## Plasma Density and Interpretation of the Downstream OTR for Runs 11182cb, 11182cc & 11182cf

The spot size variation on the downstream OTR for runs 11182cb and 11182cc have been shown often as examples of focusing in E-157. Usually 11182cb and 11182cc are combined and shown. 11182cf can be added provided the change in attenuation of the GADC digitizing the transmitted laser intensity is taken into account. The saturation of the GADC digitizing the incident energy must be accounted for in all the runs. The data are shown below in Figure 1.

These data can be used to extract the plasma phase advance and density if one treats the plasma as a thick, linear lens. Let  $\alpha_0$ ,  $\beta_0$  and  $\gamma_0$  be the Twiss parameters at the entrance to the oven. Let  $L_{oven}$  (= 1.3 m) be the length of the oven and L (=1.03 m) be the distance from the end of the oven to the downstream OTR.  $\beta$  at the downstream OTR is given by

$$\beta = C^2 \beta_0 - 2CS\alpha_0 + S^2 \gamma_0$$

where C and S are the cosine-like and sine-like trajectories starting at the plasma entrance. They are given by

$$C = \cos \varphi - \sqrt{kL} \sin \varphi, \quad S = \frac{1}{\sqrt{k}} \sin \varphi + L \cos \varphi.$$

k is the plasma focusing strength, and  $\varphi$  is the phase advance through the plasma.



Figure 1: Horizontal RMS beam size on the downstream OTR. The horizontal axis is the incident laser GADC corrected for saturation at high values. The different bins are: o = run 11182cb, x = run 11182cc and + = run 11182cf.

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 $\varphi$  is given by

$$\varphi = \sqrt{k} L_{oven}$$

and k in terms of the ion density,  $n_I$ , beam energy,  $\gamma$ , and classical electron radius,  $r_e$ , is

$$k = \frac{2\pi r_e n_I}{\gamma}.$$

k can be written in terms of T = GADC-(-1600), which is incident laser power corrected for the GADC offset. Write it as

$$k = AT$$
.

There are three unknowns,  $\beta_0$ ,  $\alpha_0$ , and A. They are determined by solving the equation

$$\frac{d\beta}{dk} = 2\left[C\beta_0 \frac{dC}{dk} - \alpha_0 \left(C\frac{dS}{dk} + S\frac{dC}{dk}\right) + S\gamma_0 \frac{dS}{dk}\right] = 0$$

at the points where the spot size is a minimum(GADC = -1500, -1000 & 500 in Figure 1) or a maximum(GADC = -1250 & -250 in Figure 1). Non-linear least squares fitting is used. The derivatives are

$$\frac{dC}{dk} = -\frac{1}{2\sqrt{k}} \left( (L + L_{oven}) \sin \varphi + LL_{oven} \sqrt{k} \cos \varphi \right)$$
$$\frac{dS}{dk} = -\frac{1}{2\sqrt{k}} \left( (LL_{oven} + \frac{1}{k}) \sin \varphi - \frac{L_{oven}}{\sqrt{k}} \cos \varphi \right)$$

The plot describing the solution is in Figure 2 below. The quantity plotted is proportional to  $d\beta/dk$ , and should equal zero at the five points indicated by o's. The solution is:  $\beta_0 = 0.80$  m,  $\alpha_0 = -0.031$ , ( $\gamma_0 = 1.26$  m<sup>-1</sup>), and  $A = 1.18 \times 10^{-2}$  m<sup>-2</sup>. The Twiss parameters and plasma phase advance vs GAC are given in Figure 3.

There are a number of conclusions.

- 1. The  $\beta$  function from the solution approximately reproduces the observed minima.
- 2. The beam is focused at  $\alpha/\gamma \sim -2.5$  cm which is just in front of the plasma.



Figure 2:  $d\beta/dk$  vs GADC. The o's are the minima and maxima listed above.





Figure 3: Twiss parameters and plasma phase advance for the solution above ( $\beta_0 = 0.80 \text{ m}, \alpha_0 = -0.031, \gamma_0 = 1.26 \text{ m}^{-1}$ , and  $A = 1.18 \times 10^{-2} \text{ m}^{-2}$ ).

3. The first minimum at GADC = -1500 corresponds to a phase advance in the plasma of  $\varphi \sim 1$  not  $\varphi \sim \pi$ . The envelope near the first minimum is shown in Figure 4 below. The beam is diverging slightly when it enters the plasma. As k increases, the beam is focused in the plasma and then diverges as it exits. When  $k > k_{min}$  it is focused strongly in the plasma, but



Figure 4: The beam envelope for k = 0.70, ..., 1.30 times the value at the first minimum (k<sub>min</sub>). The red lines are  $k < k_{min}$ , blue for  $k = k_{min}$ , and green for  $k > k_{min}$ . Oven end and downstream OTR are located at the red and magenta dashed lines, respectively.

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the resultant divergence makes the beam larger at the downstream OTR.

- 4. The second minimum is at  $\varphi \sim \pi$ , and is seen in Figure 5, the beam has one minimum within the plasma.
- 5. The minimum at the highest laser intensity, GADC = 500, corresponds to a phase advance  $\varphi \sim 2\pi$ . The corresponding k and  $n_I$  values are  $k = (GADC + 1600) * 1.18 \times 10^{-2} \text{ m}^{-2} = 24.8 \text{ m}^{-2}$  and  $n_I = 8.3 \times 10^{13} \text{ cm}^{-3}$ .



Figure 5: Beam envelopes at first three minimum spot sizes on the downstream OTR. Oven end and downstream OTR are located at the red and magenta dashed lines, respectively.