NOTE ON PHASE SHIFT IN TRANSMISSION LINES DUE TO IONIZING RADIATION

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The analysis given here represents an attempt to estimate the magnitude of the phase shift in the drive line of the Project M accelerator due to ionizing radiation from the klystrons and the accelerator. This question is especially pertinent if the drive line is placed in the accelerator tunnel (as has been proposed to ease the temperature tolerance problem).

In this analysis the simple Langmuir equation(1) for the effective dielectric constant in a weakly ionized medium ("plasma") has been used. It is given by the following expression:

$$\epsilon = \left(1 - \frac{ne^2}{m\,\epsilon_o\,\omega^2}\right)\epsilon_o \qquad (1)$$

where

n = number of electrons per cm^3.
m = rest mass of electron
e = charge of electron
ω = $2\pi \times$ frequency of propagating radiation
ϵ_o = dielectric constant of free space

Letting $p = \dfrac{ne^2}{m\,\epsilon_o\,\omega^2} = \left(\dfrac{\omega_p}{\omega}\right)^2$, where ω_p is the plasma frequency,

1. Langmuir, I., Phys. Rev. 33, 954 (1929)
Langmuir, I. and Tonks, L., Phys. Rev. 33, 195 (1929).

and substituting the values of the fundamental constants we obtain

$$p = \frac{ne^2}{m\,\epsilon_o \omega^2} = \frac{(1.6 \times 10^{-19})^2 \; 4 \times 9 \times 10^9 \times 10^6}{9.1 \times 10^{-31} \times (1.79)^2 \times 10^{20}} \; n$$

$$= 9.9 \times 10^{-12} \; n \qquad (2)$$

where we have taken the frequency to be 2856 Mc/sec.

We may rewrite Eq. (1) as

$$\epsilon = (1-p)\,\epsilon_o = \epsilon_o - p\epsilon_o \qquad (3)$$

or

$$\frac{\delta \epsilon}{\epsilon} = -p = -\,9.9 \times 10^{-12} \; n \qquad (4)$$

The guide wavelength in the transmission line is given by

$$\lambda_g = K\,\epsilon^{-\frac{1}{2}} \qquad (5)$$

where K is a constant depending upon the configuration and dimensions of the transmission line and the frequency. From this relation we obtain

$$\frac{\delta \lambda_g}{\lambda_g} = -\tfrac{1}{2}\,\frac{\delta \epsilon}{\epsilon} \qquad (6)$$

The phase shift in a transmission line of length L caused by a change $\delta\epsilon$ in the dielectric constant of the medium is given by

$$\delta\phi = -2\pi\frac{L}{\lambda_g}\frac{\delta\lambda_g}{\lambda_g} \qquad (7)$$

Substituting for $\frac{\delta\lambda_g}{\lambda_g}$ from Eq. (6) and using Eq. (4) we obtain

$$\delta\phi = -9.9 \times 10^{-12}\, n\pi\, \frac{L}{\lambda_g} \qquad (8)$$

The number of electrons per cm^3 in the transmission line is given approximately by (assuming all ions formed remain during entire pulse)

$$n = 2.08 \times 10^{9}\, R_{pulse} \qquad (9)$$

where R_{pulse} is the radiation dose in the transmission line in roentgens per pulse. Substituting Eq. (9) in Eq. (8) we have

$$\delta\phi = -64.7 \times 10^{-3}\, \frac{L}{\lambda_g}\, R_{pulse} \qquad (10)$$

Applying Eq. (10) to Project M where $\frac{L}{\lambda_g} \simeq 3 \times 10^{4}$, the result is

$$\delta\phi = -1940\, R_{pulse} \qquad (11)$$

If we want φ to be less than 0.1 radian, then

$$R_{pulse} < 5.15 \times 10^{-5} \qquad (12)$$

or

$$R_{hour} < 11.1 \quad \text{(at a repetition rate of 60 pulses/sec.)} \qquad (13)$$

In terms of electron density and plasma frequency the requirement of Eq. (12) may be written

$$n < 1.07 \times 10^{5} \text{ electrons/cm}^{3}$$

and

$$p = \left(\frac{\omega_p}{\omega}\right)^2 < 1.06 \times 10^{-8} \qquad (13)$$

or

$$f_p = \frac{\omega_p}{2\pi} < 2.94 \text{ Mc/sec.}$$

For a given radiation intensity, the electron density n is proportional to the pressure. Therefore, it may be desirable to evacuate the drive line to keep the phase shift within reasonable limits. The need to do this will depend upon the intensity of radiation in the drive line location.