Analysis of Beam Envelope Data for Cherenkov Paper

This note is an analysis of beam envelope data used in the E-157 paper on plasma diagnostics via Cherenkov radiation. The observation is that there were minima in the beam envelope, measured on the downstream OTR, at 6 and 37 μsec and a maximum at 14 μsec. The values of the density based on Cherenkov radiation are in the table 1 below.

Table 1: Time & Density at Extrema

Delay Time (msec)	Density (´10^{13} cm^{-3})	Extrema
6	6.8	Minimum
14	4.8	Maximum
37	2.5	Minimum

A comparison of the time dependence of these measurements with work reported on the 25 cm oven is in Figure 1. There is reasonable agreement at the three values of the extrema of the beam envelope.

There are a number of relationships derived in earlier notes on betatron oscillations.

1) The relationships between plasma density, quadrupole k value, and betatron phase shift across the plasma

$$k = \frac{2pr_e n_I}{g},\ j = \sqrt{k}L_{oven}$$

2) The C and S trajectories evaluated at the downstream OTR (L from the end of the oven)

$$C = \cos j - \sqrt{k}L\sin j,\ S = \frac{1}{\sqrt{k}}\sin j + L\cos j\ .$$

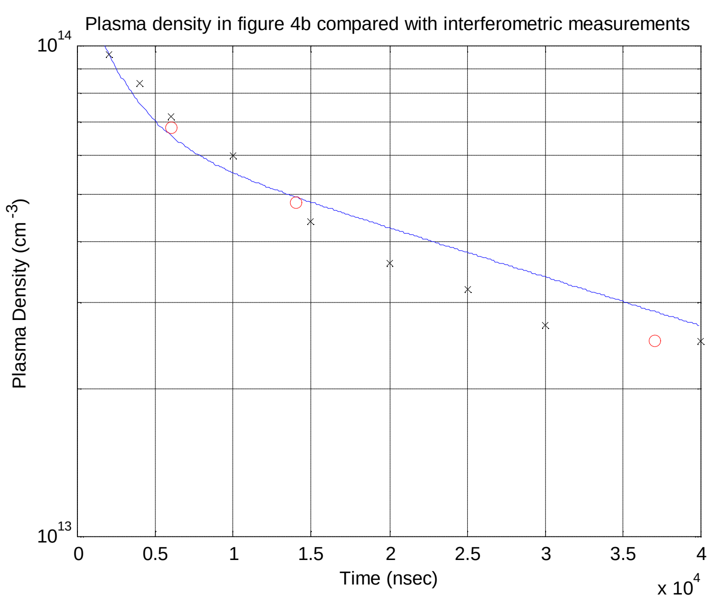

Figure 1: Time dependencies of plasma density determined by Cherenkov radiation and by interferometry (solid line). An overall scale factor is applied to the interferometry data. The o's are the densities at the extrema of the beam envelope, and the x's are other points.

3) β-function at the downstream OTR in terms of the initial Twiss parameters

$$\beta = C^2\beta_0 - 2CS\alpha_0 + S^2\gamma_0.$$

3) Derivative of the β-function at downstream OTR with respect to k

$$\frac{d\beta}{dk} = 2\left[\beta_0 C\frac{dC}{dk} - \alpha_0\left(S\frac{dC}{dk} + C\frac{dS}{dk}\right) + \gamma_0 S\frac{dS}{dk}\right],$$

$$\frac{dC}{dk} = -\frac{1}{2\sqrt{k}}\left((L + L_{oven})\sin\varphi + LL_{oven}\sqrt{k}\cos\varphi\right),$$

$$\frac{dS}{dk} = -\frac{1}{2\sqrt{k}}\left((LL_{oven} + \frac{1}{k})\sin\varphi - \frac{L_{oven}}{\sqrt{k}}\cos\varphi\right)$$

The extrema of the spot size at the downstream OTR are given by $d\beta/dk = 0$. There are three equations, one for each of the extrema, and two unknowns, β_0 and α_0. (γ_0 is given by the relationship $\beta_0\gamma_0 = 1 + \alpha_0^2$.) These equations can be solved in the least-squares sense. The solution is shown in the left-hand side of figure 2 below. The solution corresponds to β_0 = 0.232 m, α_0 = 0.482 and γ_0 = 4.32 m^{-1}. These correspond to the beam being focused 0.011 m in front of the plasma. There is an indication of another extrema at 1×10^{13} cm^{-3}. However, the beam envelope shown in the right-hand side of figure 2 shows that this solution does not agree with the measurements. The maxima and minima are interchanged; i.e. the envelope has a maximum at 2.5×10^{13} cm^{-3} (corresponding to 37 μsec) instead of a minimum as in the data. The conclusion is

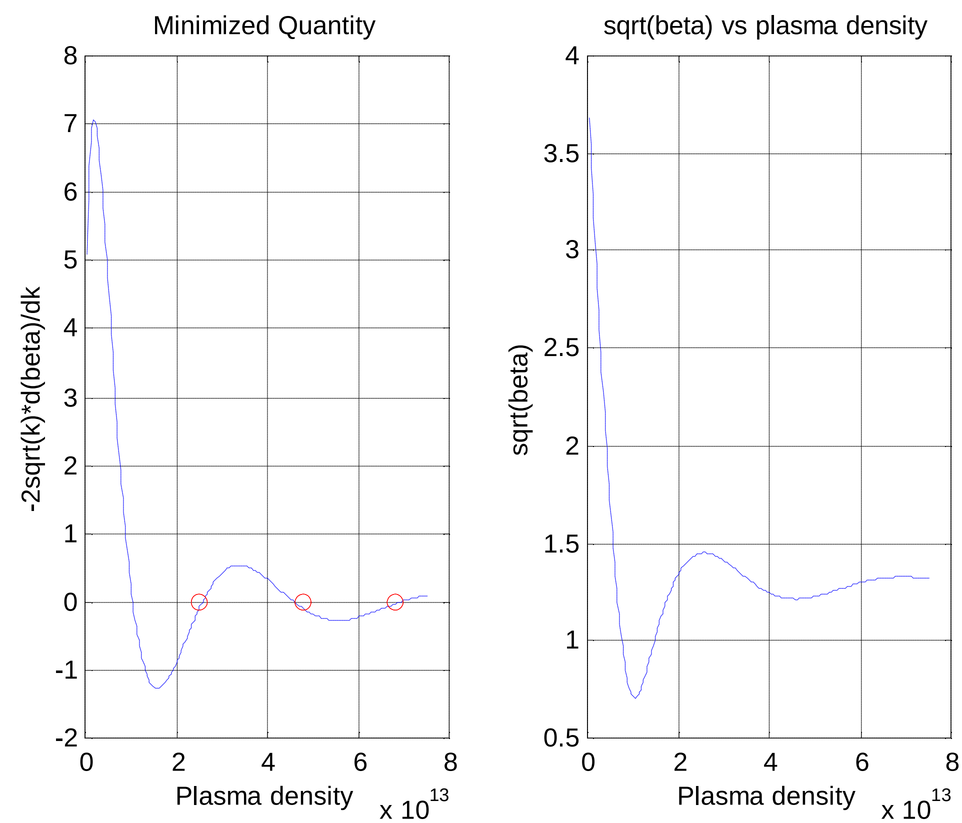

Figure 2: LHS - Least-squares solution to $d\beta/dk = 0$ at the three extrema indicated by the o's. RHS – beam envelope for this solution.

that the plasma densities in the table 1 do not describe the data.

One possible way to explain the data could be with a scale factor for the plasma density. The result is shown in figure 3 when the densities in table 1 are multiplied by 0.4. This gives the minima and maxima at the correct times but does not reproduce the envelope shape at large density (small time). The conclusion is that this does not explain the data either.

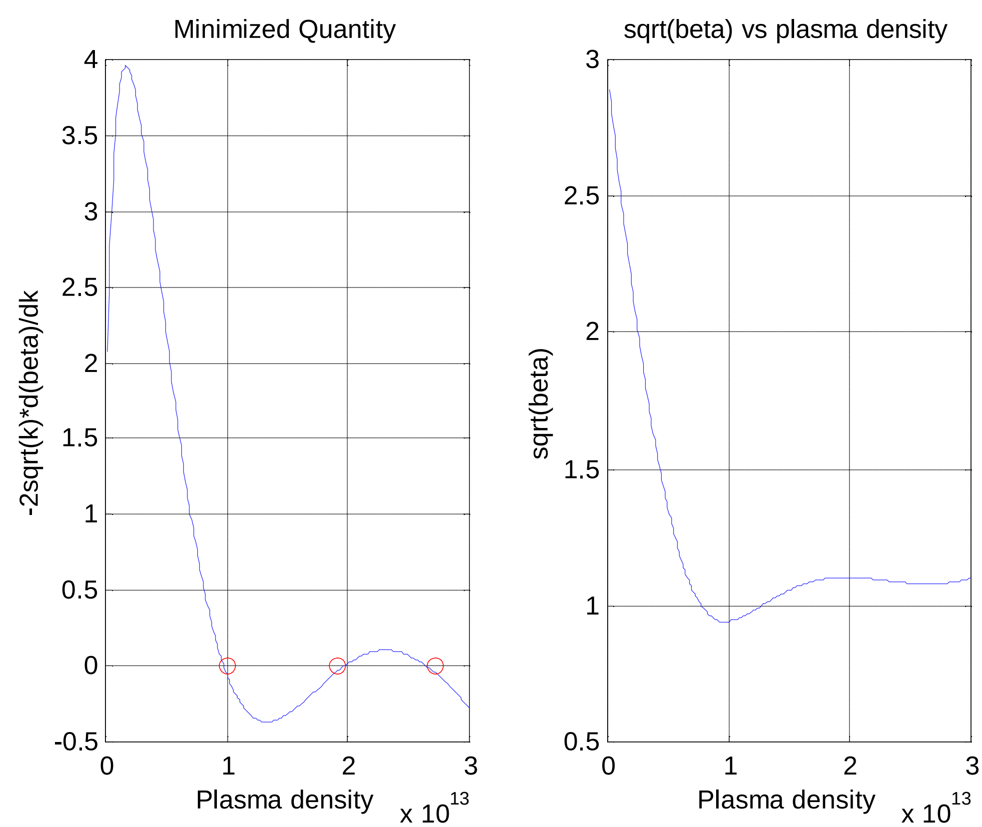

Figure 3: : Plasma densities in table 1 multiplied by 0.4. LHS - Least-squares solution to $db/dk = 0$ at the three extrema indicated by the o's. RHS – beam envelope for this solution.