Thick Quadrupole Analysis of Run 05112ck

This note describes a thick quadrupole analysis of Run 05112ck, which is a run with many betatron envelope minima and maxima. The goal is to extract the proportionality constant between the UV energy measured by the incident UV meter and the plasma density.

Connections & Offset: Just prior to this run there was work on the amplifiers and attenuators for the UV energy meters. The incident UV meter was connected to the transmitted UV channel by accident. For this run the incident UV energy is read out in the "transmitted UV" channel. The UV energy and $\sigma_y(DN)$ for the early part of the run, shown in the Figure 1 below, indicate that

i) The beam is focused by a small amount of laser power;
ii) the laser was off for shots 5 – 29;
iii) the GADC measuring the incident UV has a zero of $G_0 = 11$ channels.

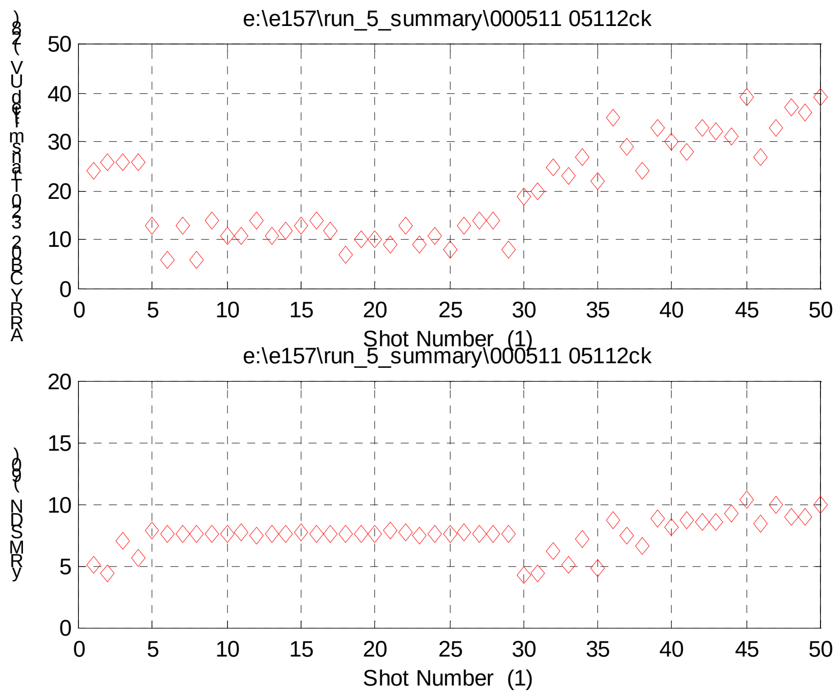

Figure 1: Incident UV and $\sigma_y(DN)$ for the early shots in the run

Algorithm: The algorithm is the same as discussed in earlier ARDB notes (ARDB-217, ARDB-221, ARDB-223, and ARDB-230). Let $\beta = \beta(L)$ denote the β-function at a downstream detector located a distance L from the end of the plasma. In terms of the initial Twiss parameters, β_0 and α_0, and the principle trajectories, C and S,

$$\beta = \beta(L) = C^2\beta_0 - 2CS\alpha_0 + S^2\gamma_0.$$

The principle trajectories are

$$C = \cos\varphi - \sqrt{k}L\sin\varphi,\ S = \frac{1}{\sqrt{k}}\sin\varphi + L\cos\varphi.$$

where φ is the phase of the quadrupole given in terms of the plasma density, n_I, beam energy, γ, and oven length, L_{oven}, by

$$\varphi = \sqrt{k}L_{oven} = \sqrt{\frac{2\pi r_e n_I}{\gamma}}L_{oven}.$$

Write

$$k = A(G - G_0);\ n_I = \frac{A\gamma}{2\pi r_e}(G - G_0) = \zeta(G - G_0).$$

to give the proportionality constants between GADC, the quadrupole k and the plasma density n_I. The equations $d\beta/dk = 0$ are solved for β_0, α_0, and A at the minima and maxima of the spot size on the detector.* A non-linear least-squares equation solver (MATLAB function `lsqnonlin`) is used. At least three data points (minima & maxima) are needed.

Data: Table 1 below gives the data for different detectors and dimensions. Note that the first minimum at low UV intensity is not included on the list. Data are shown in figures that follow. The vertical coordinate for the downstream OTR is the cleanest, particularly for $G < 400$. The vertical dimension of the Integrated Cherenkov is not analyzed further because of dispersion.

Table 1: GADC Readings for Minima and Maxima for Different Detectors and Planes

Downstream OTR				Integrated Cherenkov			
Horizontal		Vertical		Horizontal		Vertical	
Minima	Maxima	Minima	Maxima	Minima	Maxima	Minima	Maxima
65	45	62	38	52	36	100	36
140	100	126	102	115	90	250	160
250	200	230	180	220	160	440	340
450	335	415	320	410	310		
		680	560				

Results: The solution of the least squares solver depends on the starting values for the solution search. A Monte Carlo method is used to examine multiple starting points. This is best illustrated by Figure 2. Five thousand different starting points are randomly selected in the ranges: 0.1 m < β_0 < 0.3 m, -0.1 < α_0 < 0.1, 0.05 m^{-2} < A < 0.45 m^{-2}. The least squares solver is run, and the final values of β_0, α_0, and A are recorded together with the quality of the fit defined as

$$\chi^2 = \sum_n \left(\frac{d\beta(G_n)}{dk} \right)^2 .$$

where the G_n's are the values of the extrema included in the least squares solution. There are discrete solutions. The preferred one has β_0 = 0.209 m, α_0 = -0.083, A = 0.229 m^{-2}.

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* From ARDB-217, the derivative is given by

$$\frac{d\beta}{dk} = 2\left[\beta_0 C\frac{dC}{dk} - \alpha_0\left(S\frac{dC}{dk} + C\frac{dS}{dk}\right) + \gamma_0 S\frac{dS}{dk}\right],$$

$$\frac{dC}{dk} = -\frac{1}{2\sqrt{k}}\left((L + L_{oven})\sin\varphi + LL_{oven}\sqrt{k}\cos\varphi\right),$$

$$\frac{dS}{dk} = -\frac{1}{2\sqrt{k}}\left((LL_{oven} + \frac{1}{k})\sin\varphi - \frac{L_{oven}}{\sqrt{k}}\cos\varphi\right).$$

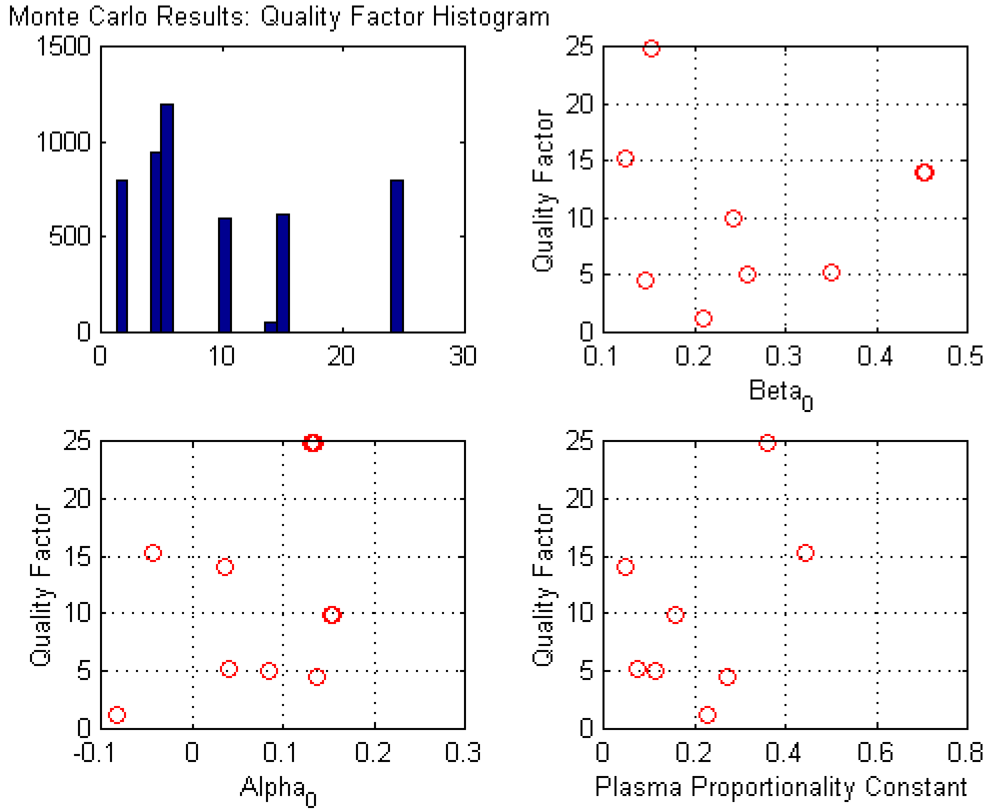


Figure 2: Results of 5,000 least squares solutions for the Downstream OTR, vertical coordinate with data up to G = 400 included. The range of starting values is given in the text. The top-left figure shows the histogram of the fit quality, and the other plots show the least squares solutions for the initial parameters (β_0, α_0 & A).

The quantity minimized, $-\sqrt{k}d\beta/dk$, is shown in Figure 3, and a comparison between the data and fit, with an overall scale factor applied to the fit, are shown in Figure 4.

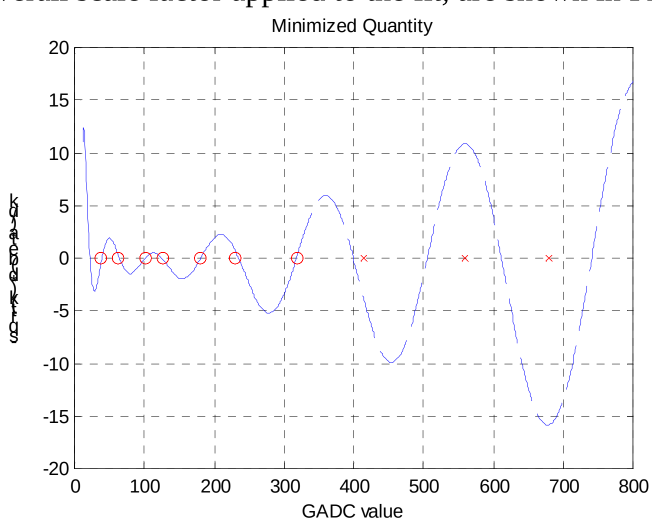
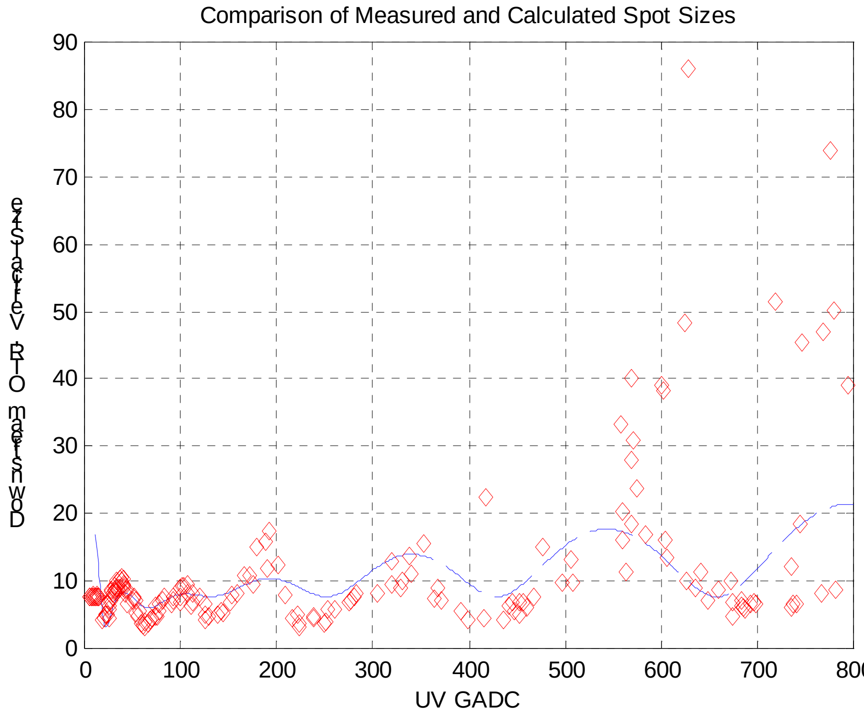


Figure 3: $-\sqrt{k}\,d\beta/dk$ for the vertical plane of the downstream OTR. The solid line is the least squares solution to the equation $d\beta/dk = 0$ at the 7 circled points. The 3 x's are points that were not included in the fit. The dashed line is an extrapolation of the fit.



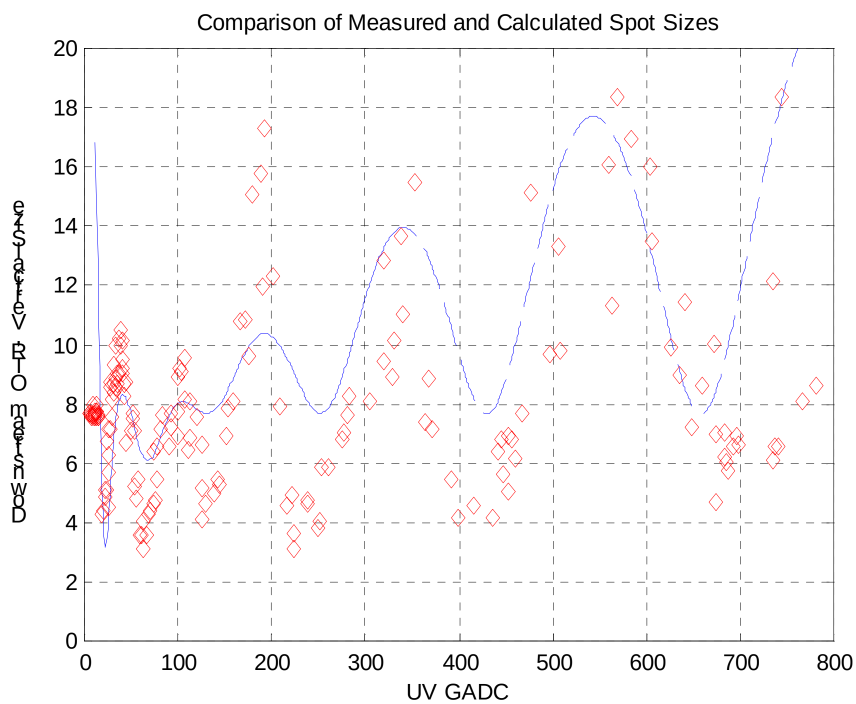


Figure 4: Comparison of $\sigma_y(DN)$ with the resultant fit for $G < 400$. Input data were the minima at [62, 126, 230, 415, 680] and the maxima at [38, 102, 180, 320, 560]. A solid line is drawn in the region that is fit. The dashed line is an extrapolation of the fit.

The same technique has been applied to the different detectors and planes and with different values of the GADC cuts. The results are presented in Table 2 below, and comparisons of solutions and data are in figures that follow. Table 2 has the solutions for the initial Twiss parameters, the proportionality constant between k and G, A, and ζ, the proportionality constant between the plasma density and GADC.

Table 2: Solutions to the least squares minimization

Device/Plane	GADC cut	β_0 (m)	α_0	γ_0 (m^{-1})	A (m^{-2})	ζ (cm^{-3})
DOWN, vert	300	0.195	-0.114	5.21	0.238	7.50E+11
DOWN, vert	400	0.209	-0.083	4.81	0.229	7.23E+11
DOWN, horiz	300	0.212	-0.050	4.73	0.210	6.64E+11
DOWN, horiz	400	0.202	-0.068	4.98	0.216	6.81E+11
CHER, horiz	320	0.227	0.062	4.42	0.227	7.15E+11

The proportionality constant of interest is

$$n_I(cm^{-3}) = \zeta(G - G_{0)}) = 7\times10^{11}(G - G_0).$$

The error on ζ is estimated as ±10% based on the range of values in the solutions in Table 2.

Direct Measurement of Plasma Density: UV calibration and white light absorption measurements were made just before run 05112ck. The result of the white light absorption measurement for the neutral density ρ_0 is (from Sho)

$$\rho_0 = 4.36\times10^{15}\ \text{cm}^{-3}$$

The UV calibration measurements are not "standard" because of the connection problem discussed above and because there is a large offset on the downstream energy meter GADC. Nonetheless, the data can be analyzed. The fit to the upstream energy meter from run 05112ci is

$$\textit{Upstream Energy} = (-16.56 + 0.5872G)\times13.13\ \mu\text{J/channel},$$

and the fit for the downstream energy meter from run 05112cj is

$$\textit{Downstream Energy} = (246.6 + 0.3982G)\times8.547\ \mu\text{J/channel}.$$

The plasma transmission equals the ratio of the slopes which is 0.4315. This implies

$$\rho_0 = 3.46\times10^{15}\ \text{cm}^{-3}$$

The UV spot size at the entrance to the lithium oven was measured on 5/09/00 to be 0.17 cm^2 (log VI, page 130). Using the measured neutral densities, the plasma density is linear with the incident UV measurement by

$$\zeta = \frac{dn_I}{dG} = \frac{dE}{dG}\frac{\lambda}{hc}\frac{\sigma}{A}\rho_0 = 0.5872\times13.13\times10^{-6}\ J/channel\,\frac{193\,nm}{1.99\times10^{-16}\,J-nm}\,\frac{1.8\times10^{-18}\,cm^2}{0.17\,cm^2}\rho_0$$

$$= 7.9\times10^{-5}\rho_0$$

$$= 2.7\times10^{11}cm^{-3}\ \text{to}\ 3.4\times10^{11}cm^{-3}.$$

The plasma density measured by the beam is two to three times higher than that measured by the UV. This is a significant disagreement that remains to be resolved.

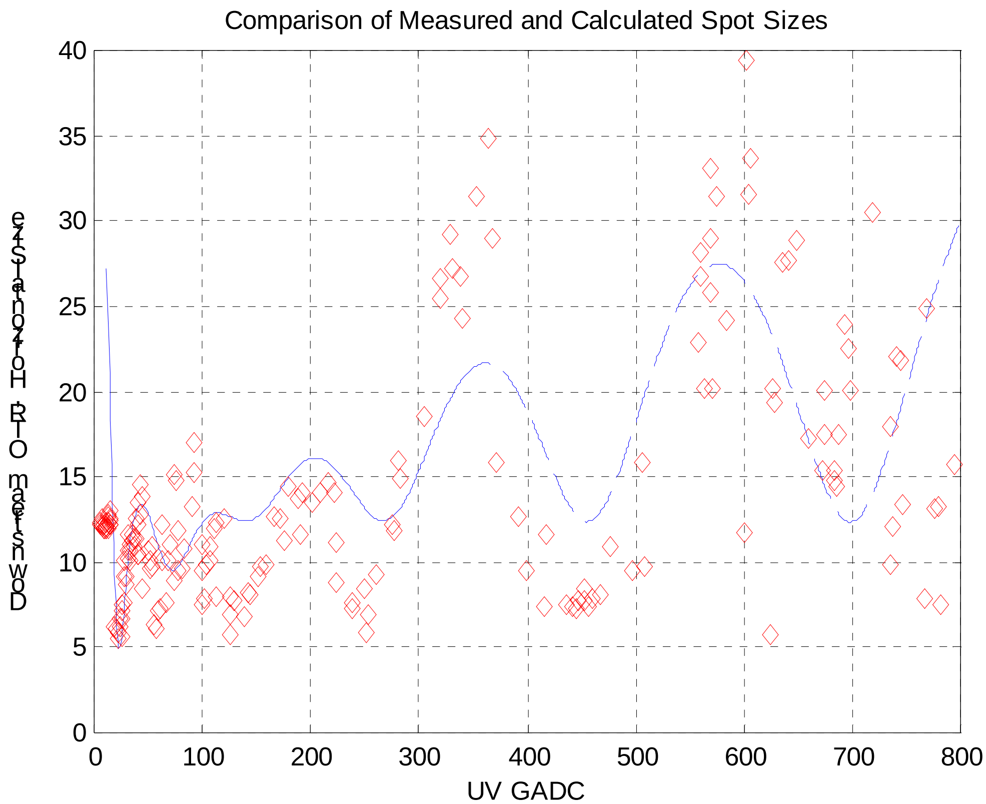


Figure 5: Comparison of $\sigma_x(DN)$ with the resultant fit for $G < 400$. Input data were the minima at [65, 140, 250, 450] and the maxima at [45, 100, 200, 335]. The solid and dashed lines have the same meaning as in Figure 4.

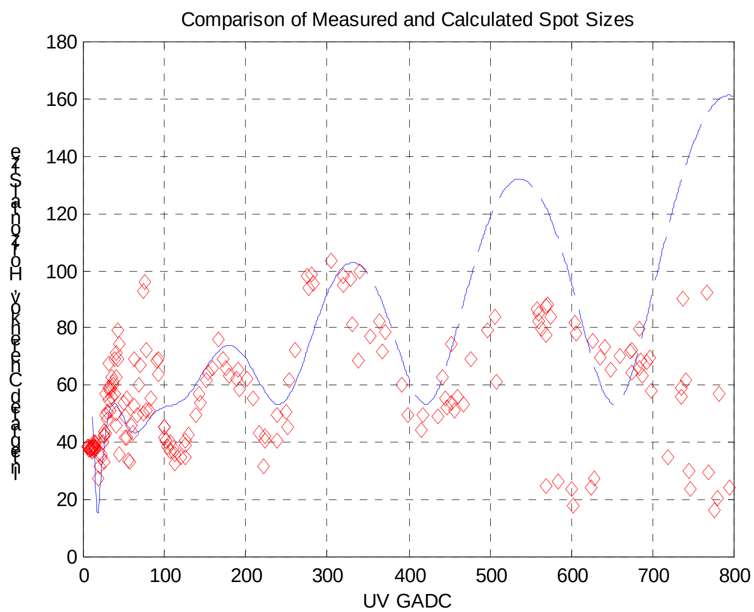


Figure 6: Comparison of $\sigma_x(CV)$ with the resultant fit for $G < 320$. Input data were the minima at [52, 115, 220, 410] and the maxima at [36, 90, 160, 310].