

## Beam optics of a self-focusing plasma lens

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In this paper we analyze the optics of a high-energy beam which is focused by its own wakefields in a plasma. We calculate the effects of lens aberrations on the focusing strength of the lens and on the dilution of the beam's phase space. From this we derive the minimum spot size achievable using a bi-Gaussian beam and, after inclusion of the beam-beam pinch effect, the luminosity enhancement that can be gained in principle. We estimate the luminosity enhancement in the case of SLAC Linear Collider beam design parameters, and discuss limitations and possible improvements in plasma-lens performance.

### I. INTRODUCTION

The plasma lens has been discussed recently as a promising candidate for a luminosity-enhancing final focus element.<sup>1-4</sup> The calculation of the plasma physics involved is somewhat intricate, but the basic physical mechanism is simply understood under certain conditions: (1) the beam is less dense than the plasma  $n_b \ll n_0$ , (2) the beam length  $\sigma_z$  is large compared to the plasma wavelength  $\lambda_p = (\pi r_e / n_0)^{1/2}$ , and (3) the beam width  $\sigma_x$  is small compared to the plasma wavelength. If these criteria are satisfied, then the plasma electrons move to approximately neutralize the beam charge, leaving the beam-current self-pinching forces unbalanced. In this case the focusing wakefields reduce, to a good approximation, to the magnetic self-fields of the beam. This is the regime of largest-focusing wakefields inside the beam, and is the most interesting case for use in final focusing systems. As a most relevant example, the design parameters of the SLAC Linear Collider (SLC) beam near its planned final focus can satisfy all these criteria if the plasma density is in the range  $n_0 = 10^{18} - 10^{19} \text{ cm}^{-3}$ . If we assume a cylindrically symmetric bi-Gaussian beam-density profile given by

$$\rho_b = n_b e^{-r^2/2\sigma_r^2} e^{-z^2/2\sigma_z^2}, \quad (1)$$

where  $n_b = N / (2\pi)^{3/2} \sigma_z \sigma_r^2$ , then the magnetic self-forces everywhere inside the beam can be easily calculated (to order  $\gamma^{-2}$ ) to be

$$F_r(r, z) = K_0 \gamma m_e c^2 e^{-z^2/2\sigma_z^2} \left[ \frac{2\sigma_r^2}{r} (1 - e^{-r^2/2\sigma_r^2}) \right]. \quad (2)$$

We have defined here a maximum focusing strength (in the core of the beam)  $K_0$ , which is calculated as

$$K_0 = \frac{Nr_e}{\sqrt{2\pi}\beta_0\epsilon_n\sigma_z}, \quad (3)$$

where  $r_e = 2.82 \times 10^{-13} \text{ cm}$  is the classical electron ra-

dius,  $\beta_0$  is the  $\beta$  function at the lens, and  $\epsilon_n$  is the normalized emittance of the beam. The radial force can be used to define a radial focusing strength, a function of position  $(r, z)$ ,

$$K_r(r, z) = \frac{F_r}{r\gamma m_e c^2} = K_0 e^{-z^2/2\sigma_z^2} \left[ \frac{2\sigma_r^2}{r^2} (1 - e^{-r^2/2\sigma_r^2}) \right]. \quad (4)$$

For a perfect lens,  $K_r$  would of course be a constant, with no dependence on position in the beam. For this to be true for the plasma lens, the beam density must be constant, as this produces a magnetic self-force linear in  $r$  and independent of  $z$  (Ref. 1). Bi-Gaussian beams yield self-forces that are less than this ideal. In this paper we calculate the effects of these aberrations on the final spot size one can achieve with a plasma lens, and estimate the possible enhancement on the luminosity, by taking into consideration both the contribution from the reduction of the spot size due to a plasma lens and that from the additional pinching due to beam-beam disruption.

We would like to employ the notation and formalism of Twiss parameters in our discussion, so we must take the expression for focusing strength in cylindrical coordinates and convert to the equivalent effect in Cartesian coordinates. We first note that for cylindrically symmetric distributions we need only examine one transverse coordinate ( $x$ ) and we have simply  $\sigma_x = \sigma_r$  and

$$K_x = \frac{F_x}{x\gamma mc^2} = \frac{F_r \cos\theta}{r(\cos\theta)\gamma mc^2} = K_r. \quad (5)$$

Thus,  $K_x$  is a function of all three coordinates  $(x, y, z)$  explicitly

$$K_x = K_0 e^{-z^2/2\sigma_z^2} \left[ \frac{2\sigma_x^2}{x^2 + y^2} (1 - e^{-(x^2 + y^2)/2\sigma_x^2}) \right]. \quad (6)$$

Since the beam distributions are assumed separable in

longitudinal and transverse coordinates, the variation in the focusing strength can be identified as arising from longitudinal and radial aberrations separately. The longitudinal aberrations are statistical in nature, since the position in  $x$  is uncorrelated to its longitudinal coordinate. The radial aberrations when projected onto the Cartesian representation have both a spherical aberration dependent on  $x$  (the focusing strength falls off at larger  $x$ ), and a statistical portion which enters in through  $y$ . Inclusion of nonzero values of both  $y$  and  $z$  degrade the calculated focusing strength.

## II. RADIAL AND LONGITUDINAL ABERRATIONS

We first investigate the severity of the radial aberrations and their effect on the final spot size. To accomplish this, we digress for a moment to derive the transformations that a transverse phase space undergoes when it traverses a thin, aberration-prone lens. We consider a beam of initial Twiss parameters  $\alpha_0$  and  $\beta_0$  and emittance  $\epsilon_0$ . The effect that any source of additional, phase-space diluting divergence  $\delta\theta$  (rms) is given by

$$\alpha = \left[ \frac{(1 + \alpha_0^2)\epsilon_0 + \beta_0\delta\theta^2}{\epsilon_0 + \beta_0\delta\theta^2} - 1 \right]^{1/2}, \quad (7)$$

$$\beta = \beta_0 \frac{(\epsilon_0^2 + \beta_0\epsilon_0\delta\theta^2)^{1/2}}{\epsilon_0 + \beta_0\delta\theta^2}, \quad (8)$$

$$\epsilon = (\epsilon_0^2 + \beta_0\epsilon_0\delta\theta^2)^{1/2}. \quad (9)$$

We now wish to calculate an rms divergence due to radial plasma-lens aberrations. The model we are employing assumes that the phase space has an extra source of divergence  $\delta\theta$  that is independent of  $x$ . This is not precisely the case, however. The rms divergence arising from an uncertainty in focusing strength  $\Delta K/\bar{K}$  ( $\bar{K} \equiv \langle K_x \rangle_y$ , the average focusing strength at point  $x$ ) generated through the random variable  $y$  at any given point in  $x$  requires an average over  $y$  as follows:

$$\delta\theta = \frac{\sigma_x}{f} \left\langle \frac{x}{\sigma_x} \frac{\Delta K}{\bar{K}} \right\rangle_y. \quad (10)$$

To calculate this quantity, we set  $z=0$  in Eq. (6) and perform the integration by Monte Carlo methods. A plot of the integral is shown in Fig. 1, which also shows the average focusing strength  $\bar{K}$  as a function of  $x$ . Note that the average focusing strength falls from approximately 0.8 near the origin to one-half that at  $x=2\sigma_x$ . The normalized rms divergence  $\langle (x/\sigma_x)(\Delta K/\bar{K}) \rangle_y$  rises from zero near the origin to approximately 0.2 over much of the bunch population. Two effects are important here, the first being that larger impact-parameter particles obviously generate a linearly larger (in  $x$ ) rms divergence than smaller impact-parameter particles. On the other hand, the quantity  $\Delta K/\bar{K}$  is largest at small  $x$ , since the random variable  $y$  has its strongest impact there, as can be deduced from inspection of Eq. (6). If we sum over all  $x$  to obtain a single parameter characterizing radial aberrations we find that  $\langle \langle (x/\sigma_x)(\Delta K/\bar{K}) \rangle_y \rangle_x \simeq 0.2$ , and that the average focusing strength is  $\langle \bar{K} \rangle_x \simeq 0.7K_0$ .

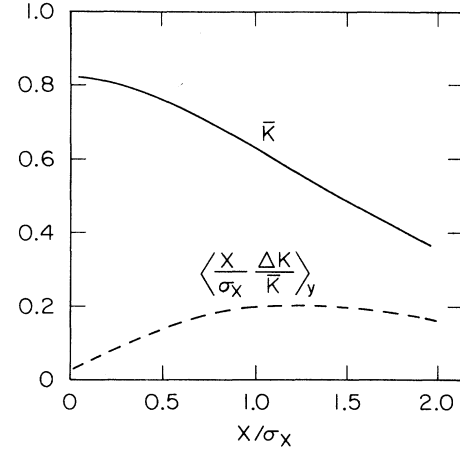


FIG. 1. The average focusing strength  $\bar{K} \equiv \langle K_x \rangle_y$ , and the rms divergence increase  $\langle x\Delta K/\sigma_x\bar{K} \rangle_y$  as a function of  $x$ .

The plasma-lens transformations on a phase space are shown explicitly in Fig. 2. The initial phase-space population ( $\beta=\epsilon=1$ ,  $\alpha=0$ ) in Fig. 2(a) is transformed by a linear (aberration-free) lens of nominal focal length  $f_0=(K_0l)^{-1}=\beta/3$  to the population in Fig. 2(b). The case of a thin plasma lens of the same focal length is shown in Fig. 2(c) for comparison, where we have only considered radial aberrations. Notice that the average focusing is noticeably smaller for the plasma-lens case, and that the rms divergence increase is indeed quite uniform over the population, with no strong dependence on  $x$ , validating our approach to the Twiss parameter transformations for radial aberrations in Eqs. (7)–(9). This result agrees quite well with the particle-in-cell simulation

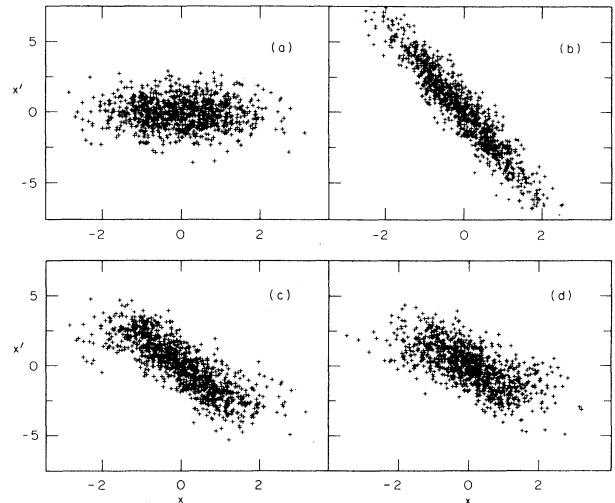


FIG. 2. (a) Initial phase-space population ( $\beta=1$ ,  $\epsilon=1$ ,  $\alpha=0$ ). (b) Transformation of phase space by a thin lens  $f=\beta/3$ , for comparison with plasma lens. (c) Transformation of phase space by a thin plasma lens  $f_0=\beta/3$ , radial aberrations included. (d) Transformation of phase space by a thin lens  $f=\beta/3$ , all aberrations included.

using the computer code PIC4 developed recently by Simpson.<sup>5</sup>

Longitudinal aberrations can be treated easily, since there is no intricate correlation with the transverse position. Unfortunately, the rms-induced divergence angle is now linear in  $x$ , since  $\Delta K/\hat{K}$  is independent of  $x$ , and the longitudinal aberrations do not fit our phase-space dilution model as well as the radial aberrations. Setting  $x=y=0$  in Eq. (6), we can calculate analytically the reduction in average focusing strength from longitudinal aberrations,

$$\begin{aligned}\hat{K} &\equiv \langle K_x \rangle_z = K_0 \frac{1}{\sqrt{2\pi}\sigma_z} \int_{-\infty}^{\infty} (e^{-z^2/2\sigma_z^2})^2 dz \\ &= \frac{1}{\sqrt{2}} K_0.\end{aligned}\quad (11)$$

We can also calculate the rms value of  $\Delta K/\hat{K}$ :

$$\begin{aligned}\left\langle \frac{\Delta K}{\hat{K}} \right\rangle_z &= \left[ \frac{1}{\sqrt{2\pi}\sigma_z} \int_{-\infty}^{\infty} (1 - \sqrt{2}e^{-z^2/2\sigma_z^2})e^{-z^2/2\sigma_z^2} dz \right]^{1/2} \\ &= (\sqrt{4/3} - \frac{1}{2})^{1/2} \simeq 0.33.\end{aligned}\quad (12)$$

The rms divergence increase comes from a final integral over  $x$ , which gives

$$\begin{aligned}(\langle x^2 \rangle_x)^{1/2} &= \left[ \frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^{\infty} x^2 e^{-x^2/2\sigma_x^2} dx \right]^{1/2} \\ &= \sigma_x.\end{aligned}\quad (13)$$

The total divergence increase is obtained by adding the longitudinal and radial aberrations in squares

$$\delta\theta^2 \simeq \left[ \frac{\sigma_x}{f} \right]^2 (0.33^2 + 0.2^2) \simeq 0.15 \left[ \frac{\sigma_x}{f} \right]^2. \quad (14)$$

A phase-space transformation for a plasma lens that includes all aberrations is shown in Fig. 2(d). For the remainder of the discussion we take this coefficient of 0.15 to be designated as the parameter  $\delta^2$ . This number has been calculated for a bi-Gaussian distribution; it would be smaller for more uniform distributions.

With this understanding of the divergence increase, we proceed with our Twiss parameter treatment of the aberrations. With  $\sigma_x^2 = \beta_0 \epsilon_0$  substituted into Eq. (14), Eqs. (7)–(9) simplify to

$$\alpha = \alpha_0/P, \quad \beta = \beta_0/P, \quad \epsilon = \epsilon_0 P, \quad (15)$$

where

$$P = \left[ 1 + \left[ \frac{\beta_0}{f} \delta \right]^2 \right]^{1/2} \quad (16)$$

is defined to be the aberration power, and the focal length  $f \simeq 2f_0$  for a bi-Gaussian bunch.

The average focusing in the thin lens changes the phase-space ellipse orientation further to give the final value

$$\alpha_f = (\alpha_0 + \beta_0/f)/P. \quad (17)$$

The total effect of the lens on the phase-space ellipse orientation is diluted by a factor of  $P$  due to the aberrations. The other beam parameters are unchanged by the average focusing in the lens.

### III. THE THICK-LENS CORRECTION

We now have all the optics tools to conceptually design a thin plasma-lens final focus system. Unfortunately, the present phase-space densities in linear collider beams may not be large enough to provide sufficient focusing in a thin lens. The normalized focal length, which should be moderately small compared to unity, is

$$\frac{f}{\beta_0} = \frac{2}{\beta_0 K_0 l} = \frac{\sqrt{8\pi}\sigma_z \epsilon_n}{N r_e l}, \quad (18)$$

where for SLC design parameters  $N = 5 \times 10^{10}$ ,  $\sigma_z = 1$  mm,  $\epsilon_n = 3 \times 10^{-5}$  mrad, and  $l$  is the length of the thin lens. Thus, we can estimate

$$\frac{f}{\beta_0} \simeq \frac{1}{9.3[l \text{ (cm)}]} \text{ for SLC.} \quad (19)$$

Note that if we want to take  $f/\beta_0 = \delta$  to maximize the focusing efficiency, we calculate that  $l = 3.6$  mm, which is not small compared to the conventional final focus beta  $\beta_0^* = 7$  mm. Thus, the thin-lens approximation, which assumes that the beam size is constant inside the lens, does not hold in this case. We are in need of a thick-lens model.

We start by writing the equation of motion for the beam  $\beta$  function, and attempt to solve analytically whatever differential equation arises. The starting point is the first-order Twiss parameter equations

$$\alpha' = K\beta - \gamma, \quad (20)$$

$$\beta' = -2\alpha, \quad (21)$$

and

$$\gamma' = 2K\alpha. \quad (22)$$

The prime indicates differentiation with respect to distance along the beamline  $s$ . From these, we can derive the familiar third-order linear differential equation for the  $\beta$  function

$$\beta''' + 4K\beta' + 2K'\beta = 0. \quad (23)$$

For the situation at hand we take

$$K = 0.7K_0 = \xi/\beta \quad (24)$$

to examine the transverse slice at  $z=0$ , where  $\xi$  is a quantity proportional to the total phase-space density of the beam which is numerically equal to  $1.3 \times 10^3 \text{ m}^{-3}$  for SLC parameters.

To solve Eq. (23) we must first integrate through the  $\delta$  function in  $K'$  at the start of the lens

$$\Delta\beta_0'' = -2K\beta_0. \quad (25)$$

The other two initial conditions are just continuity requirements  $\beta' = \beta_0'$  and  $\beta = \beta_0$ . Also note that  $\beta_0'' = 2/\beta_0^*$

just before the lens, where  $\beta_0^*$  is the value at the waist that would be formed in the absence of the lens.

Now that we have the correct initial values, we can rewrite Eq. (23), with Eq. (24) inserted, as a nonlinear third-order elliptical differential equation:

$$\beta''' + \frac{2\zeta\beta'}{\beta} = 0. \quad (26)$$

The first integral of this equation is, using the derived initial conditions,

$$\beta'' + 2\zeta[\ln(\beta/\beta_0) + 1] - 2/\beta_0^* = 0. \quad (27)$$

This nonlinear equation may also be integrated easily by multiplying by  $\beta'$ , and applying the continuous initial conditions on  $\beta$  and  $\beta'$ :

$$\left[ \frac{\beta'}{2} \right]^2 = \frac{\beta - \beta_0^*}{\beta_0^*} + \zeta\beta[\ln(\beta_0/\beta)]. \quad (28)$$

We can numerically integrate Eq. (28) to provide the correction to the thin-lens theory. As an example, we take a lens which starts 5 mm from the final focus of an SLC beam, and has a thickness  $l$  of 3 mm. The plasma density is taken to be  $n_0 = 5 \times 10^{18} \text{ cm}^{-3}$ . The effective thin-lens focal length for this case is calculated, with the thin lens placed at the midpoint of the plasma region, and found to be 3.34 mm. A naive calculation from the thin-lens formula would yield  $f = (KI)^{-1} = 3.64 \text{ mm}$ . This is in contrast with conventional thick-lens behavior, where the focal length of the lens would rise with use of the correct thick-lens expression, because of the non-negligible phase advance. The focal length drops in a thick plasma lens because the lens gets stronger as the beam pinches.

#### IV. LUMINOSITY ENHANCEMENT

There are two sources that contribute to luminosity enhancement in a final focus system invoking a plasma lens. The first enhancement, designated as  $H_{D1}$ , is associated with the reduction of the beam-spot size. The second enhancement  $H_{D2}$  comes from the beam-beam disruption effect. We shall estimate them in the following.

We wish to see what improvement can be made in ultimate spot size using a strong yet aberration-prone lens. We note first the reduction in spot size from the lens position to the next waist is, using Eqs. (15)–(17),

$$\frac{\beta^*\epsilon}{\beta_0\epsilon_0} = \frac{P^2}{P^2 + (\alpha_0 + \beta_0/f)^2}. \quad (29)$$

However, the eventual spot size would be given by  $\beta_0^*\epsilon_0$  in the absence of the added lens, with

$$\beta_0 = \beta_0^*[1 + (s/\beta_0^*)^2]. \quad (30)$$

The net compression factor (or the luminosity enhancement excluding beam-beam disruption effects) realized is thus

$$H_{D1} \equiv \frac{\beta_0^*\epsilon_0}{\beta^*\epsilon} = \frac{\beta_0^*}{\beta_0} \left[ \frac{1 + \left[ \frac{\beta_0}{f} \delta \right]^2 + \left\{ \left[ \frac{\beta_0}{\beta_0^*} \right] - 1 \right\}^{1/2} + \beta_0/f}{1 + \left[ \frac{\beta_0}{f} \delta \right]^2} \right]^2. \quad (31)$$

This expression shows clearly the limitations of an aberration-prone lens such as the plasma lens. The compression one obtains is a strongly increasing function of the parameter  $(\beta_0/f)$  in the absence of aberrations. The aberration term in the denominator is  $(\beta_0\delta/f)^2$ , however, and we note that the aberration effects are amplified by the strong compression force. For our assumed  $\delta$  and  $f < \beta_0$ , the function given by Eq. (31) is a maximum at about  $\beta_0 \approx \beta_0^* + f$ . Thus, in the case of large compression force ( $\beta_0 \gg f$ ), the optimum placement of the plasma lens is very near the minimum spot one obtains from conventional focusing and Eq. (31) becomes, for large  $\delta$ ,

$$\frac{\beta_0^*\epsilon_0}{\beta^*\epsilon} = \frac{\beta_0^*}{\beta_0} \frac{1}{\delta^2}. \quad (32)$$

Thus, the limitation on luminosity enhancement due to aberrations is about  $1/\delta^2 \approx 6.7$ . If one overdoes the focusing, i.e., makes  $\beta_0/f \gg 1/\delta$ , the consequences are more severe than merely saturation of the luminosity, however. The rms angle of the beam leaving the lens becomes very large in this case, and that poses the problem of damaging conventional final focusing elements, as well as lowering the possible luminosity boost due to beam-beam disruption. We will return to this point later.

It is of interest also to calculate the position of the next waist. We obtain in a similar manner the distance as measured from the strong lens:

$$L = \frac{\beta_0 \left\{ \left[ \frac{\beta_0}{\beta_0^*} \right] - 1 \right\}^{1/2} + \beta_0/f}{1 + \left[ \frac{\beta_0}{f} \delta \right]^2 + \left\{ \left[ \frac{\beta_0}{\beta_0^*} \right] - 1 \right\}^{1/2} + \beta_0/f}^2. \quad (33)$$

The luminosity enhancement that one calculates from application of Eq. (31) with the thick-lens corrected parameters, i.e., with  $n_0 = 5 \times 10^{18} \text{ c.c.}$ ,  $l = 3 \text{ mm}$ , and the principal plane at  $f = 3.34 \text{ mm}$ , is about  $H_{D1} \approx 4.1$ . The new waist is about 0.8 mm in front of the old waist, so there must be some adjustment of the conventional optics to get the final foci for both beams to coincide. The emittance has been blown up by a factor  $P = \sqrt{1 + 6.58^2} \approx 1.4$ , and the final  $\beta$  function  $\beta^* = 7 \text{ mm}/(4.1 \times 1.4) \approx 1.2 \text{ mm}$ .

Next we estimate the contribution from beam-beam disruption effects. The disruption effects from the interaction of round  $e^+e^-$  beams have been recently studied in detail by Chen and Yokoya.<sup>6</sup> It occurs that the

luminosity enhancement in this case is influenced by two factors: the strength of the pinch, represented by the disruption parameter  $D$ ,

$$D \equiv \frac{Nr_e \sigma_z}{\gamma \sigma_0^2} = \frac{Nr_e \sigma_z}{\gamma \beta^* \epsilon}, \quad (34)$$

and the inherent divergence of the beam, represented by the parameter  $A$ ,

$$A \equiv \frac{\sigma_z}{\beta^*}. \quad (35)$$

The luminosity enhancement is found to satisfy the empirical scaling law

$$H_D = 1 + D^{1/4} \left[ \frac{D^3}{1 + D^3} \right] \left[ \ln(\sqrt{D} + 1) + 2 \ln \left[ \frac{0.8}{A} \right] \right]. \quad (36)$$

The above expression reproduces all the computer simulation data shown in Fig. 3 to an accuracy of around  $\pm 10\%$ . The results of these simulations do not take into account the correlations between final focusing and longitudinal position in the beam due to the plasma lens. To this extent, our analysis below is approximate.

With the design parameters of SLC, we see that with the given values of  $\beta_0^*$  and  $\epsilon_0$ ,

$$D_0 = 0.51 \text{ and } A_0 = 0.14. \quad (37)$$

Therefore, from Eq. (36), one expects to have an enhancement

$$H_{D0} \approx 1.4. \quad (38)$$

On the other hand, with the insertion of a plasma lens, we expect to have

$$D = 2.1 \text{ and } A = 0.83. \quad (39)$$

The new enhancement factor would then be

$$H_{D2} \approx 1.9. \quad (40)$$

Our overall enhancement on luminosity can now be estimated easily,

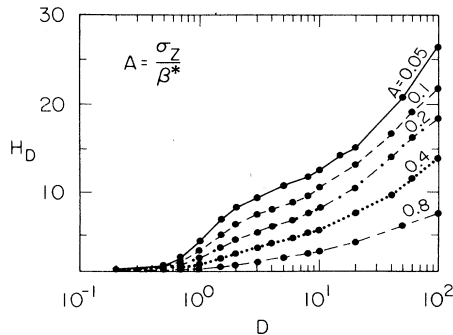


FIG. 3. Luminosity-enhancement factor as a function of  $D$ , computed with five different values of  $A$  (taken from Ref. 5). The  $A$  values are so chosen that they are equally separated on the logarithmic scale.

$$H_D = H_{D1} \frac{H_{D2}}{H_{D0}} \approx 5.6. \quad (41)$$

It can be seen that although we are able to push the disruption parameter  $D$  up by a factor of 4.1, the fact that we have substantially reduced  $\beta^*$  in turn has made the inherent divergence of the beam more severe. The net result is that one does not benefit too significantly from the mutual pinching during beam-beam interaction.

## V. DISCUSSION

To conclude, we have studied in this paper the beam optics of a self-focusing plasma lens by taking into account the aberrations due to nonlinear focusing strength and the correction due to finite thickness of the plasma lens. Furthermore, we have formulated the estimation of the luminosity enhancement, taking into account both the reduction of the effective spot size due to the plasma lens, and the pinch effect due to beam-beam interaction.

The design parameters of SLC were taken as an example to investigate the possible performance of a plasma lens. All the calculated parameters for an SLC plasma lens are listed in Table I. Our conclusion is that with the parameters so chosen, one could expect an enhancement on luminosity by a factor of around 5–6. To appreciate the performance of the plasma lens, let us consider a hypothetical strong lens which is free of aberration. The only effect that the strong lens introduces is the reduction

TABLE I. A set of plasma-lens parameters for SLC.

Plasma-lens parameters	Values
$n_0$ (cm <sup>-3</sup> )	$5 \times 10^{18}$
$l$ (mm)	3.0
Beam parameters	
$N$	$5 \times 10^{10}$
$\mathcal{E}$ (GeV)	50
$\epsilon_0$ (mrad)	$3.0 \times 10^{-5}$
$\epsilon$ (mrad)	$4.2 \times 10^{-5}$
$\sigma_z$ (mm)	1.0
$\zeta$ (m <sup>-3</sup> )	$1.3 \times 10^3$
Beam-optics parameters	
$s_0$ (mm) <sup>a</sup>	3.5
$f$ (mm)	3.3
$\alpha_0$	-0.5
$\beta_0$ (mm)	8.8
$\beta_0^*$ (mm)	7.0
$\beta^*$ (mm)	1.2
$\delta$	0.39
$P$	1.4
Luminosity enhancement	
$D_0$	0.51
$D$	2.1
$A_0$	0.14
$A$	0.83
$H_{D0}$	1.4
$H_{D1}$	4.1
$H_{D2}$	1.9
$H_D$	5.6

<sup>a</sup>Measured from lens midpoint.

of  $\beta^*$ . Now since both  $D$  and  $A$  vary as  $1/\beta^*$ , it can be seen from Eq. (36) that there is an optimum value of  $\beta^*$  below which  $H_D$  will be degraded. For the SLC parameters (except  $\beta^*$ ) this optimum  $\beta^*$  occurs at 2.24 mm, which corresponds to  $D=2.7$ ,  $A=0.45$ , and  $H_{D2}=8.59$ . This optimum value of  $H_{D2}$ , however, does not correspond to the best possible performance when the contribution from  $H_{D1}$  is also included. When the final lens is free of aberration, we have simply

$$H_{D1} = \frac{\beta^*}{\beta_0} \quad (42)$$

The variation of the combination  $H_{D1}H_{D2}$  as a function of  $\beta^*$  is plotted in Fig. 4. For the SLC parameters, the optimum is found to be  $\beta^*=1.05$  mm, and  $H_{D1}H_{D2}=12.7$ . Thus,

$$H_D = D_{D1} \frac{H_{D2}}{H_{D0}} \simeq 9.3 \quad (43)$$

With all other parameters fixed except  $\beta^*$ , the above value is the best luminosity enhancement that one could achieve for SLC, independent of the specific nature of the strong inserted lens. We thus find that the plasma lens in this case has a performance which is about 60% of that of an optimized ideal, aberration-free lens.

To improve the performance of the plasma lens, it is necessary to reduce the radial and longitudinal aberrations. As was pointed out earlier,<sup>1</sup> since the focusing strength in a plasma lens is self-induced by the beam-charge density, a proper shaping of the bunch can, in principle, mitigate the problem. One way to reduce the radial aberration is to install an octupole somewhere upstream from the plasma lens such that the transverse distribution can be more "flat-topped" than the Gaussian distribution.

For a bi-Gaussian distribution, the longitudinal aberrations have been shown to be more severe than the radial aberrations. In order to make the longitudinal distribution more uniform, we can in principle debunch the beam slightly by applying a nonlinear accelerating wave form to the beam and sending it through a transport line con-

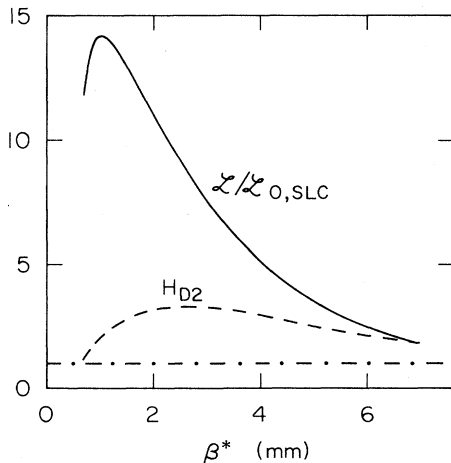


FIG. 4. The variation of  $H_{D1}H_{D2}$  as a function of  $\beta^*$ .

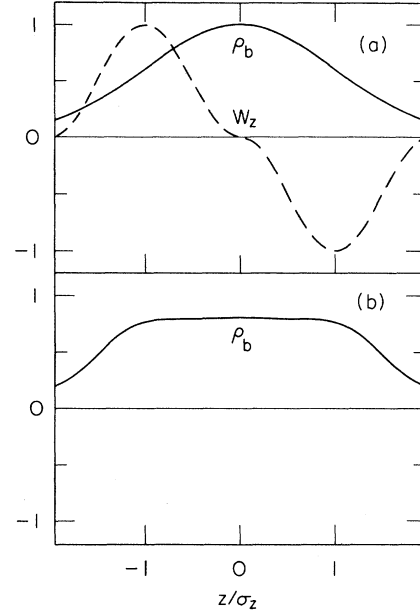


FIG. 5. (a) Longitudinal wakefields and profile for beam of length  $\sigma_z$  in iris-loaded structure with fundamental wake wavelength  $\lambda=2\sigma_z$ . (b) Longitudinal profile after wakefield debunching.

taining bend magnets with nonzero longitudinal dispersion. This could be done, for example, at the exit of the damping rings.

As an aside, we note that the nonlinear wave form could in principle be derived from the self-wakefields of the beam in an iris loaded tube.<sup>7</sup> We take the case of the beam length  $\sigma_z$  equal to one-half the fundamental wavelength of the wakefields. The initial beam profile and associated wakefields for this case are shown in Fig. 5(a). We take the initial rms momentum spread as  $\Delta p/p = 0.1\%$ . If the amplitude of the wakefield-induced momentum spread is taken to be  $1.5\Delta p/p$ , and the longitudinal dispersion of the transport line is  $\eta_z = -600\sigma_z$ , the final longitudinal distribution is flattened significantly, as shown in Fig. 5(b). Since the wave form for debunching is nearly sinusoidal this scheme is not dependent of wakefields, yet they may prove to be the handiest source of strong, short-wavelength fields.

After proposing improvements in the plasma lens, it is necessary to temper the discussion by noting that the parameters used in this paper describing the phase-space density of the beam at final focus are marginal for plasma-lens focusing. If the actual values of these parameters in any way reduce the phase-space density of the beam, the effectiveness of the plasma-lens focusing system degrades dramatically. We also have not mentioned the problems of background event generation from beam-plasma ion collisions, or the effects of misalignment due to beam jitter. Both considerations may place constraints on the effective implementation of a plasma lens.

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- <sup>1</sup>P. Chen, Part. Accel. **20**, 171 (1987).
- <sup>2</sup>P. Chen, J. J. Su, T. Katsuleas, S. Wilks, and J. M. Dawson, IEEE Trans. Plasma Sci. **PS-15**, 218 (1987).
- <sup>3</sup>J. B. Rosenzweig, B. Cole, D. J. Larson, and D. B. Cline, Part. Accel. (to be published).
- <sup>4</sup>D. B. Cline, B. Cole, J. B. Rosenzweig, and J. Norem, Proceedings of the IEEE Particle Accelerator Conference, 1987 (unpublished).
- <sup>5</sup>J. Simpson, PIC2 and PIC4, particle pushing simulation codes for plasma lens, Argonne National Laboratory Notes WF-76, 1988 (unpublished).
- <sup>6</sup>P. Chen and K. Yokoya, Phys. Rev. D **38**, 987 (1988).
- <sup>7</sup>H. Figueroa, W. Gai, R. Konecny, J. Norem, A. Ruggiero, P. L. Schoessow, and J. Simpson, Phys. Rev. Lett. **60**, 2144 (1988).