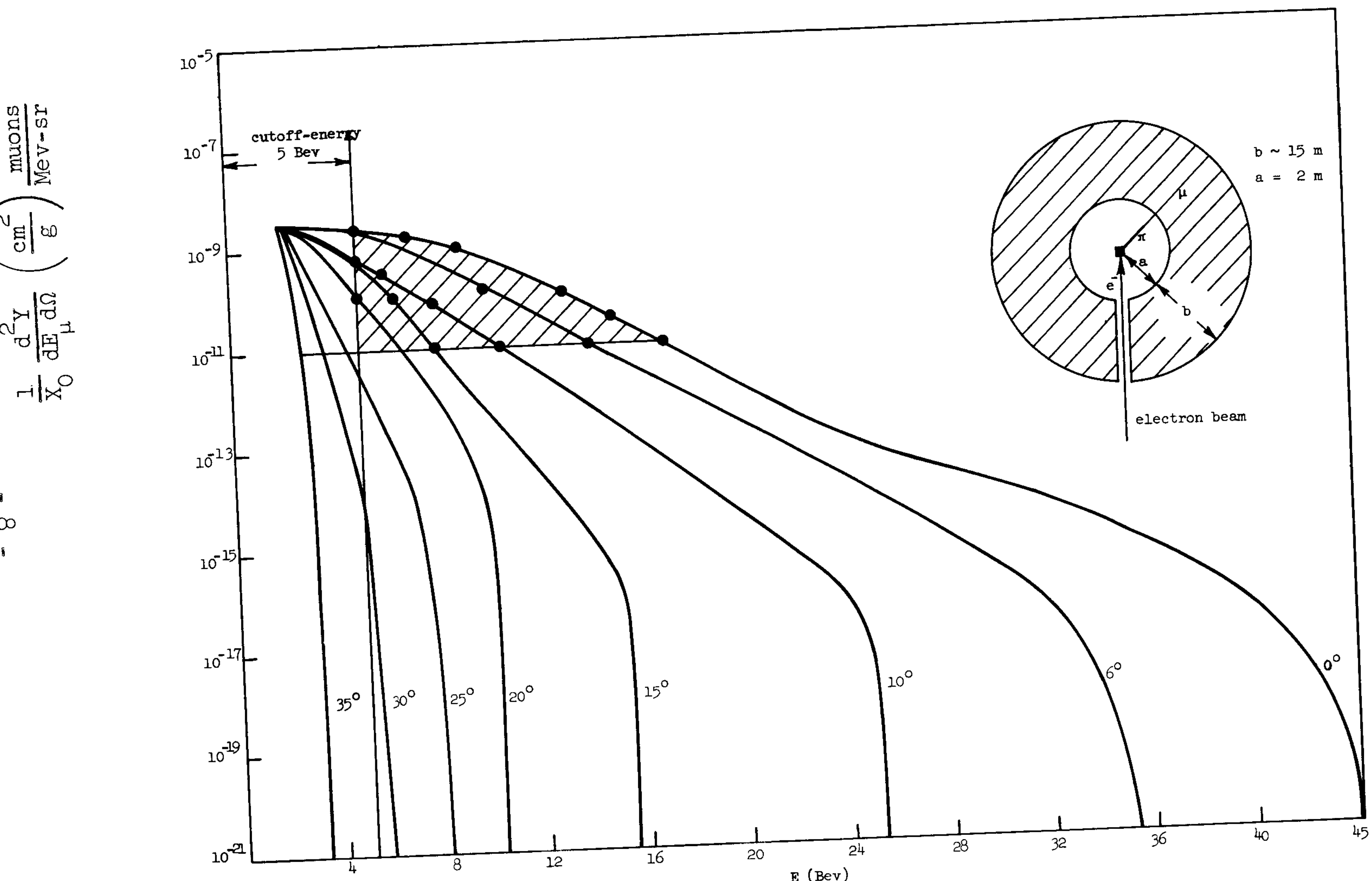


FIG. 4--Muon yield at E_0 = 45 Bev vs muon laboratory energy in Bev.

$$N_\mu = N_\pi \times \frac{34.7 \times 10^{-3}}{E_\mu(\mathrm{Bev})}$$

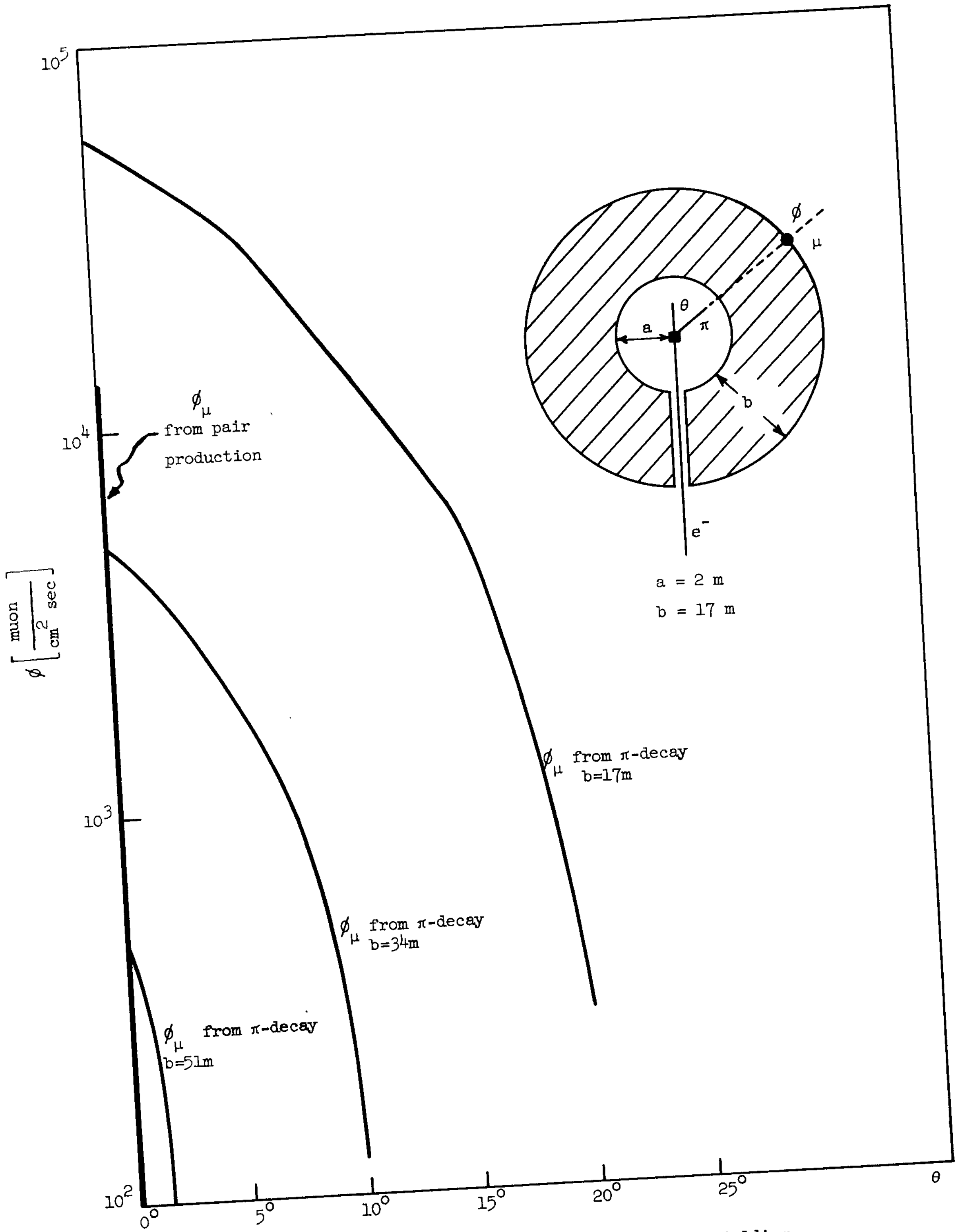


FIG. 5--Muon flux vs angle with spherical shielding.

III. ELECTROMAGNETIC PAIR PRODUCTION YIELDS

A very small fraction of the energy of an electromagnetic cascade goes into the pair production of particles heavier than electrons. The order of magnitude* of this fraction is $(m_e/m)^2$, where m is the mass of the produced particles. However, because these pairs are produced into an opening angle $\sim (mc^2/E)$, where E is the energy of the secondary, even a small production can contribute to the in-line shielding problem (Fig. 6).

The pair production yield of muons,* in units of number per Bev/c per steradian per 50 Bev electrons of both positive and negative muons, is given as follows (Fig. 7):

Secondary Energy	45 Bev	40 Bev	25 Bev	10 Bev	5 Bev
Muons	1.6×10^{-4}	5.2×10^{-4}	3.7×10^{-3}	6.4×10^{-3}	5.6×10^{-3}

For example, at 40 Bev energy these muons are produced into a mean angle of

$$M_\mu c^2/E_\mu \approx 0.1/40 = 2.5 \times 10^{-3} \text{ radians}$$

or

$$\pi \times (1/400)^2 \text{ steradian} = 1.96 \times 10^{-5} \text{ steradian}$$

If the thickness of the spherical target shielding b is 50 ft of earth (density 1.8), or 90 radiation lengths, then the multiple scattering angle is

$$\sqrt{\left\langle \theta^2 \right\rangle_{cw}} = \left(\frac{E_s}{Bcp} \right) \sqrt{\frac{b}{X_0}} = 5 \times 10^{-3} \text{ radians}$$

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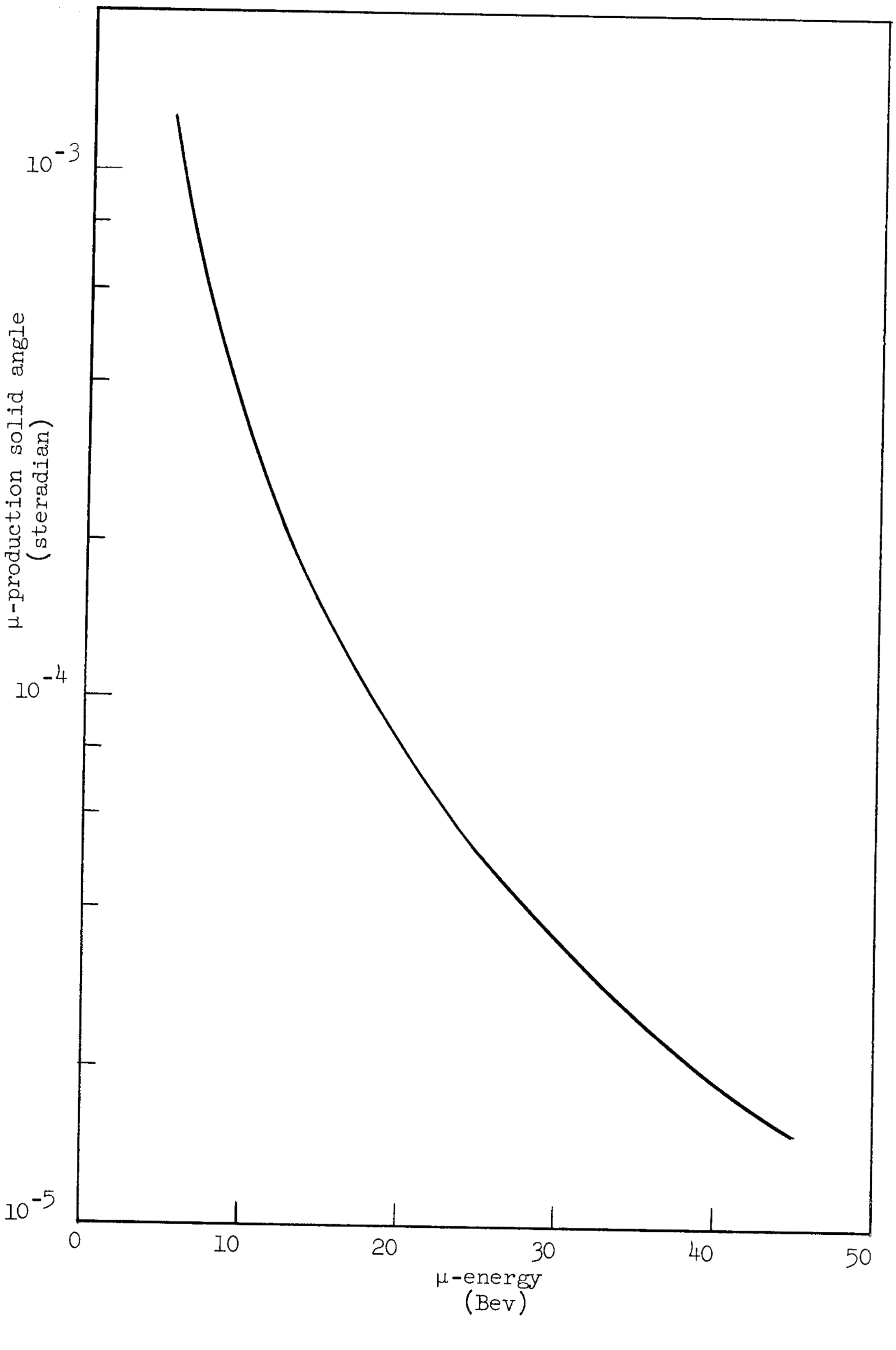
*D. M. Ritson, "Pure Electromagnetic Pair Production Yields for Mu-Mesons," M Report No. 200-3A, Stanford University, July 1960.



10^{-3}
10^{-4}
10^{-5}
μ-production solid angle
(steradian)
0
10
20
30
40
50
μ-energy
(Bev)

FIGURE 6

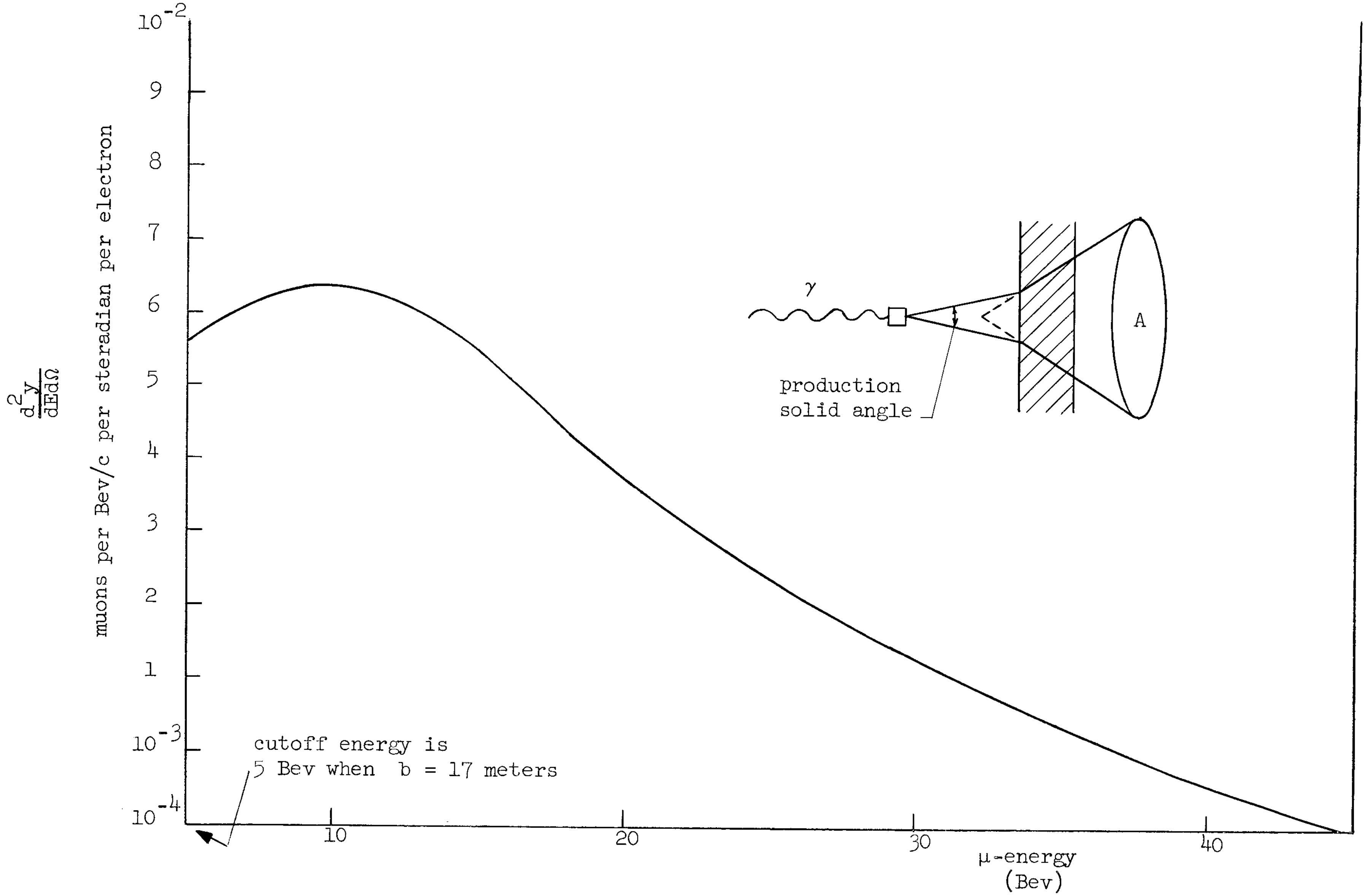


FIG. 7--Muon pair-production yield vs μ-energy.

which is larger than the production angle. At a distance of 1000 ft the area occupied by the beam will be

$$\pi \times \left[\left(5 \times 10^{-3}\right) \times 1000 \times 30\right]^2 \mathrm{cm}^2 = 7.1 \times 10^4 \mathrm{cm}^2 = A$$

and the muon flux from the integration of the yield curve will be

$$\int_{E_{min}}^{E_{max}} \frac{d^2Y}{d\Omega dE} dE \times \frac{\text{Production solid angle}}{A} \times N_e = \phi_\mu$$

For any scattering geometry the production solid angle and A are both proportional to $1/E^2$. Therefore, one can write:

$$N_e \times \frac{\text{Production solid angle}}{A} \int_{E_{min}}^{E_{max}} \frac{d^2Y}{d\Omega dE} dE = \phi_\mu$$

Then

$$\phi_\mu = 1.32 \times 10^3 \text{ muons/cm}^2 \text{ sec}$$

(see Fig. 5) if N_e is again 4×10^{14} electrons/sec. This flux is smaller by a factor of 4 than the flux from the pion decay at 0°.

The conclusion in the case of target shielding (spherical shape) is that the μ-flux from π-decay can be reduced by decreasing the distance between the target and the shielding wall. A good pion absorber, such as a heavy metal, should be located as close as possible behind the target in order to remove as many pions as possible through nuclear absorption.* The absorber should be at least 10 absorption-mean-

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*W. K. H. Panofsky, Conf. on Shielding of High Energy Accel., New York, April 1957.

free-paths long in order to reduce the pion flux to a value such that subsequent π-μ decays of the remaining pions will not add appreciably to the muon flux. The pion absorber should be iron of thickness sufficient to reduce the muon flux to radiation-worker tolerance outside the shielding wall.

The μ-flux contribution from pair production can be made smaller only when the total shielding thickness b is longer:

$$\phi_{\mu(\text{pair})} \sim \frac{1}{A} \sim \frac{1}{b}$$

Because the μ-flux is very high close to the axis and decreases rapidly with increasing θ, a tapered shield would be the most desirable for the target with increasing thickness in the forward direction.

In the case of the side (cylindrical) shielding the muon flux does not represent a special problem, and the presently specified shielding thickness (35 feet) seems adequate.