# DRIVE LINE CONSIDERATIONS FOR PROJECT M

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## A. INTRODUCTION

In studying drive line problems on Project M, the following considerations are applicable:

1) The important factor in phase stability requirements is the relative phase shift between the r-f wave in the drive line and the electron bunches at corresponding points along their respective paths. It is pertinent to note that both the effective phase of the electron bunches and the phase of the r-f drive wave at a distance z along the trajectories are functions of the driving frequency. This means that a particular drive system which provides only absolute stability of the r-f drive wave is not ideal since it does not provide for frequency tuning of the machine.

2) A certain amount of tuning in frequency is considered desirable since it is probably not feasible to ascertain exactly the operating frequency from cold test measurements due to the inability to make appropriate corrections for the heating effect of the r-f power. With care, however, it should be possible to keep the required tuning range within reasonable limits, say within ± 0.1 Mc/sec.

3) When the drive frequency is adjusted, another important factor in phase shift calculations is the variation of phase velocity in the drive line with change of frequency of the propagating wave. This "dispersion" may be due to basic considerations such as the more rapid rate of change of guide wavelength than free space wavelength (if a hollow waveguide is used as the drive line), the effect of the devices used to support the center conductor (if a coaxial drive line is used), or a change in the dielectric constant of the gaseous medium inside the drive line (unless an evacuated line is used) due to frequency variation or to ionization of the gas.

4) A major source of phase error is that due to variations either in the ambient temperature or in the local temperature along the length of the machine. Again, it is the relative temperature changes between the drive line and the accelerator which must be closely controlled. In general, a close control of relative temperature also implies a reasonably good control of the absolute temperatures of the two lines. An alternate procedure which

alleviates the severe temperature requirements is to tie the two lines together physically so that relative axial motion is prevented or greatly restricted. This will be studied further in later sections of this discussion.

B. DEVELOPMENT OF PHASE SHIFT EQUATIONS

The phase of the r-f wave in the drive line at a point $z$ down the line is given by:

$$\phi_w = \frac{\omega z}{v} \tag{1}$$

where $\omega$ is the drive frequency in radians/sec., and $v$ is the phase velocity of the wave in the drive line. The corresponding equation for the phase of the electron bunches at distance $z$ along the accelerator is:

$$\phi_e = \frac{\omega z}{c} \tag{2}$$

where $c$ is the velocity of light* in free space and the other symbols are as in Eq. (1).

Dividing Eq. (1) by Eq. (2) we obtain:

$$\frac{\phi_w}{\phi_e} = \frac{c}{v} \tag{3}$$

The differential equations obtained from Eqs. (1) and (2) are:

$$\delta\phi_w = \frac{\omega}{v}\,\delta z + \frac{z}{v}\,\delta\omega - \frac{\omega z}{v^2}\,\delta v$$

$$= \phi_w \left(\frac{\delta z}{z}\right)_w + \phi_w \frac{\delta\omega}{\omega} - \phi_w \frac{\delta v}{v} \tag{4}$$

* While the electron velocity never reaches $c$, it is sufficiently close at the energies with which we are concerned that the error from this assumption is negligible. It may be shown that $1-\beta \cong \frac{1}{2\gamma^2}$ where $\beta = \frac{v_e}{c}$ and $\gamma = \frac{V}{E_o}$ where $V$ is the total energy and $E_o$ the rest energy of the electron.

$$\delta\phi_e = \frac{\omega}{c}\,\delta z + \frac{z}{c}\,\delta\omega$$

$$= \phi_e \left(\frac{\delta z}{z}\right)_e + \phi_e \frac{\delta\omega}{\omega} \qquad (5)$$

The <u>relative</u> phase shift between the r-f drive wave and the electron bunches may be obtained by subtracting Eq. (4) from Eq. (5) and taking Eq. (3) into account:

$$\frac{\delta\phi}{\phi} = \frac{\delta\phi_e - \delta\phi_w}{\phi_e} = \left(\frac{\delta z}{z}\right)_e - \frac{c}{v}\left(\frac{\delta z}{z}\right)_w + \left(1 - \frac{c}{v}\right)\frac{\delta\omega}{\omega} + \frac{c}{v}\frac{\delta v}{v} \qquad (6)$$

Eq. (6) is a general relationship which may be used to calculate the relative phase shift due to variations in temperature, frequency, and phase velocity. This equation will now be used to study several cases.

C. <u>EFFECT OF FREQUENCY TUNING</u>

It is desirable to be able to vary the frequency to optimize the output energy without introducing phase shifts which interact with and confuse the primary adjustments. Since these frequency variations are short-term in nature, the temperature dependent terms in Eq. (6) can be omitted in this instance and the pertinent relation becomes:

$$\frac{\delta\phi}{\phi} = \left(1 - \frac{c}{v}\right)\frac{\delta\omega}{\omega} + \frac{c}{v}\frac{\delta v}{v} \qquad (7)$$

Two cases will be discussed: where (a) the drive line is a coaxial transmission line, and (b) the drive line is a hollow waveguide.

(a) Coaxial Drive Line

In this case, it is presumed that at the design frequency the phase velocity in the drive line equals the velocity of light, c . At other frequencies in the vicinity of the design frequency, the phase velocity may deviate from the velocity of light due to the presence of supporting devices for the center conductors, imperfections in the line, finite attenuation, air dielectric, etc. The resulting relative phase shift in Eq. (7) thus becomes:

$$\frac{\delta\phi}{\phi} = \frac{\delta v}{v} \qquad (8)$$

If the accelerator has a total length of 10,000 ft. (305,000 cm.), then $\phi$ = 30,000 cycles. If we wish to hold the phase shift at the end of the accelerator to a reasonable value, say 0.05 cycles, the phase velocity must be held to an accuracy of

$$\frac{\delta v}{v} \leq \frac{.05}{30,000} = \frac{1}{600,000}$$

over whatever frequency tuning range is required.

(b) Evacuated Hollow Waveguide Drive Line

An evacuated hollow waveguide offers the advantage of not requiring a center conductor with the attending support and cooling problems. However, waveguides have velocity dispersion due to the fact that guide wavelength changes more rapidly with frequency than free space wavelength. The dispersion can be reduced by increasing the physical dimensions of the waveguide, but the problem of higher mode propagation then arises. We will investigate the use of waveguide and will reserve until later a discussion of means of suppressing unwanted modes.

In hollow waveguide the phase velocity v is given by

$$\frac{v}{c} = \frac{\lambda_g}{\lambda_o} \qquad (9)$$

where $\lambda_g$ is the guide wavelength and $\lambda_o$ is the free space wavelength.

From Eq. (9) we obtain the differential expression:

$$\frac{\delta v}{v} = \frac{\delta\lambda_g}{\lambda_g} - \frac{\delta\lambda_o}{\lambda_o} \tag{10}$$

The relationship between $\lambda_g$, $\lambda_o$, and the cut-off wavelength $\lambda_c$ is

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_o^2} - \frac{1}{\lambda_c^2} \tag{11}$$

from which the differential expression,

$$\frac{\delta\lambda_g}{\lambda_g} = \left(\frac{\lambda_g}{\lambda_o}\right)^2 \left(\frac{\delta\lambda_o}{\lambda_o}\right) - \left(\frac{\lambda_g}{\lambda_c}\right)^2 \frac{\delta\lambda_c}{\lambda_c} \tag{12}$$

may be obtained.

Substituting Eq. (12) into Eq. (10) we obtain:

$$\frac{\delta v}{v} = \left[\left(\frac{\lambda_g}{\lambda_o}\right)^2 - 1\right] \frac{\delta\lambda_o}{\lambda_o} - \left(\frac{\lambda_g}{\lambda_c}\right)^2 \left(\frac{\delta\lambda_c}{\lambda_c}\right) \tag{13}$$

Since temperature variations are not pertinent in this issue which involves only phase shift due to frequency tuning, $\lambda_c$ may be treated as a constant, and the second term on the right in Eq. (13) drops out. Substituting Eq. (13) into Eq. (7) we obtain:

$$\frac{\delta \phi}{\phi} = \left(\frac{c}{v} - 1\right) \frac{\delta\lambda_o}{\lambda_o} + \frac{c}{v} \left[\left(\frac{\lambda_g}{\lambda_o}\right)^2 - 1\right] \frac{\delta\lambda_o}{\lambda_o} \tag{14}$$

$$= \left(\frac{v}{c} - 1\right) \frac{\delta\lambda_o}{\lambda_o}$$

Equation (14) gives the allowable frequency change in hollow waveguide for a given phase shift $\delta \phi$. From another point of view, it predicts that the phase shift can be reduced to any desired value by making the waveguide large enough so that v approaches c. We will now determine the dimensions which the waveguide must have in order that a given $\frac{\delta \lambda_o}{\lambda_o}$ will not cause a resultant phase shift exceeding a given $\frac{\delta \phi}{\phi}$. Solving for v/c from Eq. (14) we obtain:

$$\frac{v}{c} = 1 + \frac{\delta\phi/\phi}{\delta\lambda_o/\lambda_o} \qquad (15)$$

Solving for $\frac{\lambda_o}{\lambda_c}$ from Eq. (11) we obtain:

$$\frac{\lambda_o}{\lambda_c} = \left[ 1 - \left(\frac{\lambda_o}{\lambda_g}\right)^2 \right]^{\frac{1}{2}} = \left[ 1 - \left(\frac{c}{v}\right)^2 \right]^{\frac{1}{2}} \qquad (16)$$

Substituting Eq. (15) into Eq. (16) we have:

$$\frac{\lambda_o}{\lambda_c} = \left[ 1 - \left( \frac{\delta\lambda_o/\lambda_o}{\delta\lambda_o/\lambda_o + \delta\phi/\phi} \right)^2 \right]^{\frac{1}{2}} \qquad (17)$$

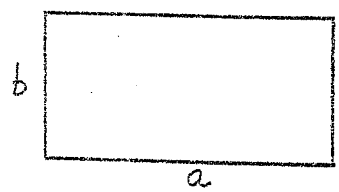
Equation (17) specifies the waveguide dimensions if the tuning range $\frac{\delta \lambda_o}{\lambda_o}$, the allowable phase shift $\frac{\delta \phi}{\phi}$ and the propagating mode are known. For example, suppose that $\frac{\delta \lambda_o}{\lambda_o} = \frac{1}{30,000}$ and $\frac{\delta \phi}{\phi} = \frac{1}{600,000}$; then

$$\frac{\lambda_o}{\lambda_c} = \left[ 1 - \left(\frac{1}{1.05}\right)^2 \right]^{\frac{1}{2}} = .305$$

Then, if $\lambda_o = 10.5$ cm.,

$$\lambda_c = \frac{10.5}{.305} = 34.4 \text{ cm.}$$

The dimensions of the waveguide having a cut-off wavelength of 34.4 cm. depend upon the shape of the guide and the propagating mode. In ordinary rectangular waveguide,

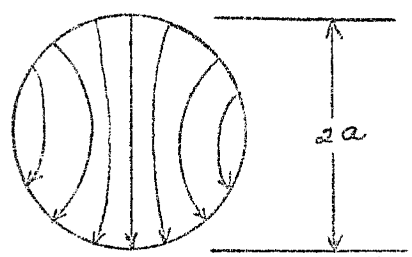



$\lambda_c$ = 2a = 34.4 cm.

a = 17.2 cm. (6.77 inches)

b = 8.6 cm. (3.38 inches)

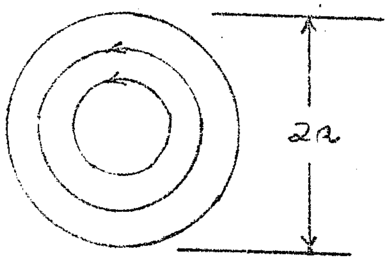
In cylindrical waveguide propagating in the dominant $TE_{11}$ mode,




$\lambda_c$ = 34.1

2a = 20.2 cm. (7.95 inches)

In cylindrical waveguide propagating in the $TE_{01}$ mode,




$\lambda_c = 1.64 \times a = 34.4$ cm.
2a = 42 cm. (16.5 inches)

The attenuation in the two modes in circular waveguide are as follows:

$TE_{11}$ mode: $\lambda_c = 34.4$ cm., 2a = 20.2 cm.

$$\alpha = \frac{R_s}{a\eta_1} \frac{1}{\sqrt{1 - \left(\frac{\lambda_o}{\lambda_c}\right)^2}} \left[\left(\frac{\lambda_o}{\lambda_c}\right)^2 + 0.420\right] \frac{\text{nepers}}{\text{meter}}$$

where $R_s$ is the surface resistivity of the waveguide walls,

$\eta_1 = \sqrt{\frac{\mu}{\epsilon}}$ inside the waveguide, $\frac{R_s}{\eta_1} = 3.7 \times 10^{-5}$ at $\lambda_o = 10.5$ cm.

With the above substitutions we obtain:

$$\alpha = 1.98 \times 10^{-4} \ \frac{\text{nepers}}{\text{meter}}$$

$$= 5.25 \times 10^{-4} \ \text{db/ft.}$$

In 10,000 feet of drive line, the total attenuation would be 5.25 db.

$TE_{01}$ mode: $\lambda_c = 34.4$ cm, 2a = 42 cm.

$$\alpha = \frac{R_s}{a\eta_1} \frac{(\lambda_o/\lambda_c)^2}{\sqrt{1 - \left(\frac{\lambda_o}{\lambda_c}\right)^2}}$$

$$= 1.72 \times 10^{-5} \ \text{nepers/meter}$$

$$= 4.57 \times 10^{-5} \ \text{db/foot}$$

In 10,000 feet, the total attenuation would be 0.457 db.

D. EFFECT OF TEMPERATURE VARIATIONS

In studying the effect of temperature variations, we must return to the general phase shift equation, Eq. (6). It is worthwhile to distinguish and discuss two cases: (1) where the frequency is held constant and the phase shift is due completely to the temperature dependent terms, (2) where the frequency is adjusted to compensate for the temperature change in the accelerator structure. In the first instance, Eq. (6) becomes:

$$\frac{\delta\phi}{\phi} = \left(\frac{\delta z}{z}\right)_e - \frac{c}{v}\left(\frac{\delta z}{z}\right)_w + \frac{c}{v}\,\frac{\delta v}{v} \qquad (18)$$

In the second case, Eq. (6) must be used in its entirety.

(a) Coaxial Drive Line

If the drive line is a coaxial line with $c/v \cong 1$, Eqs. (6) and (18) become

$$\frac{\delta\phi}{\phi} = g_e\,(\delta T)_e - g_w(\delta T)_w + \frac{\delta v}{v} \qquad (19)$$

where $g_e$ and $g_w$ are the temperature coefficients of expansion, and $(\delta T)_e$ and $(\delta T)_w$ the temperature deviations of the accelerator and drive line respectively. Should the accelerator and drive line be made of the same material, $g_e = g_w$. For copper structures $g = 1.6 \times 10^{-5}/^{\circ}C$ in which case

$$\frac{\delta\phi}{\phi} = 1.6 \times 10^{-5}\,(\delta T_e - \delta T_w) + \frac{\delta v}{v} \qquad (20)$$

In the case where $\frac{\delta v}{v} = 0$ or where $\frac{\delta v}{v}$ is negligible compared to the first term in Eq. (20) we have

$$\frac{\delta\phi}{\phi} = 1.6 \times 10^{-5}\,(\delta T_e - \delta T_w) \qquad (21)$$

Thus, to obtain the value $\frac{\delta\phi}{\phi} \leq \frac{1}{600,000}$ the temperature variation must not exceed

$$\delta T_e - \delta T_w \leq 0.1^\circ \, C$$

This is indeed a stringent requirement. We will later investigate means of easing the temperature tolerances.

(b) Evacuated Hollow Waveguide Drive Line

Substituting Eq. (13) into Eq. (6) we obtain for the waveguide drive line:

$$\frac{\delta\phi}{\phi} = \left(\frac{\delta z}{z}\right)_e - \frac{c}{v}\left(\frac{\delta z}{z}\right)_w + \left(\frac{c}{v} - 1\right)\frac{\delta\lambda_o}{\lambda_o} + \frac{c}{v}\left[\left(\frac{v}{c}\right)^2 - 1\right]\frac{\delta\lambda_o}{\lambda_o} - \frac{c}{v}\left(\frac{\lambda g}{\lambda_c}\right)^2\frac{\delta\lambda_c}{\lambda_c}$$

$$= \left(\frac{\delta z}{z}\right)_e - \frac{c}{v}\left(\frac{\delta z}{z}\right)_w + \left(\frac{v}{c} - 1\right)\frac{\delta\lambda_o}{\lambda_o} - \frac{v}{c}\left[1 - \left(\frac{c}{v}\right)^2\right]\frac{\delta\lambda_c}{\lambda_c} \qquad (22)$$

Now we will consider several possibilities.

Accelerator and Drive Line Rigid and Free to Slide

In this case,

$$\left(\frac{\delta z}{z}\right)_e = g\,\delta T_e$$

$$\frac{\delta\lambda_o}{\lambda_o} = g\,\delta T_e \quad \text{(Frequency adjustable to maximize energy)}$$

$$\frac{\delta\lambda_o}{\lambda_o} = 0 \qquad \text{(Frequency fixed)}$$

$$\left(\frac{\delta z}{z}\right)_w = \frac{\delta\lambda_c}{\lambda_c} = g\,\delta T_w$$

It is assumed the drive line and accelerator structures have the same temperature coefficient $g$. With the above conditions substituted into Eq. (22) we obtain

$$\frac{\delta\phi}{\phi} = \frac{v}{c}\, g\, (\delta T_e - \delta T_w) \;;\; \text{(optimum Frequency adjustment)} \qquad (23)$$

$$\frac{\delta\phi}{\phi} = g\,(\delta T_e - \frac{v}{c}\,\delta T_w) \;;\; \text{(Freq. fixed)} \qquad (24)$$

For the waveguide of the earlier example $\frac{\lambda_o}{\lambda_c}$ = .305 from which we obtain $\frac{v}{c}$ = 1.05. Then with

$$g = 1.6 \times 10^{-5} \text{ and } \frac{\delta\phi}{\phi} \leqq \frac{1}{600{,}000}$$

$$\delta T_e - \delta T_w \leqq 0.1^\circ\, C \;;\; \text{(opt. freq. adjustment)}$$

$$\delta T_e - 1.05\, \delta T_w \leqq 0.104^\circ\, C \;;\; \text{(freq. fixed)}$$

Accelerator and Drive Line Rigid and With Fixed Length

The conditions for fixed accelerator and drive line lengths are:

$$\left(\frac{\delta z}{z}\right)_e = \left(\frac{\delta z}{z}\right)_w = 0$$

Substituting these conditions in Eq. (22) we obtain

$$\frac{\delta\phi}{\phi} = \left(\frac{v}{c} - 1\right) g\;\; \delta T_e - \frac{v}{c}\left[1 - \left(\frac{c}{v}\right)^2\right] g\, \delta T_w \qquad (25)$$

(opt. freq. adjustment)

$$\frac{\delta\phi}{\phi} = -\frac{v}{c}\left[1-\left(\frac{c}{v}\right)^2\right] g\,\delta T_w \qquad (26)$$

(Fixed frequency

With $v/c = 1.05$, $g = 1.6 \times 10^{-5}$, $\frac{\delta\phi}{\phi} \leqq \frac{1}{600{,}000}$
we obtain:

$$\delta T_e - 1.95\ \delta T_w \leqq 2.08^\circ\ C \qquad \text{(opt. freq. adjustment)}$$

$$/\delta T_w/ \leqq 1.07^\circ\ C \qquad \text{(Fixed frequency)}$$

Thus, it is observed that by constricting the axial freedom of the accelerator and drive line, the temperature tolerances are relaxed by an order of magnitude.

Accelerator with Periodic Expansion Bellows; Drive Line Rigid; Feed Points Tied Together

In this case, absolute axial motion of the two systems is permitted but relative motion is prohibited. Assuming that the motion is governed by the drive line, the condition is:

$$\left(\frac{\delta z}{z}\right)_e = \left(\frac{\delta z}{z}\right)_w = g\,\delta T_w$$

After substitution of the above condition Eq. (22) becomes:

$$\frac{\delta\phi}{\phi} = \left(\frac{v}{c} - 1\right) g \left(\delta T_e - \delta T_w\right) \qquad \text{(opt. freq. adjustment)} \qquad (27)$$

$$\frac{\delta\phi}{\phi} = -\left(\frac{v}{c} - 1\right) g\ \delta T_w \qquad \text{(fixed frequency)} \qquad (28)$$

With $v/c = 1.05$, $g = 1.6 \times 10^{-5}$, $\frac{\delta\phi}{\phi} \leqq \frac{1}{600{,}000}$, we obtain from Eqs. (27) and (28);

$$\delta T_e - \delta T_w \leqq 2.08^\circ\ C \qquad \text{(opt. freq. adjustment)}$$

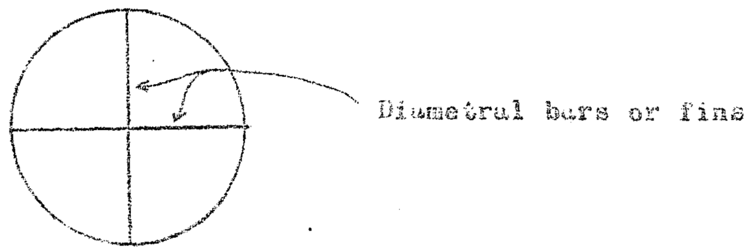
$$/\delta T_w/ \leqq 2.08^\circ\ C \qquad \text{(fixed frequency)}$$

Thus, with this system the temperature tolerances can be relaxed by an additional factor of two. However, implementation of this scheme will probably require that the drive line be located alongside the accelerator inside the accelerator tunnel. This should not cause undue difficulty. In fact, the drive line in this case might possibly be employed to serve simultaneously as the main vacuum manifold for the accelerator.

## APPENDIX I

## Use of $TE_{01}$ Mode in Circular Waveguide for Project M Drive Line with Suppression of Unwanted Modes

In the main body of this report the use of a circular waveguide operating in the $TE_{01}$ mode has been discussed. Such a guide is large enough to permit a great many other modes to be propagated. Most of these modes can be eliminated by the use of mode suppresors such as that shown below:




However, even with the suppressors shown above, the next higher mode of the same type ($TE_{02}$) can be propagated. Its cut-off wavelength is

$$\lambda_c = 0.897\ a$$

$$= 18.8 \text{ cm.}$$

for $a = 21$ cm. One method of suppressing the $TE_{02}$ mode while allowing the $TE_{01}$ mode to propagate would be to reduce the diameter of the guide over a short region at intervals along the length so as to place the $TE_{02}$ mode beyond cut-off. In these intervals the diameter should be within the limits

$$1.64\,a > 10.5 > .897\,a$$

or

$$23.4 \text{ cm.} > 2a > 12.8 \text{ cm.}$$

The above limits correspond to the cut-off diameters for the $TE_{02}$ and the $TE_{01}$ modes respectively at $\lambda_o = 10.5$ cm.

It would be desirable to locate the couplers which remove power from the guide in the regions where the diameters are reduced in order to avoid coupling to unwanted modes.