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Secondary Particle Yield from High Energy Electrons

In this note we wish to estimate the yield of secondary particles expected from high-energy electrons (10 to 45 Bev), for comparison with the yield expected from protons of comparable energy. We shall consider secondary pions explicitly, since these are probably most plentifully produced in this energy range. They are of interest not only for themselves, but also (1) because they may serve as indicators of expected yields of other mesons and hyperons, (2) they produce muons when they decay, and these must be considered for shielding, and (3) they also produce neutrinos when they decay, and these are expected to be of considerable physical interest in their own right.

Our estimate is very crude. For primary electrons, it consists in taking the track length for photons of a given energy in an electron-initiated shower, converting this photon energy to the center-of-mass system for a photon and a nucleon, assuming that this photon is absorbed in accordance with an extrapolation from the known photopion cross section in hydrogen, and finally assuming that the number of pions produced following this absorption is proportional to the photon energy. In explanation of this procedure, we offer the following comments. The track length is defined as the total distance traveled by all photons in the shower that have the particular energy. For target nuclei heavier than hydrogen, we assume that the nucleons absorb photons independently, with no distinction between protons and neutrons, and multiply the hydrogen cross section by the atomic weight; this probably overestimates the yield slightly for beryllium and carbon. The weakest assumption is perhaps that concerning the multiplicity of pion production. While it seems plausible, there is little experimental evidence on this point at the present time. Our estimate for primary protons consists in neglecting ionization loss (the range of a multi-Bev proton based on ionization loss alone is many times that based on nuclear collisions alone), and assuming that the number of pions produced is proportional to the proton energy. We further assume that this proportionality factor is the same for protons as for the absorbed photons in the electron calculation.

The track length for photons of energy between W and W + dW in a shower initiated by an electron of energy E is given approximately by the expression[1]

$$0.572(E/W^2)dW, \tag{1}$$

where distance is measured in radiation lengths X_0. For hydrogen,[2] $X_0 = 58$ gm/cm^2, and for heavier elements we use the formula[1]

$$X_0^{-1} = (4N/137A)(Zr_0)^2 \ln(183/Z^{\frac{1}{3}}), \tag{2}$$

where N is Avogadro's number, Z and A are the atomic number and atomic weight of the target atoms, and r_0 is the classical electron radius $e^2/mc^2 = 2.82 \times 10^{-13}$ cm. Equation (2) gives the following values for X_0 in gm/cm^2:

Be: 85, C: 52, Pb: 5.9.

For the photopion absorption cross section per nucleon we make two alternative assumptions. The more conservative of these is

$$\sigma = 2 \times 10^{-28}(W_0/W')^2 \text{cm}^2 \text{ for } W' > W_0, \tag{3a}$$

and assume that σ is negligibly small for $W' < W_0$. Here, W_0 = 300 Mev is the peak of the first photopion resonance, and W' is the energy of a photon of energy W in the center-of-mass system of the photon and nucleon of mass M:

$$W' = W(1 + 2W/M)^{-\frac{1}{2}}. \tag{4}$$

The less conservative assumption is to follow (3a) up to the energy

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1. B. Rossi and K. Greisen, Revs. Mod. Phys. 13, 240 (1941).

2. J. A. Wheeler and W. E. Lamb, Phys. Rev. 55, 858 (1939); D. Bernstein and W. K. H. Panofsky, Phys. Rev. 102, 522 (1956).

$W = 650$ Mev or $W' = 425$ Mev, where (3a) gives a cross section of $1 \times 10^{-28}\mathrm{cm}^2$, and assume that

$$\sigma = 1 \times 10^{-28}\mathrm{cm}^2 \tag{3b}$$

for all higher energies.

We thus obtain from (1) an expression for the yield of pions per primary electron:

$$Y_e = \int_{W_0}^{E} 0.572(E/W^2)(X_0N/A)\sigma(W'/E_0)A\,dW, \tag{5}$$

where σ is given by (3a) or (3b), N/A is the number of nuclei per gram of target material, W'/E_0 is the multiplicity of pion production for an absorbed photon of center-of-mass energy W', and A is the number of nucleons per nucleus; E_0 is assumed to be of order 150 to 300 Mev. With (3a) and (4), Eq. (5) gives

$$Y_e = 4 \times 10^{-5}(X_0E/E_0) \tag{6}$$

when E is large in comparison with W_0, as is actually the case. When (3a) is replaced by (3b), the yield (6) is approximately doubled.

In accordance with our assumption, the yield per primary proton with the same energy E is $Y_p = (E/E_0)$. Thus the ratio of pion yields per primary electron and proton with the same energy lies somewhere between $4 \times 10^{-5}X_0$ and twice this value. The smaller values are listed in the following table:

element	H	Be	O	Pb	
Y_e/Y_p	2.3	3.4	2.1	0.2	$\times 10^{-3}$

It requires roughly ten radiation lengths for a shower to develop from a primary electron with an energy in the range considered here. Now a radiation length in liquid hydrogen (density 0.07 gm/cm^3) is about 8.3 meters, so that a full shower would require a prohibitively large amount of hydrogen. However, a radiation length in beryllium (density 1.84 gm/cm^3) is about 46 cm, and a radiation length in graphite (density 2.25 gm/cm^3) is about 23 cm. Thus it would be quite feasible to use graphite, and perhaps possible to use beryllium.

We conclude then that a primary proton would be expected to be between 150 and 500 times as effective as a primary electron of the same energy in producing secondary particles. In comparing protons and electrons of different energies, we expect the secondary particle yield to be roughly proportional to the primary particle energy in the range under consideration.

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