Theoretical Comparison for S_{11} Measurement on a Constant Impedance Travelling-Wave Structure With Tuning Plunger on One Port

D. H. Whittum

Introduction

This note is to provide comparison for certain bench measurements on the 7-cell W-Band muffin tin structure. These measurements were made under the condition of a shorting plane attached to the waveguide that connects to the output cavity. The basic theory for a multicell structure has been summarized in a separate technical note, and this problem requires only small modifications. The picture is that of Fig.1

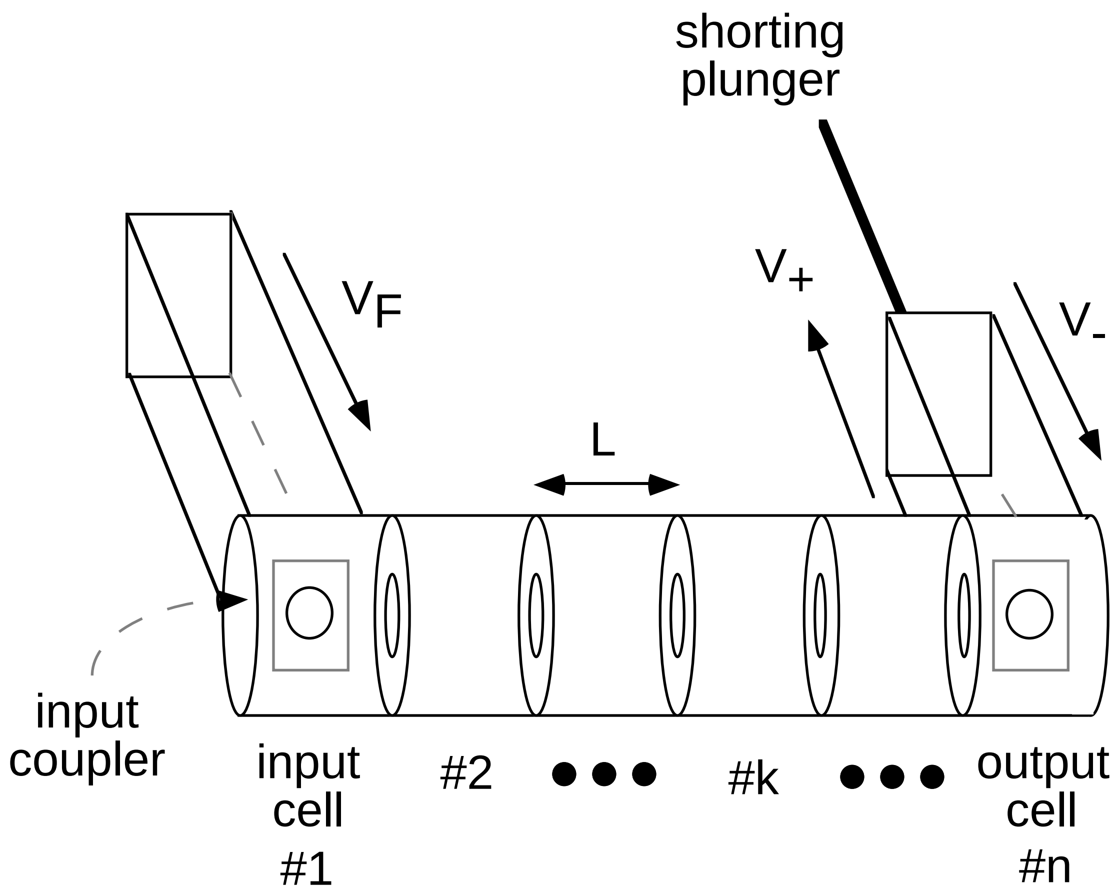

Figure 1. We consider a multi-cell structure, with constant impedance interior cells (n=2,3,...N-1), and with the output connecting guide terminated in a moveable shorting plane. The input guide is matched, and subject to an incident signal provided by a network analyzer.

Our model of this structure consists of N coupled oscillators, driven by means of excitation of the coupler cavities,

$$-\frac{1}{2}\omega_n^2\kappa_{n-1/2}\tilde{V}_{n-1}+\left(j\Omega\frac{\omega_n}{Q_n}+\omega_n^2-\Omega^2\right)\tilde{V}_n-\frac{1}{2}\omega_n^2\kappa_{n+1/2}\tilde{V}_{n+1}=2j\Omega\frac{\omega_n}{Q_{en}}\left(\tilde{V}_F\delta_{n,1}+\tilde{V}_+\delta_{n,N}\right).$$

A voltage $\tilde{V}_F$ is incident on cell #1, and a voltage $\tilde{V}_+$ is incident on cell #N. As described elsewhere, this system is straigtforward to solve numerically by solution of a tridiagonal matrix equation, or analytically in the case that the interior cells are identical --- all provided, however, one knows the terms on the right-side of the equation. The new term here is for $n=N$. To determine it, we make use of the continuity condition,

$$\tilde{V}_n=\tilde{V}_+ +\tilde{V}_-,$$

with $\tilde{V}_-$ the outgoing phasor on the connecting guide. In addition we make use of the shorting-plane condition,

$$\tilde{V}_+e^{-\gamma L}+\tilde{V}_-e^{\gamma L}=0,$$

with the waveguide propagation constant,

$$\gamma=j\beta+\alpha,$$

β the guide wavenumber at this frequency,

$$\beta=\left\{\left(\frac{2\pi}{\lambda}\right)^2-\left(\frac{\pi}{a}\right)^2\right\}^{1/2},$$

a=10/100”=2.54mm for WR10, and λ is the free-space wavelength. The theoretical attenuation constant is

$$\alpha=\pi\frac{R_s}{Z_0}\left[\frac{2}{b\lambda^2}+\frac{1}{a^3}\right]\frac{\lambda}{\beta},$$

with b the waveguide height, 0.05”=1.27mm for WR10. Z_0˜376.7Ω, and the theoretical surface resistance is R_S˜79mΩ for room temperature copper. The length L is relative to a choice of reference plane, and increases as the shorting plane moves away from the structure.

With the shorting-plane condition, we have

$$\tilde{V}_N=\tilde{V}_+ +\tilde{V}_- =\tilde{V}_+\left(1-e^{-2\gamma L}\right)\Rightarrow\tilde{V}_+=\frac{\tilde{V}_N}{\left(1-e^{-2\gamma L}\right)},$$

and our result for the N-th cell voltage,

$$\left(j\Omega\frac{\omega_N}{Q_N}+\omega_N^2-\Omega^2\right)\tilde{V}_N=\frac{1}{2}\omega_N^2\kappa_{N-1/2}\tilde{V}_{n-1}+2j\Omega\frac{\omega_N}{Q_{eN}}\tilde{V}_+,$$

may be expressed as

$$\left(j\Omega\frac{\omega_N}{Q_N} + \omega_N^2 - \Omega^2 - 2j\Omega\frac{\omega_N}{Q_{eN}}\frac{1}{\left(1-e^{-2\gamma L}\right)} \right)\tilde{V}_N = \frac{1}{2}\omega_N^2\kappa_{N-1/2}\tilde{V}_{n-1}.$$

In this form, the system is again reduced to a tridiagonal matrix equation, and can be solve directly.

To make more explicit the behaviors to expect, consider the case of negligible waveguide attenuation,

$$1-e^{-2\gamma L} = 1-e^{-2j\beta L} = e^{-j\beta L}\left(e^{j\beta L} - e^{-j\beta L}\right) = 2je^{-j\beta L}\sin(\beta L),$$

so that

$$\left(j\Omega\frac{\omega_N}{Q_N} + \omega_N^2 - \Omega^2 - \Omega\frac{\omega_N}{Q_{eN}}\frac{e^{j\beta L}}{\sin(\beta L)} \right)\tilde{V}_N = \frac{1}{2}\omega_N^2\kappa_{N-1/2}\tilde{V}_{n-1}.$$

Using

$$\frac{1}{Q_N} = \frac{1}{Q_{eN}} + \frac{1}{Q_w},$$

this is just,

$$\left(j\Omega\frac{\omega_N}{Q_w} + \omega_N^2 - \Omega^2 - \Omega\frac{\omega_N}{Q_{eN}}\cot(\beta L) \right)\tilde{V}_N = \frac{1}{2}\omega_N^2\kappa_{N-1/2}\tilde{V}_{n-1}.$$

Thus as far as the circuit is concerned, the effect of the shorting plunger is to remove the external coupling, and to tune the output cell, roughly according to

$$\frac{\delta\omega_N^2}{\omega_N^2} \approx -\frac{1}{Q_{eN}}\cot(\beta L).$$

Finally, recall from a previous note that in the case of uniform initial cells, the *S*-Matrix is

$$S_{11}(\Omega) + 1 = 2j\frac{\omega_1\Omega}{Q_{e1}}\frac{1}{\Xi}\left(\left[\Delta_N - \tfrac{1}{2}\omega_N^2\kappa_N e^{-\gamma}\right]e^{(N-1)\gamma} - \left[\Delta_N - \tfrac{1}{2}\omega_N^2\kappa_N e^{\gamma}\right]e^{-(N-1)\gamma}\right),$$

where

$$\Xi = \left(\Delta_1 - \tfrac{1}{2}\omega_1^2\kappa_1 e^{-\gamma}\right)\left[\Delta_N - \tfrac{1}{2}\omega_N^2\kappa_N e^{-\gamma}\right]e^{(N-1)\gamma} - \left(\Delta_1 - \tfrac{1}{2}\omega_1^2\kappa_1 e^{\gamma}\right)\left[\Delta_N - \tfrac{1}{2}\omega_N^2\kappa_N e^{\gamma}\right]e^{-(N-1)\gamma}.$$

The notation is, for the propagation constant along the structure (not to be confused with that in the waveguide) $\gamma = j\theta + \Gamma$, with

$$\cos\theta(\Omega) = \frac{\omega_0^2 - \Omega^2}{\omega_0^2\kappa}, \qquad \Gamma(\Omega) = \frac{\Omega/\omega_0}{\kappa Q_w \sin\theta(\Omega)}.$$

The quantity,

$$\Delta_k = j\frac{\omega_k \Omega}{Q_k} + \omega_k^2 - \Omega^2,$$

except that for $k=N$, this is instead

$$\Delta_N = j\Omega\frac{\omega_N}{Q_w} + \omega_N^2 - \Omega^2 - \Omega\frac{\omega_N}{Q_{eN}}\cot(\beta L) = \Delta_N(L).$$