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INTRODUCTION TO RELIABILITY

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RELIABILITY

Summary: A few basic definitions are given and some elementary concepts are made. The long-range cost of equipment is deduced and an advisable purchasing procedure related to reliability is examined.

Reliability is the probability of survival of a given component or equipment. This short sentence defines the problem in its mathematical and practical aspects.

A characteristic common to both animated or lifeless objects is that they are certainly going to fail, i.e., loose their initial properties which make them useful. This general statement comprising without distinctions the two classes of entities, so different in nature, like the animated and the lifeless, allows a first graphic description of that phenomenon which is "failure". A diagram known to insurance statisticians is the one given in Fig. 1, representing the failure rate during the whole useful lifespan of anything subject to use, like machinery, its components, or an animal being. The diagram is the result of experimental observation and therefore it is statistical in nature, i.e., it is a diagram of averages.

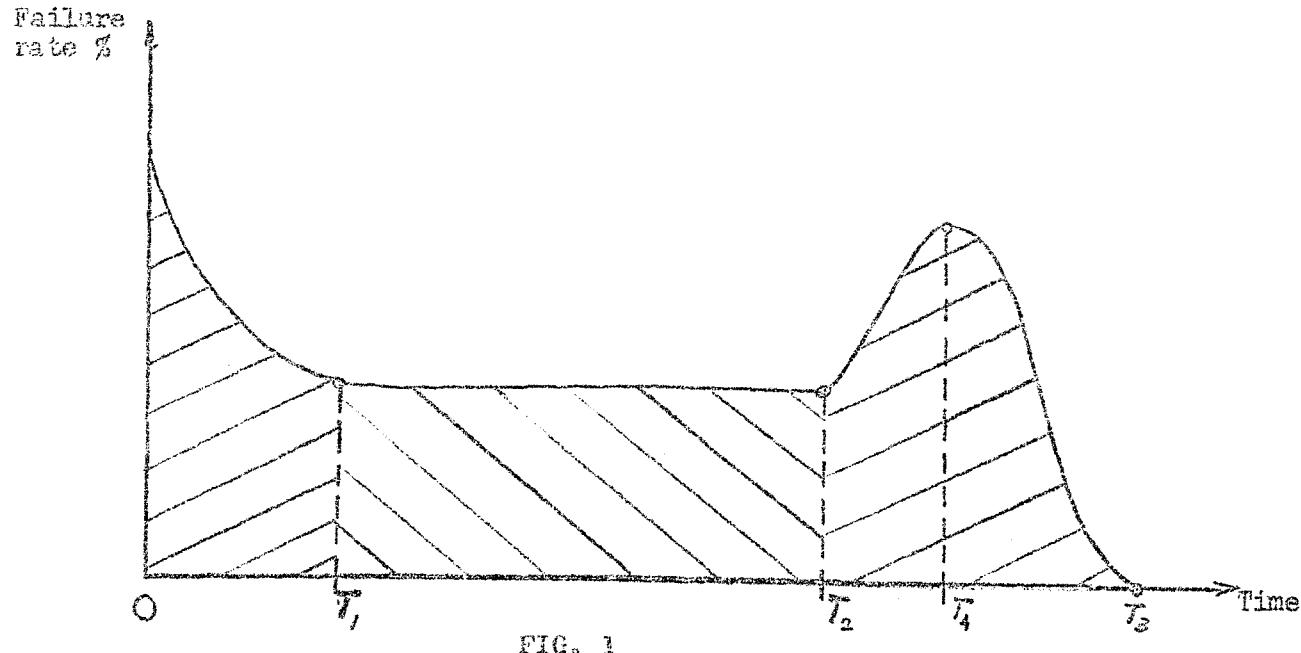


FIG. 1

Along the y-axis number of failures at a given time are indicated in percent of the total number of subjects under observation. Three characteristic regions may be distinguished. The first goes from time zero (beginning of operation or birth of the individual) to time T_1 . During this period there is a high rate of failures due to natural selection. The successive period going from time T_1 to T_2 is characterized by a nearly constant rate of failures. This is the useful life span of the equipment, component part, or animal. The successive time period, going from T_2 to T_3 , is earmarked by an increasing failure rate reaching a peak at a time T_4 and then decreasing gradually until all the initially existent individuals are "dead".

Objective of reliability studies is mainly to determine the useful life (T_1 to T_2 in Fig. 1) of equipment if statistical data on failure rates of the components in given conditions (temperature, storage, voltage, etc.) are known. Still better, the "mean life", defined later, is determined.

In order to define more precisely the important concept of "failure rate", let us write the expression of useful life probability of N individuals. It is an exponential function of time t :

$$P = e^{-\lambda t} \quad (1)$$

where $\lambda = N/T$ is the statistical failure rate, frequently called, "random failure rate", and T is the time interval during which N failures occur. Usually λ is given in percent per 1,000 hours, the only reason for this type of unit definition being that not too many decimals are involved. Obviously, the failure rate may be given in "pieces per year", too, and this may be useful in certain instances.

Now, reliability is by definition equal to the life expectancy given under (1) so that we write:

$$R \triangleq P = e^{-\lambda t} \quad (2)$$

At birth, reliability (probability of life) is unimpaired and is represented by the numerical value 1. This is total reliability. A component which failed has zero reliability. These are the limits of variation of R .

Equipment reliability is the resultant of component reliability. Reliability being defined as it is under (2), i.e., as a probability, it is obvious that the resultant reliability is the product of the single component reliabilities:

$$R_r = R_1 R_2 R_3 \dots = \prod_{i=1}^n R_p = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \dots) t} \quad (3)$$

so that the resultant failure rate is the sum of the component failure rates:

$$\lambda_r = \lambda_1 + \lambda_2 + \lambda_3 + \dots = \sum_{i=1}^n \lambda_p \quad (4)$$

On the other hand, individual component failure rates are the resultant of the intrinsic failure rate, the failure rate due to effects of usage in a circuit or in a machine and of the failure rate determined by the environment in which the component operates:

$$\lambda_p = \lambda_{INT} + \lambda_{CIRC} + \lambda_{ENV} \quad (5)$$

At this point it must be explained that the resultant reliability given under (3) as a product of individual reliabilities applies to the case of components "in series". This term is used to indicate those components, for which any failure causes a total interruption of the operation of the system of which they are a part.

Components are "in parallel" if in case of failure the functions of one of them are overtaken by another. Normally, this is not due to coincidence but to a planned redundancy. Such components are materially in parallel in an electric circuit. The reliability is improved inasmuch as the probability of simultaneous failure of both components in parallel is greatly reduced. A mathematical expression giving the value of the reliability in this case is:

$$R = 1 - (1 - R_1)(1 - R_2) \quad (6)$$

R_1 and R_2 are the reliabilities of the two components in parallel, both able to perform the same function. $1 - R_1$ and $1 - R_2$ are the related "unreliabilities" (probabilities of failure). These two probabilities multiply in order to give the probability of failure of their parallel connection and the difference of the product with respect to 1 (total reliability) supplies the composite reliability of the parallel combination.

It is worthwhile to illustrate the increase of reliability of parallel combinations with a numerical example. So, if two identical components are in parallel, for example, two relays, each characterized by a reliability factor 0.9, the composite reliability according to (6) increases to 0.89, i.e., the probability of failure during the useful lifespan is reduced from 10% to only 1%. Thus, redundancy is one of the most effective means of failure prevention.

Now, long-range cost of equipment will be analyzed in terms of reliability. Generally, it is not reliability itself which appears in practical calculations, but the failure rate related to the former through:

$$-\log R = \lambda t \quad (7)$$

Let us consider the case of an equipment built up of n different components. Each component p is represented in the complete set by N_p pieces so that the total number of components used is:

$N = N_1 + N_2 + N_3 + N_4 + N_p + \dots + N_n$. Each component p is characterized by a failure rate λ_p during its useful lifespan T_p .

Furthermore, a "mean life" t_m is defined such that at the end of it only 37% of the initial components or individuals remain:

$$t_m = \frac{1}{\lambda_p} \text{ for which } R = \frac{1}{e} = 37\% \quad (8)$$

We will now consider a period of t years during which the equipment is planned to be used and we will call:

$$h_p = \frac{t}{t_m} \quad (9)$$

the ratio of time intervals comprised in t years to the mean life of the component p in question. This ratio indicates the number of times that component, as an average, must be substituted during t years.

Furthermore, k_p is the unit cost of component p , so that this later is represented in the total initial cost of the equipment C_{in} by the ratio:

$$r_p = \frac{k_p N_p}{C_{in}} \quad (10)$$

With these definitions it is easy to see that the total cost of the equipment in case of failures of component p only during t years of use will be:

$$C_{tot} = C_{in} (1 + b_p r_p \lambda_p t) \quad (11)$$

while, if all the different components are taken into consideration, each with its own failure rate λ_p and the related coefficients b_p and r_p ,

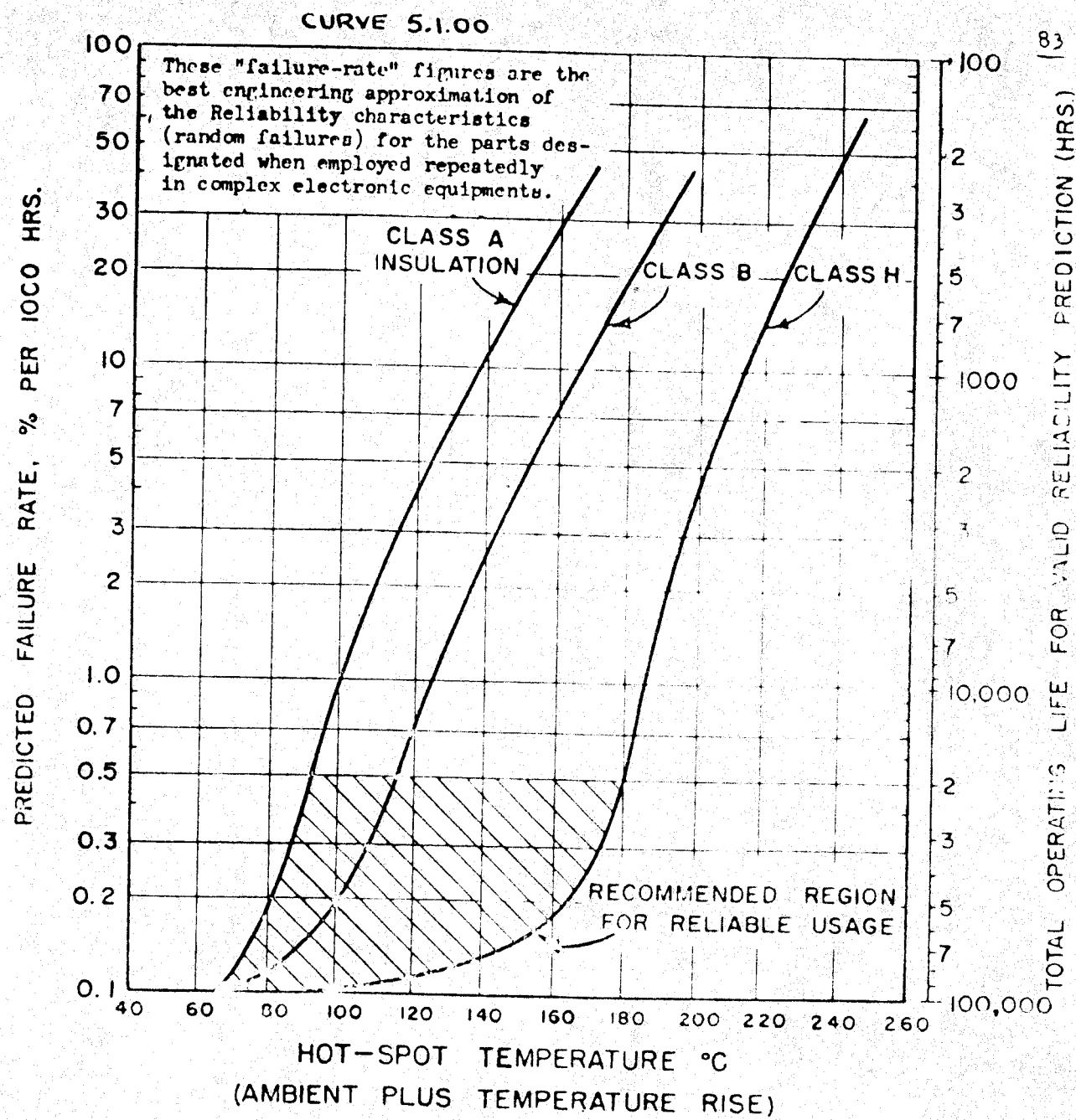


FIG. 3

PREDICTED FAILURE RATES FOR
TRANSFORMERS, CHOKES AND COILS

CURVE 4.1.10

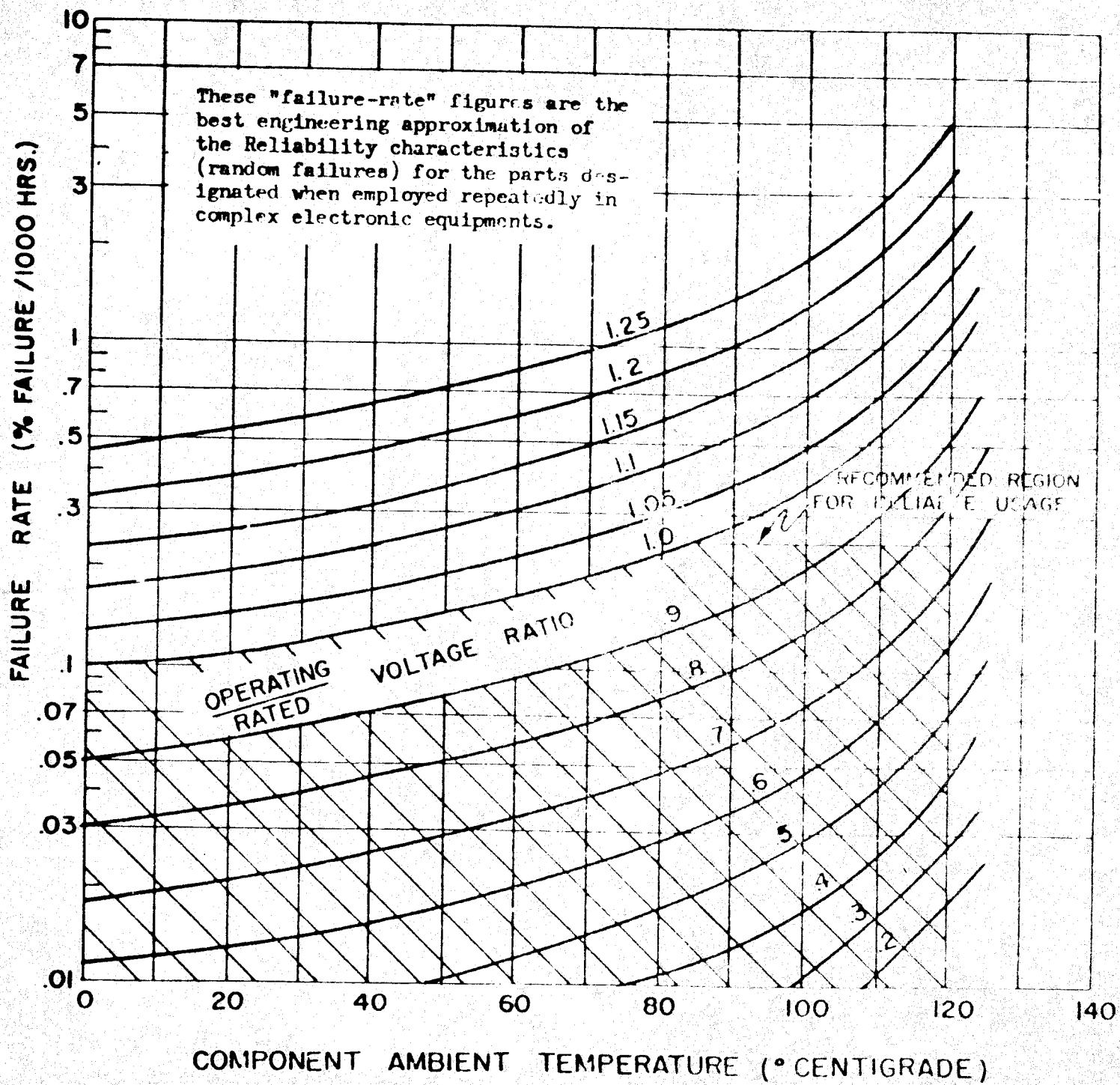


FIG. 4

PREDICTED FAILURE RATES FOR PAPER CAPACITORS
(MIL-C-25A CHARACTERISTIC K)

the total cost, equal initial cost, plus maintenance due to failure, (less servicing, which is not accounted for), amounts to:

$$C_{\text{tot}} = C_{\text{in}} \left(1 + \sum_{p=1}^n h_p r_p \lambda_p t \right) \quad (12)$$

The function

$$\frac{C_{\text{tot}}}{C_{\text{in}}} - 1 = h_p r_p \lambda_p t \quad (11.1)$$

is represented in Fig. 2. This is a universal long-range cost curve family, each being characterized by a different and counterclockwise increasing product $h_p r_p \lambda_p$. The total long-range cost of an equipment of known composition can be deduced from it by calculating the long-range cost for each group of components and adding the results. Failure rate λ_p of the various component parts of the equipment must be known.

More for purposes of illustration than anything else two diagrams are included, Figs. 3 and 4, from which failure rates of condensers and transformers can be determined. Operating temperature, voltage and wattage are the most common parameters entering in such diagrams. Other failure characterizes are reported in a comprehensive report published by RCA* containing further discussions concerning electronic equipment reliability. The initial considerations of this note related to Formulas (1) to (8) have been taken from it.

A final word in relation with λ_p must be said. The figures indicated in the above mentioned diagrams of Figs. 3 and 4 refer to the useful life of the components (time interval $T_2 - T_1$ in Fig. 1) and not to the "debugging period" (0 to T_1) or the "wear-out period" ($T_3 - T_2$). At the beginning of the actual operation of the equipment the debugging period and its decreasing failure rate is of no concern because the components which

* Reliability Stress Analysis for Electronic Equipment,
U.S. Dept. of Commerce, PB 13168, 1956

failed are not in the equipment. Not so for the wear-out period. This means that the failure rate given by the diagrams must be increased by a certain percent in order to take into account the fact that at the end of the useful life period the remaining components will be eliminated and therefore lost for purposes of useful operation. For doing this the complete failure rate diagram of the component is needed. If N_w is the number of components left at time T_2 the virtual failure rate due to the above mentioned circumstances is:

$$\lambda'_p = \lambda_p + \frac{N_w}{T_2 - T_1} \quad (13)$$

This formula can be derived from Fig. 1 through a simple geometrical consideration.

Finally, implications of formula (5) should be taken into account in the determination of the total value of the failure rate.

Formula (12) can be applied to the particular case of sampled data transmission links. Let us suppose that the speed of transmission is of P pulses per second and that there are G groups of transmitter-receiver sets to handle this information. This means that if X is the number of channels transmitted and received per each group, the relationship linking these quantities is: $P = X.G$. Each group of transmitters and receivers is composed of N component parts and there are $N_1 N_2 \dots$ components in each category, like condensers, tubes, etc., i.e., all together there are $N = N_1 + N_2 + N_3 + \dots + N_n$ components in a group, $G.N$, in the whole system of G groups. Now:

$$G.N = \frac{P}{X} (N_1 + N_2 + N_3 + \dots + N_n) = \frac{P}{X} - N \quad (14)$$

and if in formula (12) the number of component parts is made to appear explicitly by considering formula (10), we find:

$$C_{\text{tot}} = C_{\text{in}} \left(1 + \sum_{p=1}^n h_p r_p N_p \frac{P}{X} \lambda_{pt} \right) \quad (15)$$

The diagram of Fig. 2 can be used for long-range cost calculations in the case of a data link like the one now considered if the quantity $h_p r_p \frac{P}{X} \lambda_{pt}$ is entered in place of $h_p r_p \lambda_p$. Thus, the long-range cost of sampled data transmission links increases with the speed of transmission, while the number of channels per group cannot be increased beyond the limits of stable operation.

This informative note will be concluded with a consideration which is a direct result of all what has been said of Fig. 1. For more clarity, this later has been repeated in Fig. 5. It is obvious that the purchaser wishes to buy equipment (1) which has been adequately tested during a period of time not inferior to the debugging period and (2) if he purchases a number of pieces of the same equipment, he will pretend uniformity of quality, i.e., failure rates characteristic for the useful lifespan contained within certain limits. Such circumstances may be expressed in the following contractual purchase clauses:

Conditions of Purchase of Equipment

1. The manufacturer will deliver the equipment purchased after a continuous test period equal to the initial average period of time (debugging period) during which the failure rate of the equipment concerned is higher than the normal. Manufacturer and purchaser will define before signature of the purchase order the length and circumstances of this test period.
2. The manufacturer will indicate the average normal failure rate and will guarantee that the actual failure rate will not be greater than the former by more than a given percent.
3. The manufacturer will indicate the lifespan of the equipment during which the failure rate will be maintained within the limits defined under (2).
4. The manufacturer will take care of or will be charged for all the repair work and/or substitutions of parts and/or entire equipment that will occur above the level guaranteed for and agreed upon, as under (2).
5. The manufacturer will supply free of charge a quantity of stand-by sets and parts thereof as part of the guarantee mentioned under (2).
6. The accompanying diagram defines the failure rates and the useful lifespan.

