PHASING A LINEAR ACCELERATOR FROM RF PHASE SHIFT DUE TO BEAM LOADING INTERACTION

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Several methods of phasing long linear accelerators have been discussed in P.A. 1 status reports (ML Nos. 505, 520, and 557). Another method will be discussed below. This method is based on measurement of the phase shift of the impressed rf wave due to interaction with the electron beam in the accelerator section. The phase shift may be obtained by comparison of the input and output phases of the accelerator section with and without the presence of the beam. The following analysis calculates the sensitivity which can be obtained with this method.

There are two electric waves present in the accelerator section. The amplitude of the impressed rf wave from the klystron power source will be designated as E_w while that excited by the electron beam will be called E_e. When the phasing is correct, these waves will be oriented as shown in Fig. 1(a). However, if the phasing is incorrect and there is a phase error θ of the electron with respect to the wave, the vector diagram is as shown in Fig. 1(b). The phase of the resultant rf wave, E_R, lies at an angle ϕ with respect to that of the unperturbed rf wave. From the law of cosines we have

$$|E_R|^2 = E_e^2 + E_w^2 - 2\, E_e E_w \cos\,\theta \quad . \tag{1}$$

The rate-of-change of phase angle of the resultant wave relative to the electron phase angle θ may be calculated. From the law of sines,

$$\frac{\sin\phi}{E_e} = \frac{\sin\theta}{E_R} \quad . \tag{2}$$

Substituting E_R from Eq. (1) into Eq. (2) we have

$$\sin \phi = \frac{\sin \theta}{\left[1 + \left(\frac{E_w}{E_e}\right)^2 - 2\frac{E_w}{E_e}\cos\theta\right]^{1/2}} . \qquad (3)$$

Taking the differential of Eq. (3) we obtain:

$$\frac{d\phi}{d\theta} = \frac{(A)^{1/2}\cos\theta - \frac{E_w}{E_e}(A)^{-1/2}\sin^2\theta}{\cos\phi\,(A)} \qquad (4)$$

where $A = 1 + \left(\frac{E_w}{E_e}\right)^2 - 2\frac{E_w}{E_e}\cos\theta$. Let us consider the limit of $\frac{d\phi}{d\theta}$ as θ approaches zero; then we have

$$\lim_{\theta \to 0} \left|\frac{d\phi}{d\theta}\right| = \left|\frac{1}{\frac{E_w}{E_e} - 1}\right| . \qquad (5)$$

From the values of the field amplitudes at the end of the acceleration section,

$$E_e = ir\,(1 - e^{-IL}) \qquad (6)$$

$$E_w = E_0 e^{-IL} . \qquad (7)$$

we obtain for the ratio $\frac{E_w}{E_e}$

$$\frac{E_w}{E_e} = \frac{E_0 e^{-IL}}{ir\,(1 - e^{-IL})}$$

$$= \sqrt{\frac{P_0}{i^2 Lr}} \cdot \frac{\sqrt{2\,IL}}{e^{+IL} - 1} \quad . \qquad (8)$$

Consider the case:

$$P_0 = 18 \text{ Mw}$$

$$i = 5 \times 10^{-2} \text{ amps}$$

$$L = 305 \text{ cm}$$

$$r = .47 \times 10^{6} \ \Omega/\text{cm}$$

$$IL = 0.9 \quad .$$

This gives

$$\frac{E_w}{E_e} = 5.9 \quad .$$

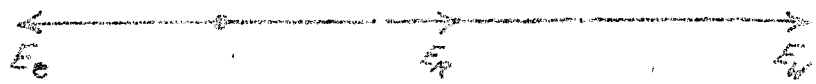
Hence, we have from Eq. (5)

$$\left|\frac{dI/I}{dE}\right| = \frac{1}{5.9 - 1} = 0.2 \quad . \qquad (9)$$

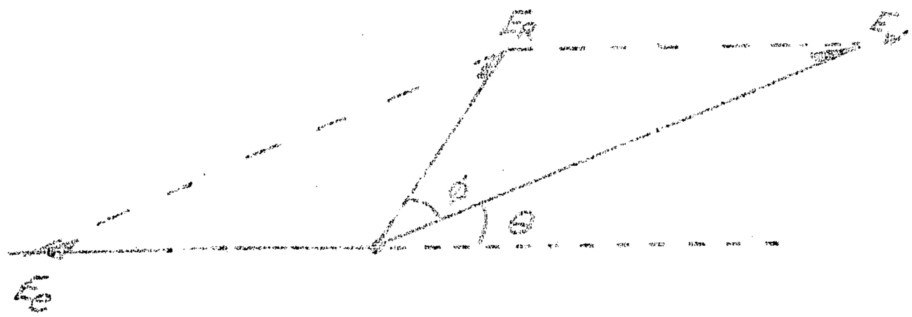
It may be noted from Eq. (8) that, for a given accelerator structure, the sensitivity of this method increases with a decrease of the input power P_0 or with an increase of electron current i .

An important point to consider is that since the power extracted from the beam is a function of the position along the machine, i.e., distance of beam interaction, it is best to consider the wave under steady-state conditions.

One technique which may be employed in phasing an accelerator section with the above mentioned method is illustrated in Fig. (2).



(a) Orientation of field vectors when phasing is correct.



(b) Orientation of field vectors when phasing is incorrect.

FIG 1 — Vector diagram illustrating principle of phasing accelerator section.

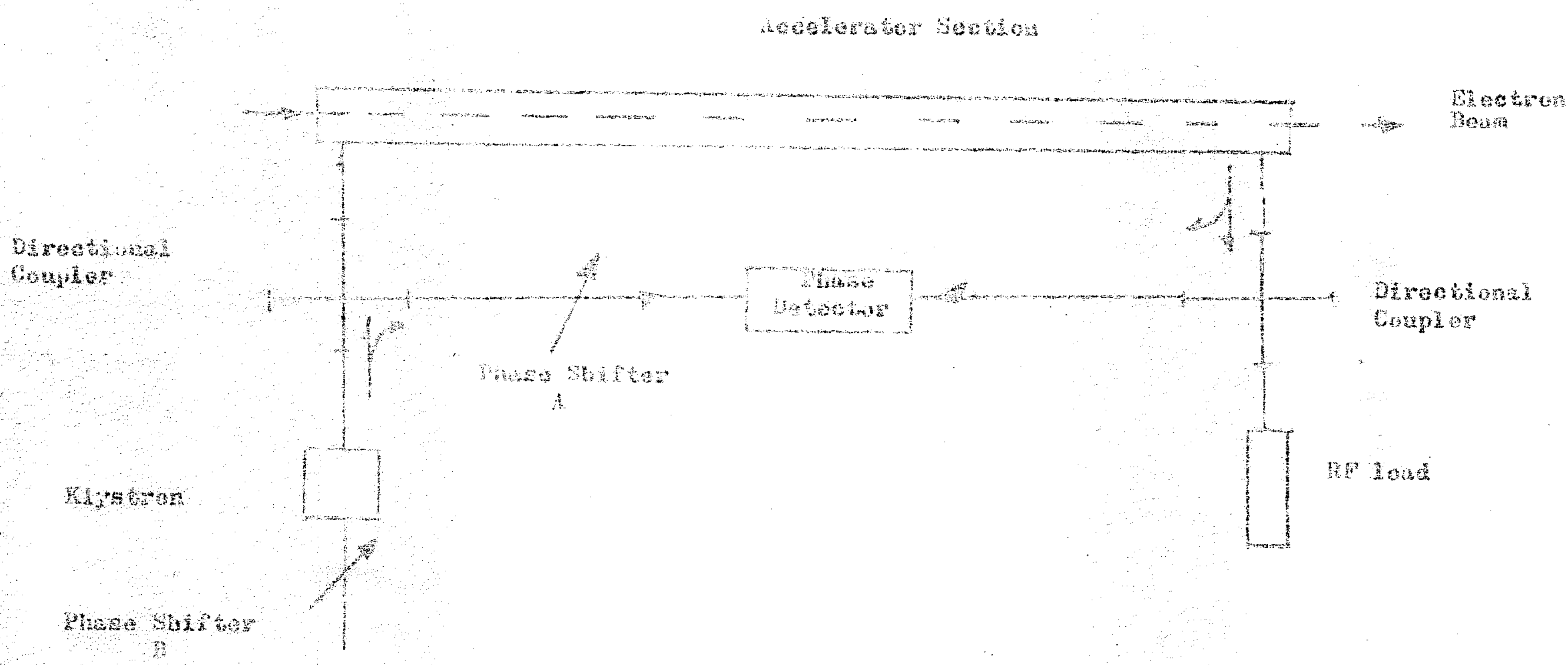


FIG. 2 — Method of Phasing Accelerator Section.
Possible method of operation

1. With electron beam off, adjust phase shifter "A" for null on phase detector.
2. Turn on beam and adjust phase shifter "B" for null on phase detector.