Photonic Band Gap Fiber Accelerator Efficiency

Eddie Lin's Photonic Band Gap Fiber Accelerator, (PBGFA) Figure 1, is an optical equivalent of a RF-driven, near-field structure with the boundary conditions from nearby material supporting a traveling wave mode with phase velocity $v_\varphi = c$.[1] The characteristic impedance relating the accelerating gradient, G_0, and the input laser power, P, is

$$Z_c = \frac{G_0^2 \lambda^2}{P}. \tag{1}$$

Z_c is determined by the accelerator structure geometry, and material damage limits the input fluence and laser power. The unloaded gradient follows from eq. (1).

Beam Loading

Beam-loading in the fundamental mode has been analyzed as it is for an RF cavity.[1] This is an extension to include the wakefield associated with Cherenkov radiation. The impedance for Cherenkov radiation in a dielectric rod with an infinite outer radius and with an inner hole of radius r_0 is[2]

$$Z_H = Z_0 \frac{1}{2\pi \left(r_0/\lambda \right)^2}. \tag{2}$$

The effect of the photonic bad gap structure is to confine radiation at the accelerating mode frequency. The approximation being made for this analysis is that the fiber does not confine other frequencies. Perhaps an effective dielectric constant should be used to account for the

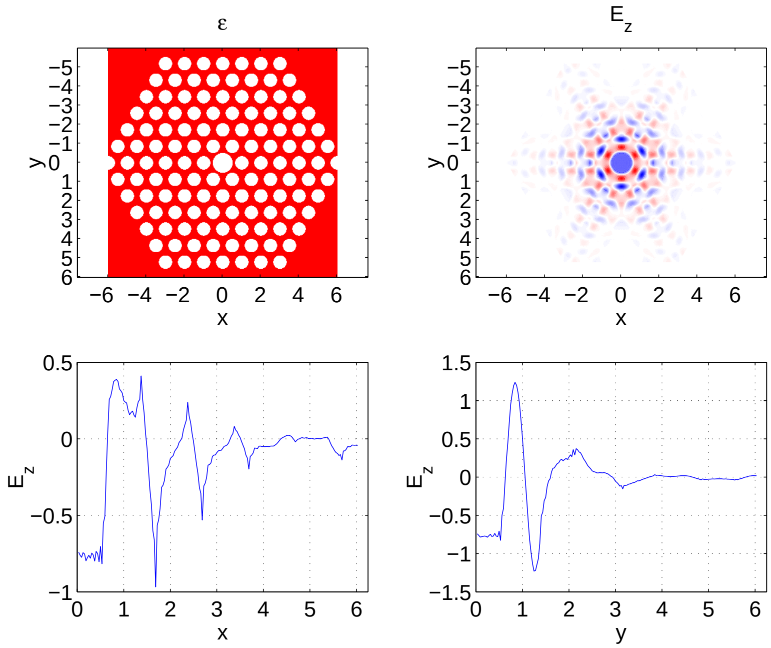


Figure 1: The dielectric distribution and the field distribution of the defect mode trapped by 2D PBG fiber.[1] In the top right plot, color coding, blue and red, represents negative and positive values respectively. The rod-to-rod spacing is $a/\lambda = 8.2/2\pi$, the radius of the central hole is $r_0 = 0.52a = 0.678\lambda$, and the radius of the other air cores is $r = 0.35a = 0.457\lambda$. This structure has a characteristic impedance $Z_c = 19.5\Omega$, a group velocity $\beta_g = v_g/c = 0.58$, and phase velocity $v_\varphi = c$.

Photonic Band Gap Fiber Accelerator at Maximum Efficiency

Quantity	General Expression	For Figure 1 ($Z_c = 19.5\ \Omega$, $Z_H = 130\ \Omega$)
Charge for max. efficiency	$q_{\max} = \dfrac{\sqrt{PZ_c}\lambda}{2cZ_H}$	$q_{\max}(pC) = 6\times 10^{-5}\sqrt{P(W)}$
Maximum efficiency	$\eta_{\max} = \dfrac{\beta_g Z_c}{4Z_H\left(1-\beta_g\right)}$	$\eta_{\max} = 0.052$
Loaded gradient at max. efficiency	$G = \dfrac{\sqrt{PZ_c}}{2\lambda}$	$G(Gev/m) = 2.08\times 10^{-3}\sqrt{P(W)}$
Beam energy gain at max. efficiency	$\dfrac{dU_{beam}}{dL} = \dfrac{PZ_c}{4cZ_H}$	$\dfrac{dU_{beam}(J/m)}{dL} = 1.25\times 10^{-10} P(W)$

fraction of the structure that is glass and the fraction that is vacuum, but the impedance doesn't depend on the dielectric constant in the limit $\gamma \to \infty$. Alternatively, the effective radius could be larger than the geometric radius, and that would have an important impact.[3] $Z_H = 130\Omega$ for the structure in Figure 1.

The loaded gradient is

$$G = G_0 - \frac{qcZ_H}{\lambda^2} = \frac{\sqrt{PZ_c}}{\lambda} - \frac{qcZ_H}{\lambda^2}, \tag{3}$$

and the gain in kinetic energy of the beam in a structure of length L is $\Delta U_{beam} = qGL$. The laser pulse length has to be the difference between the times it takes the beam and laser to travel length L, which is

$$\tau = \frac{L}{\beta_g c} - \frac{L}{c} = \frac{L}{c}\frac{1-\beta_g}{\beta_g} \tag{4}$$

The laser energy that was needed to produce this gradient is $\Delta U_{laser} = P\tau = P\left(1-\beta_g\right)L\big/\beta_g c$. The efficiency is

$$\eta = \frac{\Delta U_{beam}}{\Delta U_{laser}} = \frac{q\beta_g c}{P\left(1-\beta_g\right)}\left(\frac{\sqrt{PZ_c}}{\lambda} - \frac{qcZ_H}{\lambda^2}\right). \tag{5}$$

The efficiency can be maximized by finding the charge, q_{max}, for which $d\eta/dq = 0$. The results for various quantities at maximum efficiency are in the table.

- The maximum efficiency of 5.2% implies that 10 MW of average beam power, which is typical for a collider, would require 190 MW of average laser power.
- The beam radiates Cherenkov radiation. When $q = q_{max}$ equal amounts of energy are gained by the beam and radiated as Cherenkov radiation. The beam does not gain any energy for $q = 2q_{max}$, but a fraction $2\eta_{max}$ of the laser energy is emitted as Cherenkov radiation.

Damage & Raman Scattering

Laser-induced damage gives a gradient limit. For pulse length τ and fluence F in the fiber with dielectric constant = ε_r and index of refraction = n

$$Intensity = \frac{F}{\tau} = \frac{1}{2}\varepsilon_r\varepsilon_0\frac{c}{n}E^2 = \frac{\sqrt{\varepsilon_r}}{2}Z_0E^2. \tag{6}$$

This gives the electric field amplitude in terms of the fluence as

$$E = \sqrt{\frac{2FZ_0}{\sqrt{\varepsilon_r}\,\tau}}. \tag{7}$$

The maximum electric field in the fiber is 2.1 times the gradient,[1] so

$$G_0 = \frac{1}{2.1}\sqrt{\frac{2FZ_0}{\sqrt{\varepsilon_r}\,\tau}}. \tag{8}$$

Eqs. (1) and (8) can be combined to give an expression for the fluence in terms of the laser power

$$F = 2.21\frac{PZ_c\tau}{\lambda^2 Z_0}\sqrt{\varepsilon_r} \tag{9}$$

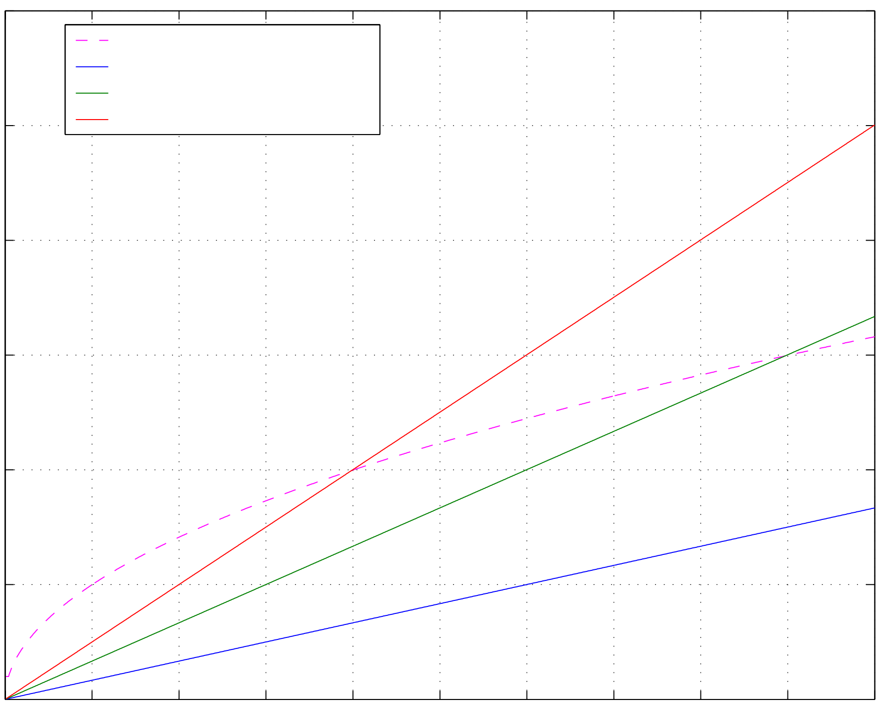
The effective area of the mode is given by eq. (9)

$$A_{eff} = \frac{P\tau}{F} = \frac{\lambda^2 Z_0}{2.21\sqrt{\varepsilon_r}\,Z_c} = 6\times 10^{-12}\,m^2 \tag{10}$$

There is a simple model for the damage fluence[4]

$$F(J/cm^2) \le \begin{cases} 3.16\sqrt{\tau(p\sec)} & \tau > 0.4\,p\sec \\ 2 & \tau \le 0.4\,p\sec \end{cases}. \tag{11}$$

These equations plotted together are in the figure below for powers that give unloaded gradients in the 0.5 GeV/m range. **7/2002 – SEE NOTE BELOW**



Laser Power	**10 kW**	**20 kW**	**30 kW**
G_0, Unloaded gradient	0.44 GeV/m	0.62 GeV/m	0.77 GeV/m
G, Loaded gradient at optimum efficiency	0.22 GeV/m	0.31 GeV/m	0.38 GeV/m
q_{max}, Optimum charge	6×10^{-3} pC (3.75×10^{4} e-'s)	8.5×10^{-3} pC (5.30×10^{4} e-'s)	10.4×10^{-3} pC (6.50×10^{4} e-'s)
f, Bunch frequency for 1×10^{14} e-'s	2.67 GHz	1.89 GHz	1.54 GHz

NOTE – THIS DAMAGE MODEL ISN'T A VERY GOOD FIT TO THE DATA. A BETTER MODEL WOULD HAVE BEEN

$$F(J/cm^2) \leq \left\{ \begin{array}{cc} 1.44\sqrt{\tau(p\sec)} & \tau > 10p\sec \\ 2510\sqrt[4]{\tau(p\sec)} & 10p\sec > \tau > 0.4p\sec \\ 2 & 0.4p\sec > \tau \end{array} \right\}. \quad (12)$$

The critical power for stimulated Raman scattering is defined as the input pump power at which the power at the Stokes line is equal to the pump power. It is[1]

$$P_0^{cr} = \frac{16}{g_2}\frac{A_{eff}}{L} \quad (13)$$

where $g_2 = 1\times10^{-13}$ m/W. This gives the critical fluence $F_0^{cr} = 16/g_2\beta_g c = 92\ J/cm^2$. The fluences in the figure above are well below that value.

Power Estimates

It is known from other collider designs that average beam powers ~ 10 MW are necessary. The table gives the optimum charge and bunch frequency for accelerating 10^{14} electrons, which is 8 MW of average beam power at 500 GeV. (This requires an average laser power of 0.36 GW when $\eta = 0.022$). Bunch frequencies must be several GHz. This can be accomplished through multiple bunches/laser pulse and/or laser repetition rate.

The two issues that are immediately clear are the average laser power and the high bunch frequency.

Open Questions

The low efficiency is leading to a configuration of the PBGFA fiber being incorporated into a laser cavity so the laser energy can be recovered and reused. The laser is likely to have a frequency in the 50 – 100 MHz range, so there must be multiple bunches/laser pulse to obtain the GHz bunch frequency needed. Among the many issues still be addressed.

1) A configuration that is consistent with staging and vacuum requirements.
2) Coupling of laser light into and out of the fiber in a manner consistent with a passage for the beam
3) The efficiency that is possible if the PBGFA is incorporated into a laser cavity.
4) Gradient limitation from ionization.[5]
5) Phase locking of the drive lasers. Can the techniques developed now be applied to other systems, e.g. fiber lasers? Is there a role for beam synchronization as in Levi's active media concept?
6) The Cherenkov impedance of the complex PBGFA structure.

[1] Xintian Eddie Lin, "Photonic Band Gap Fiber Accelerator" *Physical Review Special Topics – Accelerators and Beams* **4**, 051301 (2001).
[2] R. Siemann, ARDB-279.
[3] Levi Schächter, private communication.
[4] B. C. Stuart, M. D. Feit, A. M. Rubenchik, B. W. Shore, and M. D. Perry, *Phys. Rev. Lett*. **74**, 2248 (1995).
[5] P. Sprangle *et al*, Phys. Rev. E **55**, 5964 (1997).