Beam Envelopes & Trajectories

This is a continuation of the ideas developed in notes ARDB-217 and ARDB-221.

The plasma is treated as a thick lens of strength, k, which depends on the ion density, n_I, beam energy, γ, and classical electron radius, r_e, as

$$k = \frac{2\pi r_e n_I}{\gamma}.$$

The cosine-like and sine-like trajectories, C and S respectively, are the principle trajectories. They are elements of the **R**-matrix obtained by multiplying the transport matrices for elements making up the beam line

$$\mathbf{R}(s) = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix}.$$

C and S give the motion of individual particles in terms of the initial position and slope

$$y(s) = y_0 C(s) + y_0' S(s),$$

and they determine the beam envelope. The β-function is given in terms of C and S and the initial Twiss parameters, α_0, β_0 and γ_0 is

$$\beta(s) = C(s)^2 \beta_0 - 2C(s)S(s)\alpha_0 + S(s)^2 \gamma_0 .$$

For this note $s = 0$ is taken to be the upstream end of the plasma.

First, the effect of the quad installed near the end of run 5 is explored in Figure 1 below. The reduction in beam size is dramatic at all values of the plasma density.

"Tail-flipping" corresponds to the motion of individual particles. That motion depends on the initial conditions y_0 and y_0'. To look at a range of initial conditions, fix the betatron amplitude to a nominal value of one, and look at different phases, ψ. The initial position and

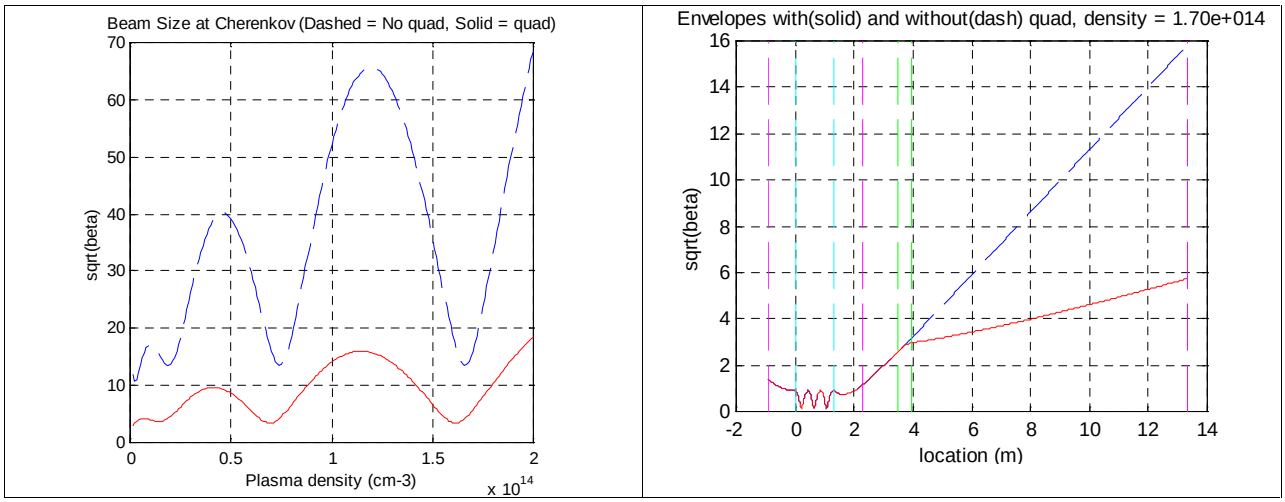

Figure 1: Effect of quadrupole with a field integral of 380 kG. The panel on the left shows the envelope with and without the quad. The panel on the right shows the beam envelope for a plasma density $n_I = 1.7\times10^{14}$ cm^{-3}. The initial Twiss parameters are $\beta_0 = 0.8$ m, $\alpha_0 = 0$. The magenta lines indicate locations of the upstream OTR, downstream OTR, and Cherenkov. The cyan lines indicate the plasma, and the green lines indicate the quad.

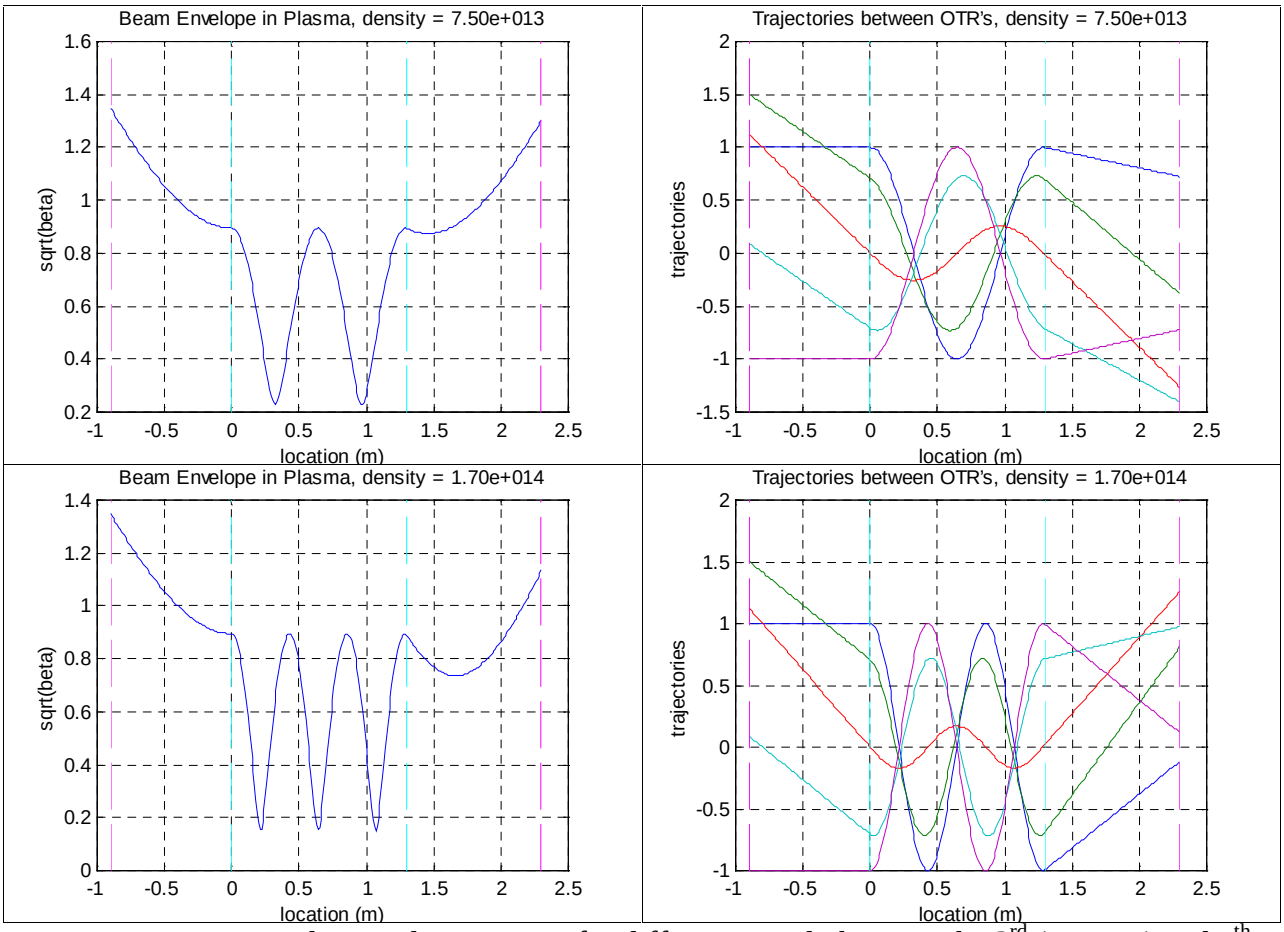

Figure 2: Beam envelope and trajectories for different initial phases at the 3rd (top row) and 4th (bottom row) pinches. The different color trajectories are: ***blue*** $\psi = 0$; ***green*** $\psi = \pi/4$; ***red*** $\psi = \pi/2$; ***cyan*** $\psi = 3\pi/4$; ***magenta*** $\psi = \pi$. The initial Twiss parameters are $\beta_0 = 0.8$ *m*, $\alpha_0 = 0$.

angle are given by

$$y_0 = \cos\psi\,, \quad y_0' = -\frac{1}{\beta_0}(\alpha_0 \cos\psi + \sin\psi)\,.$$

Figure 2 shows the beam envelope and trajectories in the region between the OTR's for $\psi = 0, \pi/4, \ldots, \pi$. at the 3rd and 4th pinches. Comparing the same initial conditions at the two pinches, trajectories change from being above (or below) the mid-plane on the 3rd pinch to the opposite on the 4th pinch. However, comparing the blue, green and red trajectories at either pinch shows that being above the mid-plane at the upstream OTR does not predict the trajectory location on the downstream OTR. For example, the blue and red trajectories are both above the mid-plane on the upstream OTR. On the 3rd pinch, the blue one is above the mid-plane on the downstream OTR, but the red one is below the mid-plane.

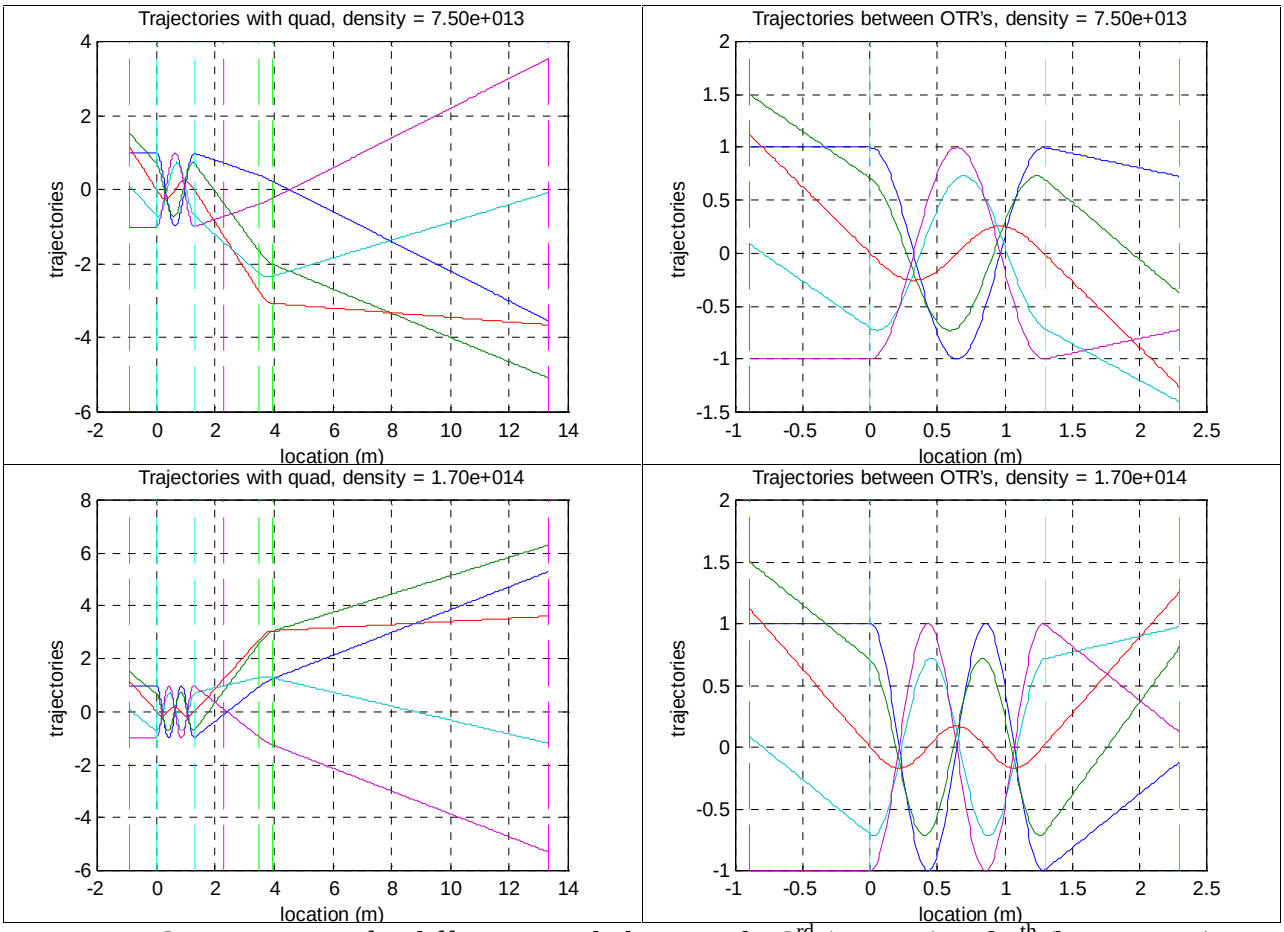

Figure 3: Trajectories for different initial phases at the 3rd (top row) and 4th (bottom row) pinches in the region from the upstream OTR to the Cherenkov and between the OTR's. Colors and initial Twiss parameters are the same as in Figure 2.

This unfortunate situation changes by the time one reaches the Cherenkov. Figure 3 shows the trajectories extended to the Cherenkov. Tail-flipping is clear, and the position on the Cherenkov reflects the initial position on the upstream OTR.

Figure 4 is a comparison of trajectories for different initial Twiss parameters. These plots were generated by keeping β at the waist (not the plasma entrance) fixed, specifying the location of the waist, and calculating α_0 to put the waist at that location. Note that the different color trajectories do not represent the same initial conditions in the rows of this figure since the relation between position and angle changes as β_0 and α_0 change. The conclusion is that the qualitative behavior of the trajectories is no affected strongly by the initial Twiss parameters.

The program used to produce these results is ***ardb223*** located in the RHS private folder on the NT server.

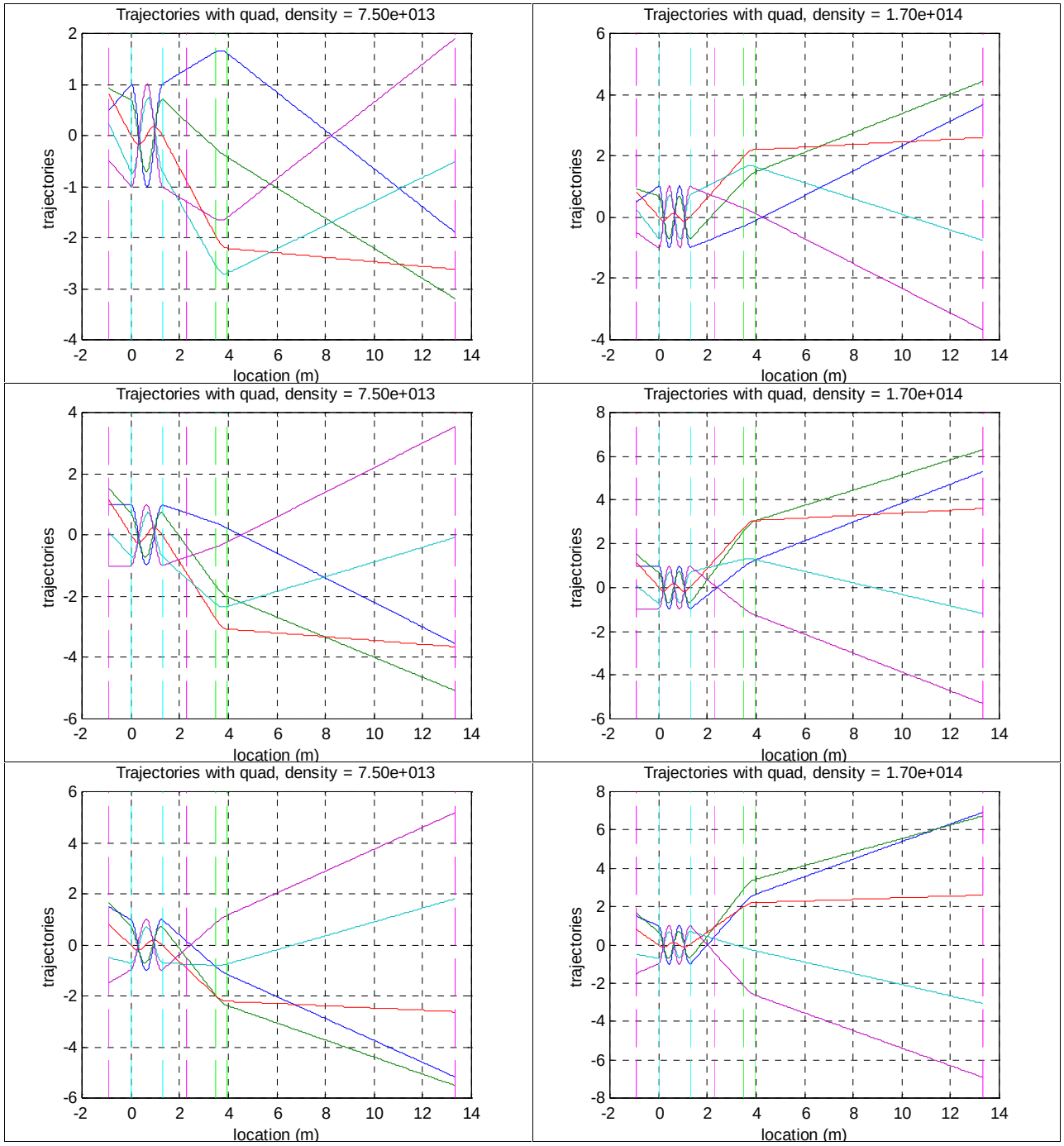

Figure 4: Trajectories for the waist located 0.5 m upstream of the plasma entrance (top row), at the plasma entrance (middle row), and 0.5 m downstream of the plasma entrance (bottom row). These trajectories are for the 3rd (left column) and 4th (right column) pinches. The β-function at the waist was β_w = 0.8 m.