SPEAR-43
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SYNCHROTRON LIGHT GEOMETRY

In the design of synchrotron light monitors and synchrotron light masks and collimators, one often encounters the problem of determining the point of origin of the light and its angle with respect to the local orbit. I treat the problem of an observer in a straight section looking into an adjacent bending magnet. Figure 1 shows a typical situation. The distance d, from the orbit center in the straight section is known, and the distance ℓ_0 from the end of the bending-magnet field is also known. Figure 2 shows the geometry of the situation. Figures 3 and 4 are the graphs of the solutions for 2δ and ℓ_1.

$$\ell_0 = \overline{AB} \qquad R = \overline{AD} = \overline{ED}$$

$$\ell_1 = \overline{EC} \qquad m = \overline{EF} = \overline{FA}$$

$$d = \overline{CB} \qquad S = \angle EDF = \angle FDA$$

$$\overline{EC} \perp \overline{ED}$$

$$\overline{DA} \perp \overline{AB}$$

$$m/R = \tan\delta \qquad m = R\tan\delta$$

$$\overline{FB} = \ell_0 + m$$

$$d/\overline{FB} = \tan 2\delta \qquad d = \overline{FB}\tan 2\delta$$

$$d = (\ell_0 + m)\tan 2\delta$$

$$d = (\ell_0 + R\tan\delta)\tan 2\delta$$

using the trig identity

$$\tan 2\delta = \frac{2\tan\delta}{1 - \tan^2\delta} .$$

Substituting $x = \tan\delta$

$$d = \frac{2x}{1 - x^2} + x(\ell_0) - d = 0 .$$

This reduces to the quadratic

$$x^2(2R + d) + x(\ell_0) - d + 0 .$$

Solving, we have

$$\delta = \tan^{-1} \frac{\ell_0 \pm \sqrt{\ell_0^2 + d(2R + d)}}{2R + d}$$

The negative square root gives the correct angle ℓ_1.

The distance to the emission point, is easily seen to be

$$\ell_1 = \frac{d}{\sin 2\delta} + R\tan\delta \ .$$

Example:

At the beginning of a normal straight section we have

$$\ell_0 = 1.19 \text{ m}$$

$$d = 7.78 \text{ cm}$$

$$R = 12.72 \text{ m}$$

$$2\delta = 2^{\circ}56'20'' = 0.05126$$

$$\ell_1 = 1.85 \text{ m}$$

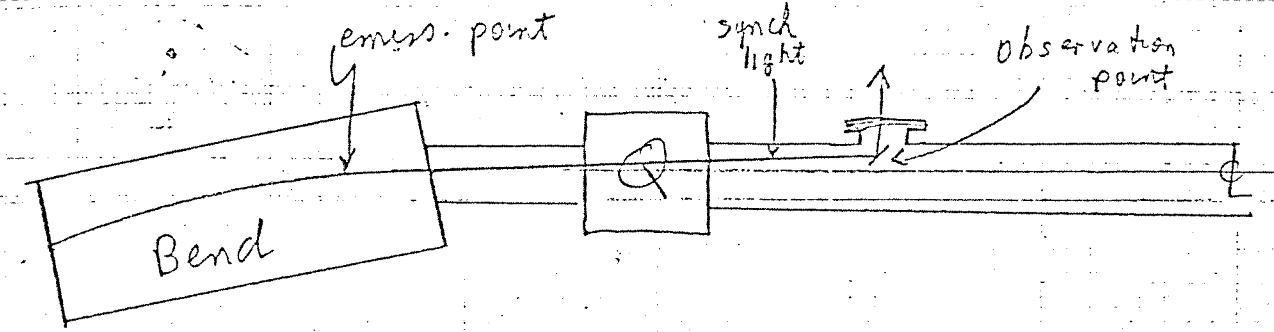
emiss. point
synch light
observation point
Bend
Q
℄

FIG. 1

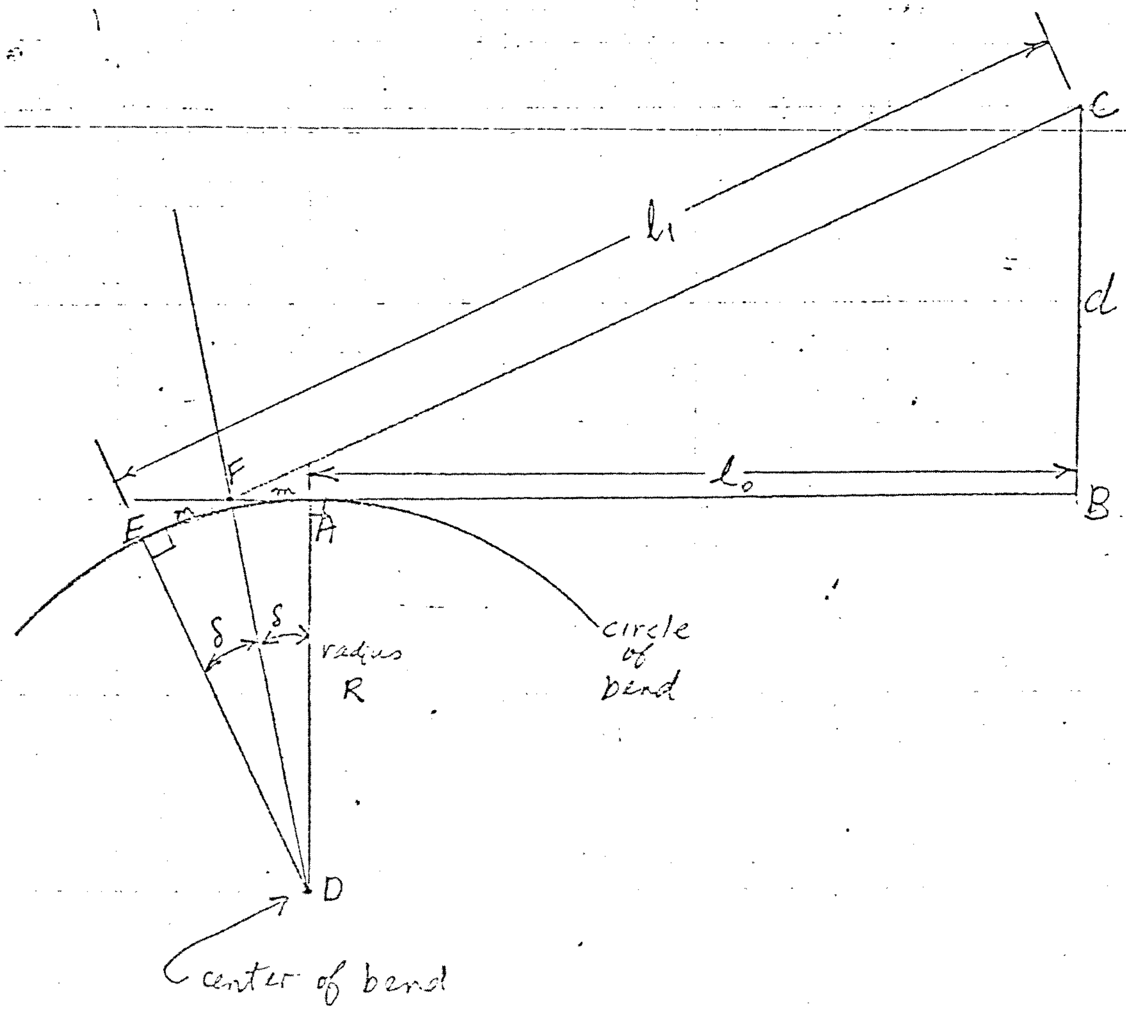
C
l_1
d
F
m
m
l_0
E
A
B
δ
δ
radius R
circle of bend
D
center of bend

FIG. 2

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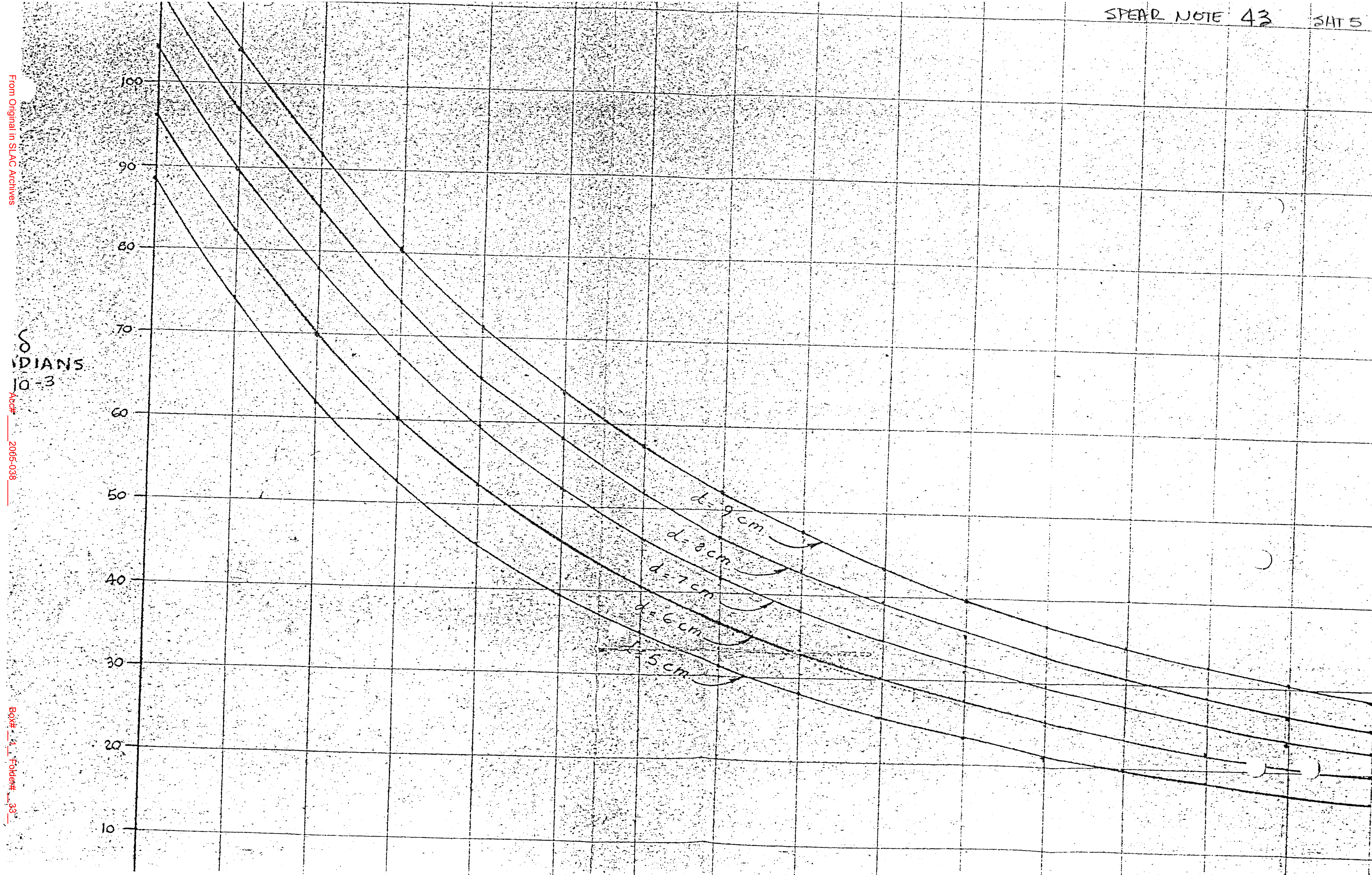

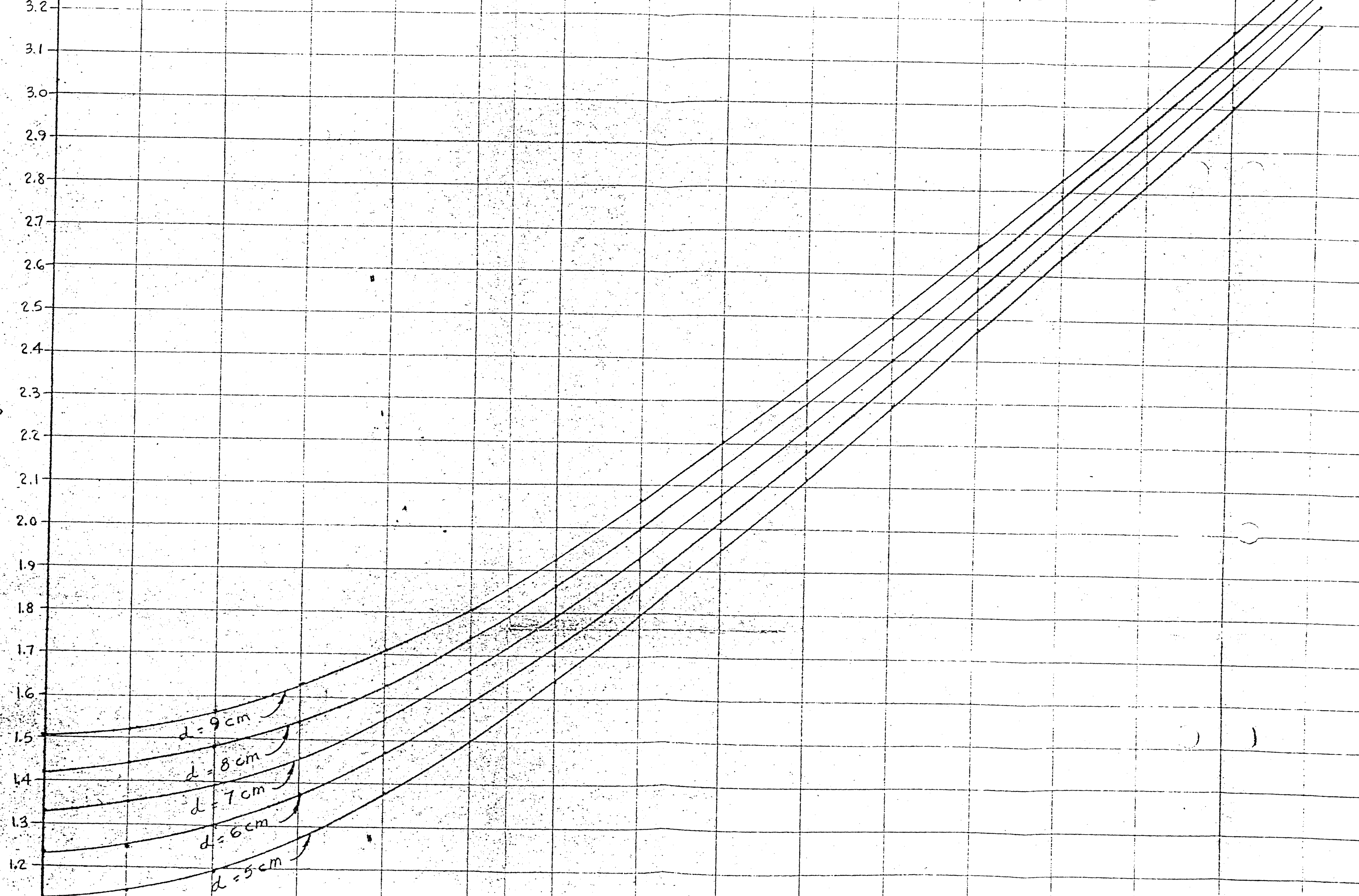
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
ETERS
d = 9 cm
d = 8 cm
d = 7 cm
d = 6 cm
d = 5 cm