M-227
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DEUTERON MODEL CALCULATION OF PHOTONUCLEON YIELDS

By
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Levinger's[1] deuteron model for the high energy nuclear photo-effect has provided a successful explanation of experiments where photon energies between 90 Mev and 340 Mev are employed[2]. The model states that the nuclear cross-section is a multiple of the deuteron photo cross-section. We shall assume that this model is valid at the high photon energies expected in the proposed Stanford accelerator and shall calculate the photonucleon yields produced by the soft-shower photons.

The deuteron cross-section will be taken to be isotropic at high photon energies[3], although this is not strictly true. Experimental values of the total cross-section are shown in Fig. 1. These results were all obtained at Cal Tech.[3,4] In Wilson's[5] shielding calculations for the Cambridge Electron Accelerator, the total deuteron cross-section is represented by

$$\sigma_0 (E_\gamma) = \begin{cases} 70 \text{ microbarns}; & 50 \text{ Mev} < E_\gamma < 300 \text{ Mev} \\ 6/E_\gamma^2 \text{ microbarns}; & E_\gamma > 300 \text{ Mev} \\ (E_\gamma \text{ in Bev}) & \end{cases} \quad (1)$$

[1] J. S. Levinger, Phys. Rev. 84, 43 (1951)

[2] Odian et al, Phys. Rev. 102, 837 (1956)

[3] See for example: J. C. Keck and A. V. Tollestrup, Phys. Rev. 101, 360 (1956)

[4] R. Gomez (private communication).

[5] R. Wilson, "A Revision of Shielding Calculations," CEA-73, May 12, 1959.

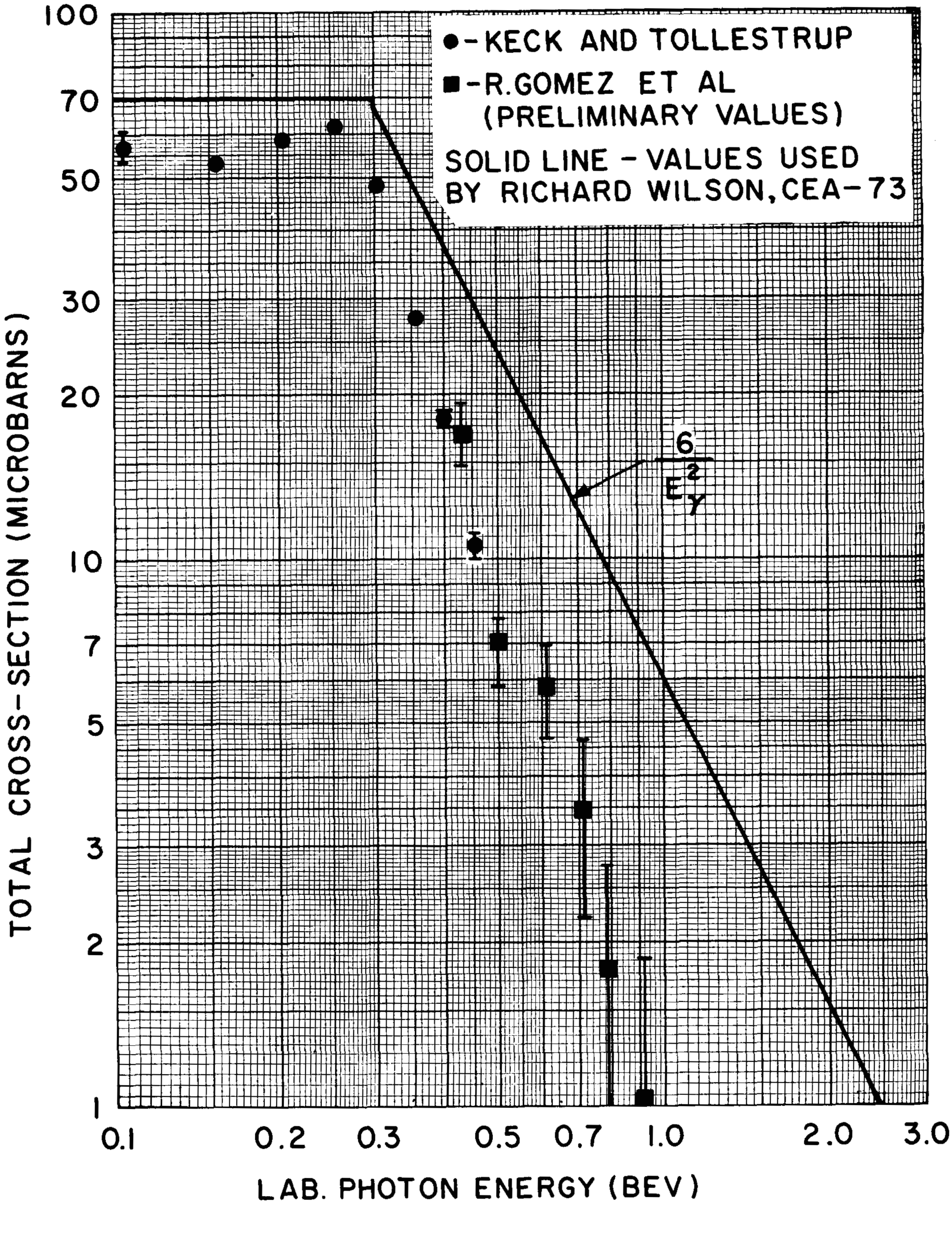

FIG. 1 Deuteron photoeffect cross section vs laboratory photon energy.

This cross-section is also shown in Fig. 1 and is clearly a conservative representation over the range of photon energies shown. We use Eq. (1) in the calculations to follow.

A further assumption is that a photon of energy E_γ yields a unique value of laboratory photonucleon energy $E_n(E_\gamma, \theta)$ at a given laboratory angle θ. This means that the momentum distribution of nucleons within the nucleus is ignored. Hence E_n is calculated using relativistic kinematics appropriate to the situation where a photon of energy E_γ is incident on a "quasi-deuteron", and we finally have two photonucleons in the laboratory. In Fig. 2 the photonucleon energies E_n are plotted vs laboratory angle θ for several values of E_γ. Figure 3 shows the angle transformation between the center-of-mass and laboratory systems.

The distribution of photonucleons in the laboratory is given by[6]

$$W(E_n, \theta) = W'(E_\gamma, \theta')\left[\left(\frac{\partial E_n}{\partial E_\gamma}\right)_\theta \left(\frac{\partial(\cos\theta)}{\partial(\cos\theta')}\right)_{E_\gamma}\right]^{-1} \qquad (2)$$

where $W(E_n, \theta)$ is the photonucleon spectrum in the laboratory (photonucleons per second per unit laboratory energy E_n per unit laboratory solid angle), and $W'(E_\gamma . \theta')$ is the rate of production of these photonucleons which occur in the corresponding unit solid angle in the center of mass and are due to photons with laboratory energies between E_γ and $E_\gamma + dE_\gamma$. The derivatives in Eq. (2) are calculated from the Lorentz transformations giving $E_n(E_\gamma, \theta)$ and $\theta(E_\gamma, \theta')$.

All that remains, then, is the problem of calculating $W'(E_\gamma, \theta')$, the distribution in the center of mass. As mentioned before, the photons arise from a soft shower. The spectrum is given by the familiar

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[6] You'll have to figure out how to get this expression all by yourself (hint: see M-226).

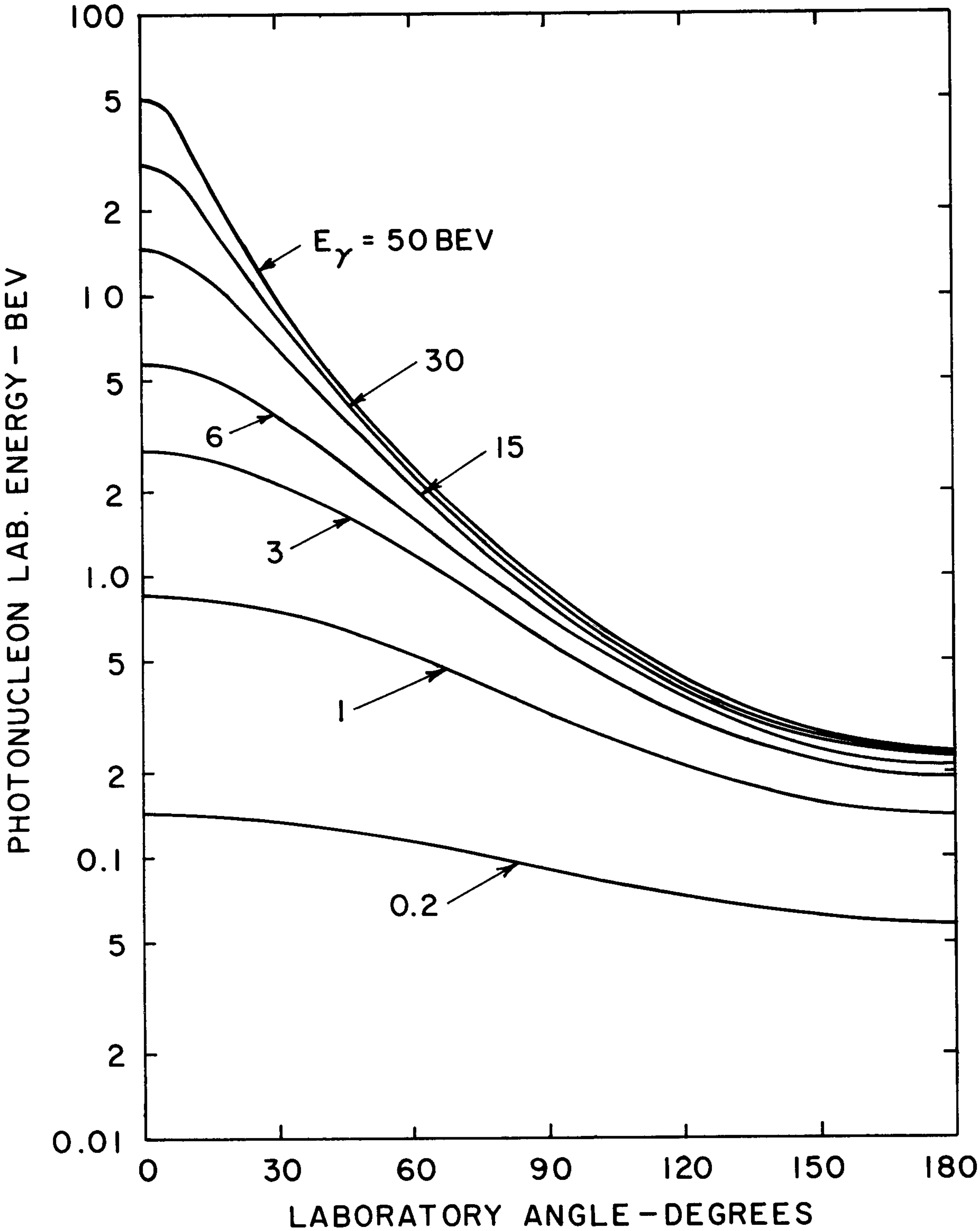

FIG. 2 Deuteron model photoeffect kinematics.

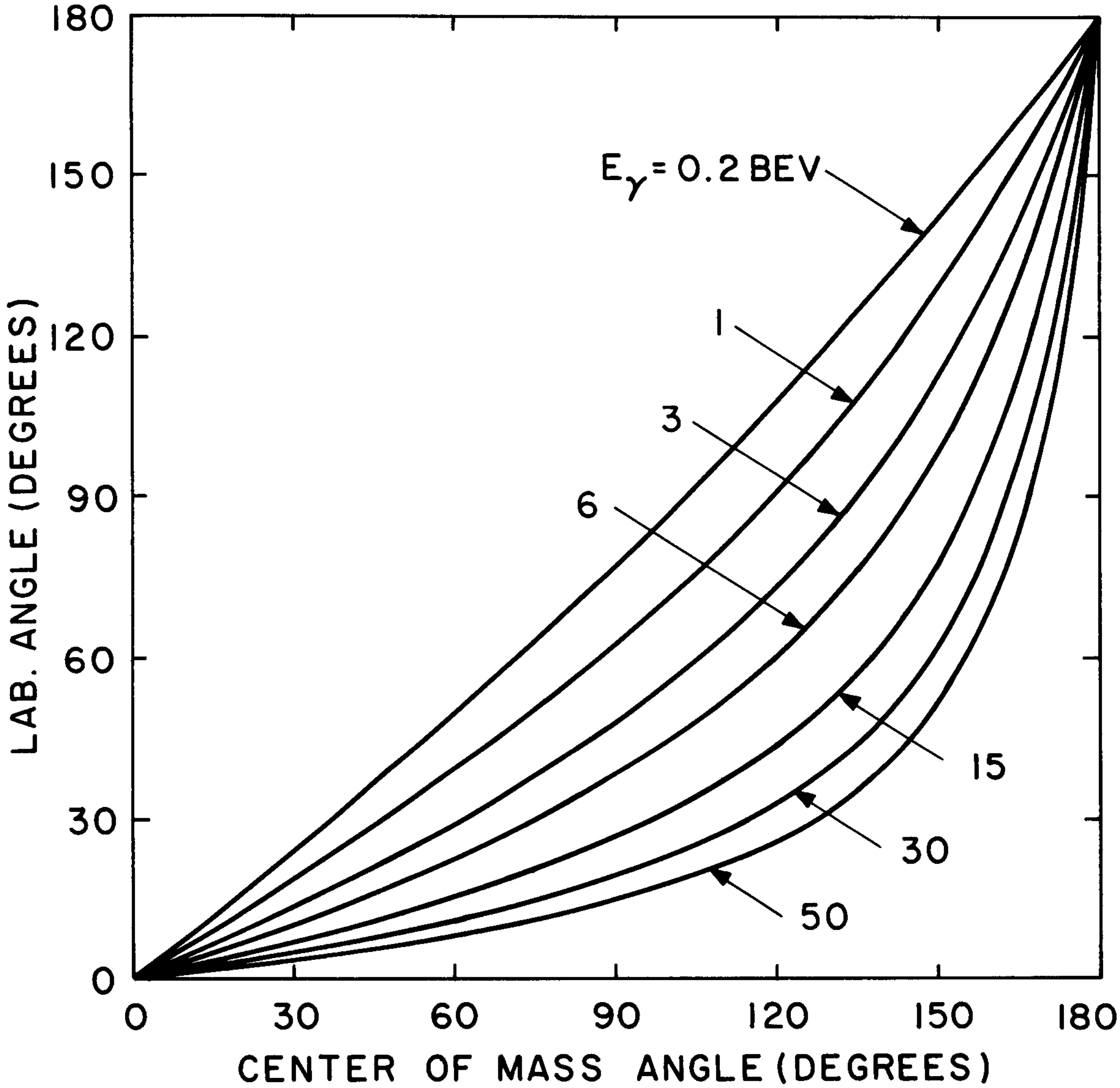

FIG. 3 Angle transformation for quasi-deuteron photoeffect kinematics.

track length expression[7]

$$(0.572)\,\frac{E_o}{E_\gamma^2}\,dE_\gamma \qquad (3)$$

where E_o is the kinetic energy of the incident electron. This expression gives the track length in radiation lengths of the photons with energies between E_γ and $E_\gamma + dE_\gamma$. Multiplication of Eq. (3) by (X_o/ρ) gives the track length in centimeters, where X_o is the value of the radiation length in grams/cm^2, and ρ is the target density in grams/cm^3. The yield $W'(E_\gamma, \theta')$ is then given by

$$W'(E_\gamma, \theta') = N_1(0.572)\,\frac{X_o}{\rho E_\gamma^2}\,\sigma_a(E_\gamma, \theta')\,E_0 \qquad (4)$$

where N_1, is the number of nuclei per cm^3 in the target, and $\sigma_a(E_\gamma, \theta')$ is the photoeffect differential cross-section of these nuclei. N_1 is given by $(N_o\rho/A)$, where N_o is Avogadro's number, and A is the atomic weight.

According to the deuteron model,

$$\sigma_a(E_\gamma, \theta') = \alpha\,\frac{NZ}{A}\,\sigma_d(E_\gamma, \theta') \qquad (5)$$

where N is the number of neutrons in the nucleus, Z is the atomic number, A is the mass number, and α is a constant which is different for all nuclei[8]. We shall take α to be 1.5 for all nuclei.

Equations (2), (4) and (5) are now combined to give

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[7]See for example: B. Rossi, High Energy Particles, Prentice-Hall, New York, 1952, p. 244.

[8]See references 2 and 5 for further discussion.

$$W(E_n,\theta) = (0.572)(1.5)\,\frac{N_o X_o E_o}{A E_\gamma^2}\left(\frac{NZ}{A}\right)\frac{\sigma_o(E_\gamma)}{4\pi}\left[\left(\frac{\partial E_n}{\partial E_\gamma}\right)_\theta \left(\frac{\partial(\cos\theta)}{\partial(\cos\theta')}\right)_{E_\gamma}\right]^{-1} \qquad (6)$$

where Eq. (1) for $\sigma_o(E_\gamma)$ will be used in the calculations based on Eq. (6). This expression gives the number of neutrons in unit solid angle in the laboratory per unit nucleon kinetic energy E_n when one electron is incident on the target.

We are mainly interested in the yields from a copper target, since many electrons are expected to strike the accelerator walls on their long journey.

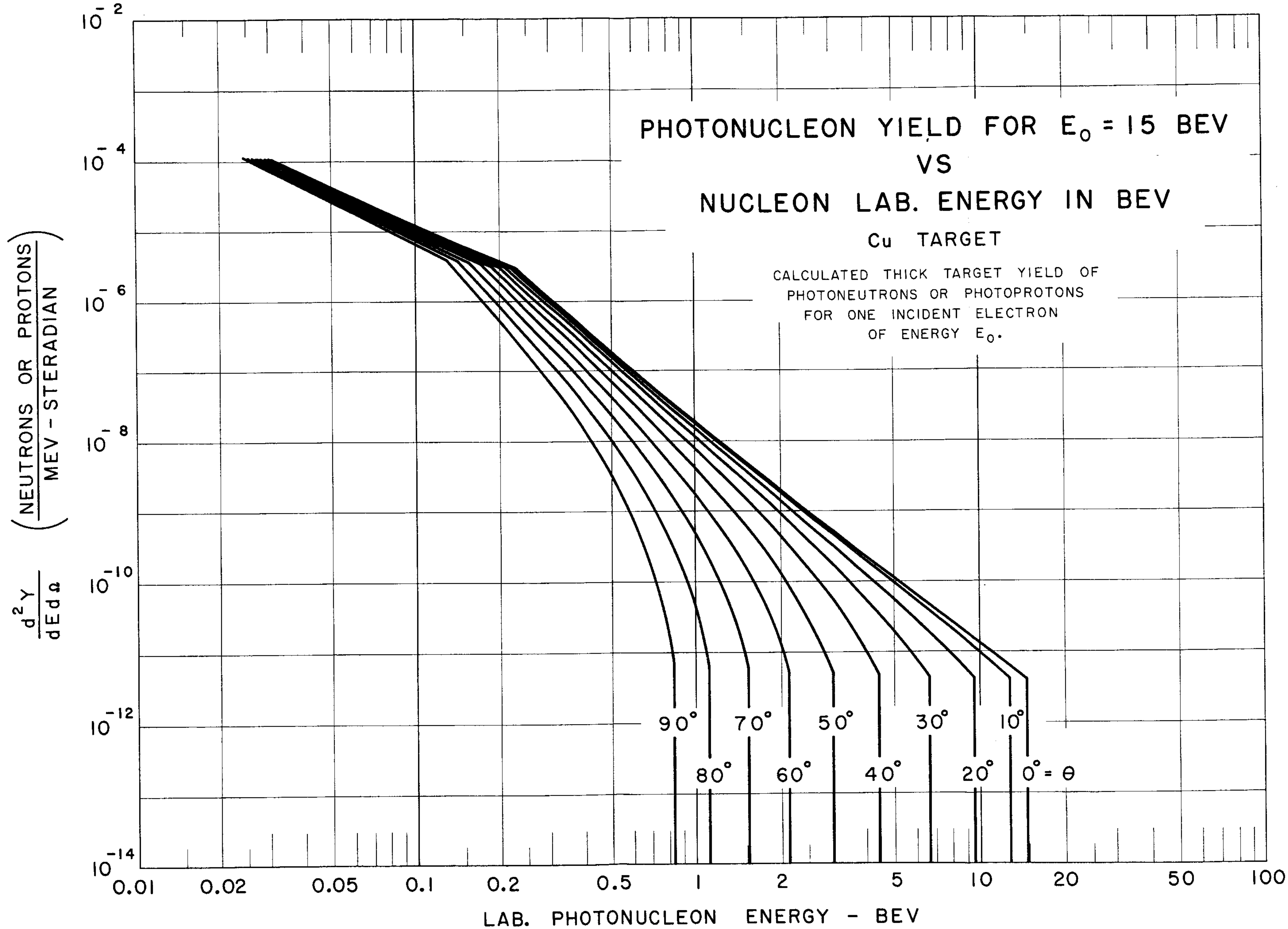
Figures 4, 5 and 6 show the results obtained for E_o = 15, 30 and 45 Bev in the case of a copper target.

Example:

What is the flux of photoneutrons at laboratory angle $\theta = 40^\circ$ with energies between E_n = 2 Bev and 2.10 Bev when a 30 Bev, one micro-ampere electron beam strikes a copper target?

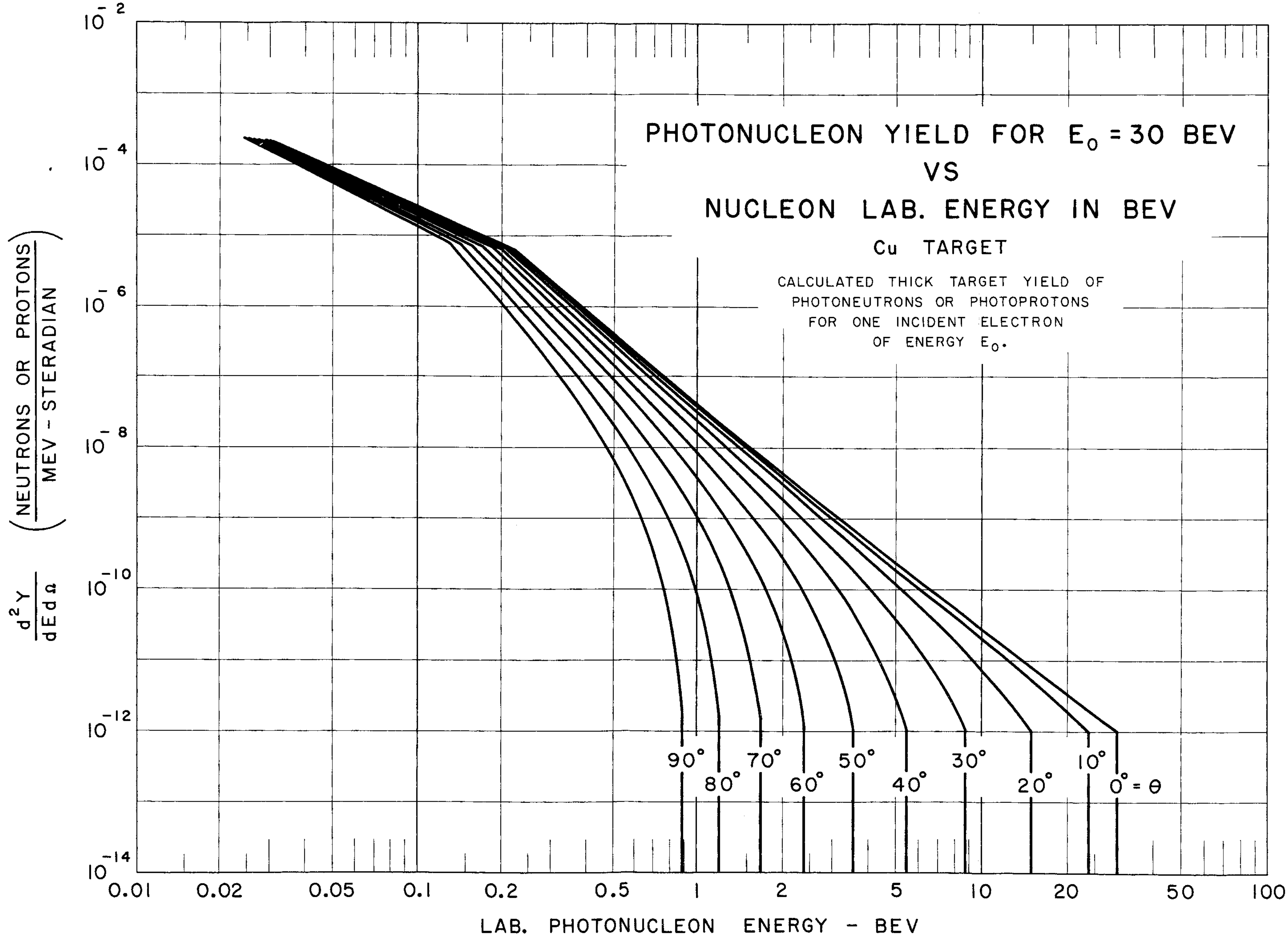
Use Fig. 5, find 10^{-9} neutrons/Mev-steradian for one incident electron. The flux is then

$$\frac{100}{1.6 \times 10^{-19}} \times 10^{-9} = 6.25 \times 10^{11} \text{ neutrons/steradian}$$



PHOTONUCLEON YIELD FOR E_0 = 15 BEV
VS
NUCLEON LAB. ENERGY IN BEV
Cu TARGET
CALCULATED THICK TARGET YIELD OF PHOTONEUTRONS OR PHOTOPROTONS FOR ONE INCIDENT ELECTRON OF ENERGY E_0.
$\frac{d^2Y}{dE\,d\Omega}$ $\left(\frac{\text{NEUTRONS OR PROTONS}}{\text{MEV - STERADIAN}}\right)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
0.01
0.02
0.05
0.1
0.2
0.5
1
2
5
10
20
50
100
90°
80°
70°
60°
50°
40°
30°
20°
10°
0° = θ
LAB. PHOTONUCLEON ENERGY - BEV

FIG. 4



PHOTONUCLEON YIELD FOR E_0 = 30 BEV
VS
NUCLEON LAB. ENERGY IN BEV
Cu TARGET
CALCULATED THICK TARGET YIELD OF PHOTONEUTRONS OR PHOTOPROTONS FOR ONE INCIDENT ELECTRON OF ENERGY E_0.
$\frac{d^2Y}{dE\,d\Omega}$ $\left(\frac{\text{NEUTRONS OR PROTONS}}{\text{MEV - STERADIAN}}\right)$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
90°
80°
70°
60°
50°
40°
30°
20°
10°
0° = θ
0.01
0.02
0.05
0.1
0.2
0.5
1
2
5
10
20
50
100
LAB. PHOTONUCLEON ENERGY - BEV

FIG. 5

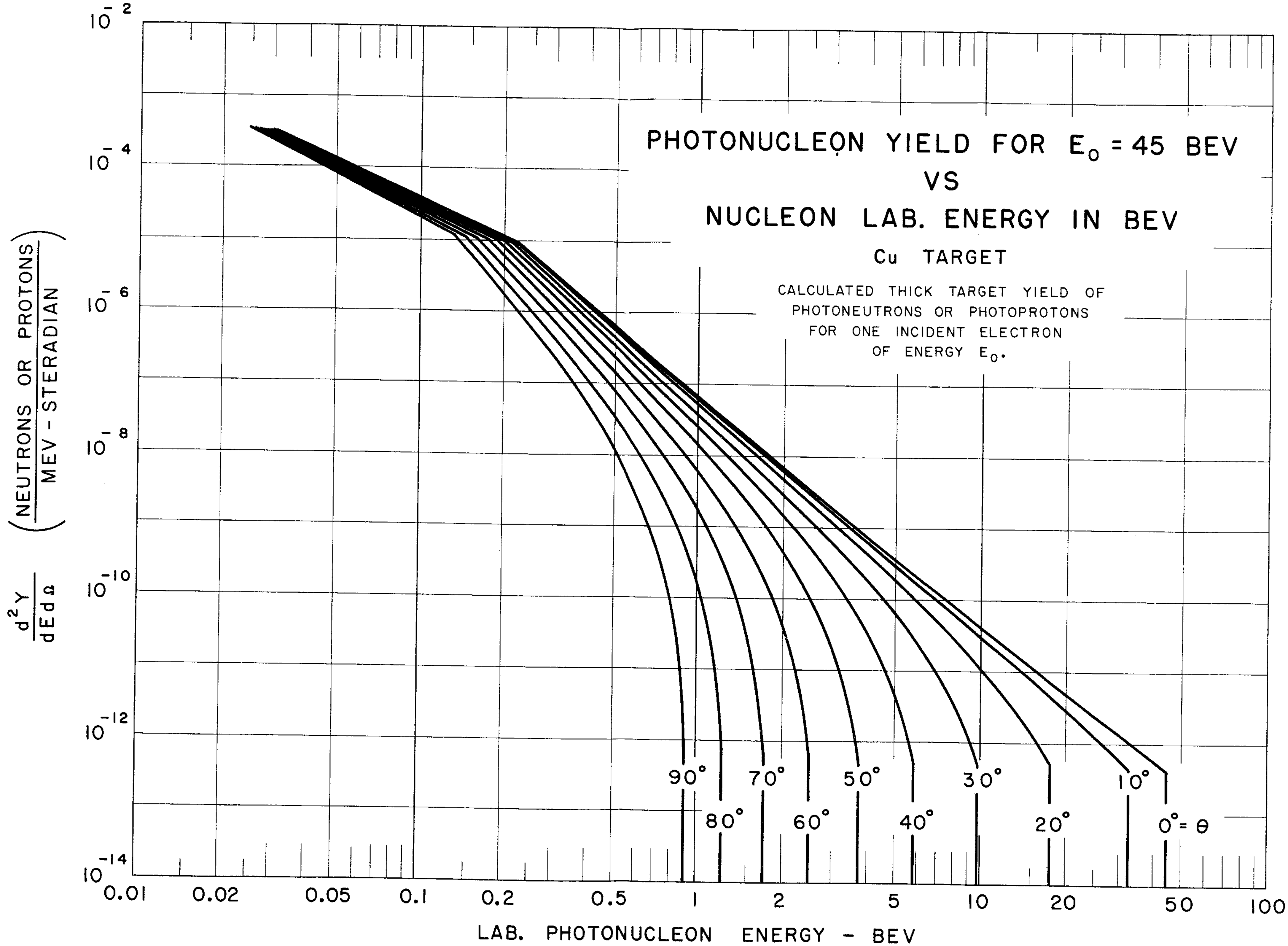

PHOTONUCLEON YIELD FOR E0 = 45 BEV
VS
NUCLEON LAB. ENERGY IN BEV
Cu TARGET
CALCULATED THICK TARGET YIELD OF
PHOTONEUTRONS OR PHOTOPROTONS
FOR ONE INCIDENT ELECTRON
OF ENERGY E0.
d²Y/dEdΩ (NEUTRONS OR PROTONS / MEV - STERADIAN)
10⁻²
10⁻⁴
10⁻⁶
10⁻⁸
10⁻¹⁰
10⁻¹²
10⁻¹⁴
0.01
0.02
0.05
0.1
0.2
0.5
1
2
5
10
20
50
100
90°
80°
70°
60°
50°
40°
30°
20°
10°
0° = θ
LAB. PHOTONUCLEON ENERGY - BEV

FIG. 6