Asymmetric Gaussian as Used in E-157

This short note documents the asymmetric Gaussian function used in E-157. The asymmetric Gaussian is written as

$$f(x) = Back + \frac{C}{\sqrt{2\pi}\sigma}\left\{[1-\theta(x-<x>)]\exp\left(\frac{-(x-<x>)^2}{2\sigma_L^2}\right) + \theta(x-<x>)\exp\left(\frac{-(x-<x>)^2}{2\sigma_H^2}\right)\right\}.$$

where $\theta(u) = \begin{Bmatrix} 0 & u<0 \\ 1 & u>0 \end{Bmatrix}$ is a step function. The variables in this expression and their position in the array that is used in the asymmetric Gaussian routines are as follows.

Variable	Meaning	Array Element
$<x>$	Peak of asymmetric Gaussian	p(1)
σ	Average RMS width $\sigma = \frac{\sigma_L + \sigma_H}{2}$	p(2)
A	Asymmetry $A = \frac{\sigma_L - \sigma_H}{\sigma_L + \sigma_H}$; $\sigma_L = \sigma(1+A)$; $\sigma_H = \sigma(1-A)$	p(3)
C	Area under curve, i.e. the distribution is normalized, see below	p(4)
$Back$	Background	p(5)

The distribution is normalized. I.e. the amplitude C is the area under the curve. Ignoring the background

$$\int_{-\infty}^{\infty} f(x)dx = \frac{C}{\sigma}\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}\exp\left(\frac{-(x-<x>)^2}{2\sigma_L^2}\right)dx + \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\exp\left(\frac{-(x-<x>)^2}{2\sigma_H^2}\right)dx\right\}$$

$$= \frac{C}{\sigma}\left\{\frac{\sigma_L}{2} + \frac{\sigma_H}{2}\right\} = C$$