

Theoretical Comparison for S_{11} and S_{12} Measurements on Constant Impedance Travelling-Wave Structures

D. H. Whittum

In this note we review the theory of travelling wave accelerating structures, from the transmission line and coupled-cavity points of view. We make use of the coupled-circuit model to match bench measurements made on the 7-cell W-Band structure.

Introduction

The motivation for this note is the need to understand anomalous loss seen in recent results of bench measurements on the 7-cell W-Band muffin tin structure.

Review of Constant-Impedance Structures

A “travelling wave linac” is a multi-cavity accelerator, with critically coupled input to the first cell, and a critically coupled output waveguide attached to the last cell, with the output waveguide terminated in a high-power matched load. The picture is that of Fig. 1.

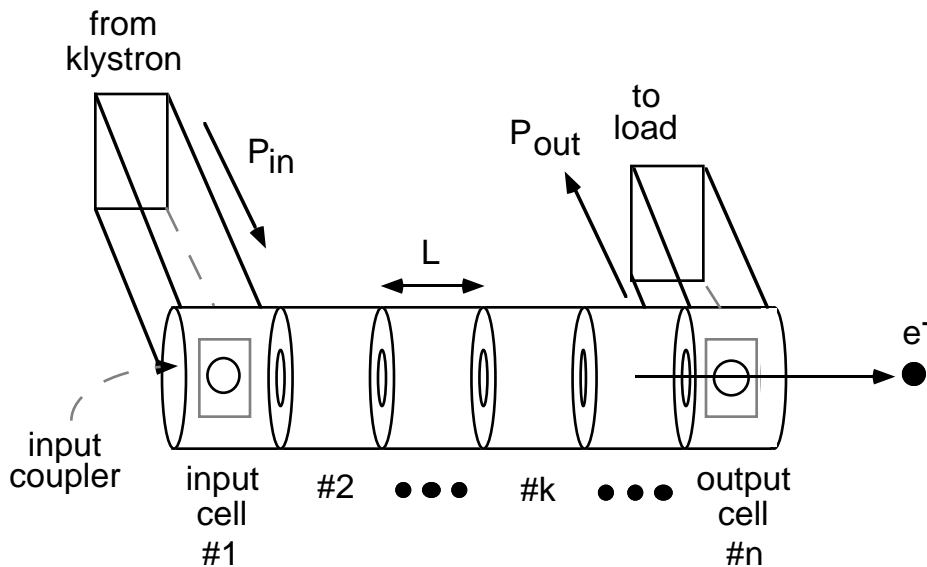


FIGURE 1. Schematic of a travelling wave accelerating structure.

The performance of such a structure is most simply described in the language of a transmission line. Let us consider the steady-state established with an input power P_{in} . If the energy stored in cell $\#k$ is U_k , then we may speak of an energy per unit length at $z=z_k=(k-1)L$ given by $u(z_k)=U_k/L$, with L the cell length. The power flowing past the point z is $P(z)=V_g u(z)$, with V_g the local group velocity. In steady-state the energy density stored in one cell is constant in

time, and thus

$$0 = dU = \text{change in stored energy in time } dt = (P_- - P_+)dt - \frac{\omega_0 U}{Q_w} dt,$$

where Q_w is the wall Q of a single cell. The power flowing into the cell from the left is P_- , and the power flowing out is P_+ . Thus,

$$P_- - P_+ \approx L \frac{dP}{dz} = -\frac{\omega_0 U}{Q_w} = -\frac{\omega_0 u L}{Q_w},$$

or

$$\frac{dP}{dz} = -\frac{\omega_0 u}{Q_w} = \frac{d}{dz}(V_g u). \quad (1)$$

A periodic structure has constant V_g . For such a *constant impedance* structure,

$$\frac{d}{dz}(V_g u) = V_g \frac{du}{dz} = -\frac{\omega_0 u}{Q_w},$$

so that

$$u(z) = u(0) \exp(-2\alpha z),$$

where

$$\alpha = \frac{\omega_0}{2Q_w V_g}.$$

The energy stored in the first cell and this may be determined from the input power P_{in} ,

$$u(0) = P_{in} / V_g(0).$$

This also provides a simple estimate of the external Q required of the input coupler geometry in the first cell,

$$P_{in} = \frac{\omega_0}{Q_e} u(0)L = V_g u(0) \Rightarrow Q_e \approx \frac{\omega_0 L}{V_g},$$

and this external Q roughly characterizes the transmission bandwidth of the structure, although not the useful bandwidth for acceleration. This relation can also be expressed as $Q_e \approx \theta / \beta_g$, where $\theta = \omega_0 L / c$ is the transit angle, and $\beta_g \approx V_g / c$. Thus for example, one would expect a “0.09c, $2\pi/3$ ” structure to have $Q_e \sim 22$, and a bandwidth on the order of $1/Q_e$ or 4%, and this corresponds to 4 GHz at W-Band.

We will be interested in acceleration with this device so let us suppose the shunt impedance

for a single cell is R , and define the *shunt impedance per unit length*,

$$r = \frac{R}{L} = \frac{Q_w \left[\frac{R}{Q} \right]}{L} = \frac{Q_w |V|^2}{L \omega_0 U},$$

where V is the voltage gain in one cell. Maximum $[R/Q]$ for a W-Band cell is 32V/pC; with beam ports achievable values are lower; a low value would be 16V/pC, and on the high side 25V/pC. Wall Q 's are in the range of 2000-2700, and cell lengths consistent with a favorable transit angle factor at 0.328cm free-space wavelength are 820 μ m to 1.64mm. Putting these numbers together, one can expect to find shunt-impedances extending perhaps as high as 390M Ω /m.

The result for shunt impedance per unit length may be expressed in terms of the accelerating *gradient* G , according to $V=GL$, so that,

$$r = \frac{Q_w (GL)^2}{L \omega_0 U} = \frac{Q_w}{\omega_0 u} G^2 = \frac{G^2}{-dP/dz}. \quad (2)$$

Thus the gradient at a distance z along the structure is

$$G(z) = \left(\frac{\omega u}{Q_w} r \right)^{1/2} = G(0) \exp(-\alpha z).$$

and the net accelerating voltage is just the integral of this,

$$V_{NL} = \int_0^{L_s} G(z) dz = G(0) \frac{1 - e^{-\alpha L_s}}{\alpha},$$

where $L_s=NL$ is the structure length. We define the *attenuation parameter*, τ ,

$$\tau = \alpha L = \frac{\omega_0 L_s}{Q_w V_g}.$$

Note that the attenuation parameter determines the power to the load,

$$P(L_s) = P(0) e^{-2\tau}, \quad (3)$$

and the *fill time*,

$$T_f = \int_0^{L_s} \frac{dz}{V_g} = \frac{L_s}{V_g} = \frac{2Q_w}{\omega_0} \tau. \quad (4)$$

Note that the result for attenuation may be expressed in terms of insertion loss

$$IL(dB) = -10 \log_{10} \left(\frac{P(L_s)}{P(0)} \right) = 20 (\log_{10} e) \tau \approx 8.686 \tau.$$

For example, a 7-cell, $2\pi/3$ -mode, $0.09c$ structure at W-Band, with $Q_w \sim 2200$ corresponds to $\tau \sim 7.4 \times 10^{-2}$, insertion loss of 0.64dB, and a fill-time of 0.57ns.

We may express the accelerating voltage directly in terms of the input power using,

$$G(0) = \left(\frac{\omega_0}{Q_w} ur \right)^{1/2} = \left(\frac{\omega_0}{Q_w} \frac{P_{in}}{V_g} r \right)^{1/2} = (2\alpha P_{in} r)^{1/2} = \frac{1}{L_s} (2\tau P_{in} R_s)^{1/2},$$

where we have introduced the shunt impedance of the structure as a whole, $R_s = rL_s$, with L_s the structure length, just NL , with N the number of cells. The voltage with no beam present ("no-load voltage") is then,

$$V_{NL} = (2\tau P_{in} R_s)^{1/2} \frac{1 - e^{-\tau}}{\tau} = (P_{in} R_s)^{1/2} (1 - e^{-\tau}) \left(\frac{2}{\tau} \right)^{1/2}. \quad (5)$$

One can show that the maximum no-load voltage for fixed input power and shunt impedance in a constant impedance structure occurs for $\tau \sim 1.26$, and is $V_{NL} \approx 0.9(P_{in} R_s)^{1/2}$. In this case, with $Q_w \sim 2700$, the fill-time is 11.8ns.

As an example application of these scalings, let us suppose we have 30-cell, $2\pi/3$ -mode structure with group velocity of $0.09c$, and shunt-impedance per unit length of $200\text{M}\Omega/\text{m}$. The structure length is 3.28cm, so that the fill-time is 1.2ns and the shunt impedance is $6.6\text{M}\Omega$. For a wall Q of 2200, the attenuation parameter is 0.16. In this case, $V_{NL} \approx 0.5(P_{in} R_s)^{1/2}$. If we wish to provide a net-voltage of 5MV, we will need to provide 14MW. This voltage corresponds to an average accelerating gradient of 150MV/m, and a peak gradient about 10% larger.

Coupled-Cavity Model

While these scalings describe the basic features of performance of such structures, it is helpful in practice to have a more detailed picture. Such a structure may be viewed as a collection of coupled oscillators (see notes on Microwave Linacs). Interior cells, $n=2,3,\dots,N-1$ satisfy,

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_0}{Q_w} \frac{\partial}{\partial t} + \omega_0^2 \right) V_k = \frac{1}{2} \omega_0^2 \kappa (V_{k-1} + V_{k+1}).$$

To analyze the behavior of such a system it is convenient to consider the problem in the frequency domain. Let us suppose, $V_k \propto \tilde{V}_k e^{j\omega t}$, so that

$$\left(j \frac{\omega \omega_0}{Q_w} + \omega_0^2 - \omega^2 \right) \tilde{V}_k = \frac{1}{2} \omega_0^2 \kappa (\tilde{V}_{k-1} + \tilde{V}_{k+1}).$$

Solutions to a recursion relation such as this with constant coefficients correspond to left and right-going waves. Let us consider such a wave, $V_k \propto e^{j\omega t - k\gamma}$, with $\gamma = j\theta + \Gamma$. Then

$$j \frac{\omega \omega_0}{Q_w} + \omega_0^2 - \omega^2 = \frac{1}{2} \omega_0^2 \kappa (e^{-j\theta - \Gamma} + e^{j\theta + \Gamma}),$$

and if $Q_w \gg 1$, then the attenuation per cell, Γ is small, and we may expand,

$$j \frac{\omega \omega_0}{Q_w} + \omega_0^2 - \omega^2 \approx \frac{1}{2} \omega_0^2 \kappa (e^{-j\theta} [1 - \Gamma] + e^{j\theta} [1 + \Gamma]),$$

and, equating real and imaginary parts, we find

$$\cos \theta = \frac{\omega_0^2 - \omega^2}{\omega_0^2 \kappa}, \quad (6)$$

and an attenuation in nepers per cell of

$$\Gamma \approx \frac{\omega / \omega_0}{\kappa Q_w \sin \theta}.$$

One can also show that group velocity is given by

$$\beta_g = \frac{L}{c} \frac{d\omega}{d\theta} = \frac{1}{2} \frac{\kappa \theta_0 \sin \theta_0}{(1 - \kappa \cos \theta_0)}, \quad \text{or} \quad \kappa = \frac{\beta_g}{\frac{1}{2} \varphi \sin \theta + \beta_g \cos \theta}, \quad (7)$$

where $\varphi = \omega L / c = \theta$ for synchronous phase-advance. The attenuation parameter $\tau = N\Gamma$. In this way, given the macroscopic parameters characterizing the structure (group velocity, phase-advance per cell, attenuation parameter), one can determine the interori-cell circuit parameters required for the coupled-cavity model. The end cells require special attention. The voltage in the input cell satisfies,

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_1}{Q_1} \frac{\partial}{\partial t} + \omega_1^2 \right) V_1 = \frac{1}{2} \omega_1^2 \kappa_1 V_2 + 2 \frac{\omega_1}{Q_{e1}} \frac{\partial V_F}{\partial t},$$

where the forward-going voltage in the connecting guide has been transformed to to V_F , and reverse waveform V_R , satisfying, $V_1 = V_F + V_R$. The loaded Q of the first cell is

$$\frac{1}{Q_1} = \frac{1}{Q_{e1}} + \frac{1}{Q_{w1}}.$$

The input cell resonance frequency, and external Q , Q_{e1} are adjusted to insure no reflected signal in steady-state ($V_I = V_F$), corresponding to a match on the transmission line to a forward-wave with phase-advance per cell θ_0 . Thus

$$\left(-\omega^2 + \frac{\omega_1}{Q_1} j\omega + \omega_1^2 \right) = \frac{1}{2} \omega_1^2 \kappa_1 e^{-j\theta_0} (1 - \Gamma) + 2 \frac{\omega_1}{Q_{e1}} j\omega,$$

with ω the design drive angular frequency for synchronism. Equality permits us to solve for the input coupler cell parameters,

$$\omega_1 = \frac{\omega}{\sqrt{1 - \frac{1}{2}\kappa_1(1-\Gamma)\cos\theta_0}}, \quad \frac{1}{Q_{e1}} = \frac{1}{Q_w} + \frac{1}{2} \frac{\kappa_1(1-\Gamma)\sin\theta_0}{\sqrt{1 - \frac{1}{2}\kappa_1(1-\Gamma)\cos\theta_0}}. \quad (8)$$

Similarly, the output cell should be matched, and this requires,

$$\left(-\omega^2 + \frac{\omega_N}{Q_N} j\omega + \omega_N^2\right) = \frac{1}{2} \omega_N^2 \kappa_N e^{j\theta_0} (1 + \Gamma),$$

This implies,

$$\omega_N = \frac{\omega}{\sqrt{1 - \frac{1}{2}\kappa_N(1+\Gamma)\cos\theta_0}}, \quad \frac{1}{Q_{eN}} = -\frac{1}{Q_w} + \frac{1}{2} \frac{\kappa_N(1+\Gamma)\sin\theta_0}{\sqrt{1 - \frac{1}{2}\kappa_N(1+\Gamma)\cos\theta_0}}. \quad (9)$$

Based on this analysis, one may expect to find that input and output cells are detuned from those in the interior of the structure.

These results determine the circuit parameters from the macroscopic quantities: wall Q , phase-advance per cell, group velocity, angular frequency. With them one is freed from dependence on the transmission line picture, and can engage in realistic modelling of the structure behavior, in particular, observables, such as transient waveforms viewed from couplers at the input and output, no-load voltage under transient conditions, effects of cell-tuning errors, and the like.

S-Matrix

Perhaps the first application one might make of the circuit model is the calculation of the S-Matrix for the structure. Let us drive a perfectly tuned structure from the input, at angular frequency Ω , (we use the symbol Ω to distinguish the drive frequency from the design drive frequency ω). The S-Matrix elements are

$$S_{11}(\Omega) = \frac{\tilde{V}_1(\Omega)}{\tilde{V}_F(\Omega)} - 1, \quad S_{21}(\Omega) = \frac{\tilde{V}_N(\Omega)}{\tilde{V}_F(\Omega)}. \quad (10)$$

In this notation, the we have transformed the impedance of the connecting guide to that of the structure. In general one is interested to solve N 2nd order differential equations,

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega_n}{Q_n} \frac{\partial}{\partial t} + \omega_n^2\right) V_n = \frac{1}{2} \omega_n^2 (\kappa_{n-1/2} V_{n-1} + \kappa_{n+1/2} V_{n+1}) + \frac{2\omega_{en}}{Q_n} \frac{\partial V_F}{\partial t} \delta_{n,1},$$

with $\delta_{n,l}$ the Kronecker delta function, $\delta_{n,l}=0$ unless $n=l$, in which case $\delta_{n,l}=1$. In the frequency domain at drive frequency Ω , we have

$$-\frac{1}{2} \omega_n^2 \kappa_{n-1/2} \tilde{V}_{n-1} + \left(j\Omega \frac{\omega_n}{Q_n} + \omega_n^2 - \Omega^2\right) \tilde{V}_n - \frac{1}{2} \omega_n^2 \kappa_{n+1/2} \tilde{V}_{n+1} = 2j\Omega \frac{\omega_{en}}{Q_n} \tilde{V}_F \delta_{n,1}.$$

and this is a tri-diagonal matrix equation that one call solve easily, numerically, using a modification of the routine `tridag`, in Numerical Recipes. We will consider a perfectly tuned

structure for illustration, where the problem is solvable analytically. Given that one is most often interested in assessing the effect of *errors* in tuning, the primary value of such a result will be as a check of the numerical circuit solution.

We may compute the terms explicitly as follows. The cell excitations in general consist of forward and backward waves,

$$\tilde{V}_k = Ae^{-(k-1)\gamma} + Be^{(k-1)\gamma},$$

where A , B and γ are functions of Ω . We wish to determine A and B , the propagation constant we know is $\gamma = j\theta + \Gamma$, with

$$\cos\theta(\Omega) = \frac{\omega_0^2 - \Omega^2}{\omega_0^2 \kappa}, \quad \Gamma(\Omega) = \frac{\Omega/\omega_0}{\kappa Q_w \sin\theta(\Omega)}.$$

The forward and backward wave amplitudes, A and B , may be computed from the conditions on the end cells,

$$\begin{aligned} \left(j \frac{\omega_1 \Omega}{Q_1} + \omega_1^2 - \Omega^2 \right) (A + B) &= \frac{1}{2} \omega_1^2 \kappa_1 (Ae^{-\gamma} + Be^{\gamma}) + 2j \frac{\omega_1 \Omega}{Q_{e1}} \tilde{V}_F, \\ \left(j \frac{\omega_N \Omega}{Q_N} + \omega_N^2 - \Omega^2 \right) (Ae^{-(N-1)\gamma} + Be^{(N-1)\gamma}) &= \frac{1}{2} \omega_N^2 \kappa_N (Ae^{-(N-2)\gamma} + Be^{(N-2)\gamma}). \end{aligned}$$

Abbreviating

$$\Delta_k = j \frac{\omega_k \Omega}{Q_k} + \omega_k^2 - \Omega^2,$$

we may express this as an equation for the two unknowns, A and B ,

$$\begin{pmatrix} \Delta_1 - \frac{1}{2} \omega_1^2 \kappa_1 e^{-\gamma} & \Delta_1 - \frac{1}{2} \omega_1^2 \kappa_1 e^{\gamma} \\ \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{\gamma} \right] e^{-(N-1)\gamma} & \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{-\gamma} \right] e^{(N-1)\gamma} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = 2j \frac{\omega_1 \Omega}{Q_{e1}} \tilde{V}_F \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

with the solution,

$$\begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{\Xi} \begin{pmatrix} \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{-\gamma} \right] e^{(N-1)\gamma} & - \left[\Delta_1 - \frac{1}{2} \omega_1^2 \kappa_1 e^{\gamma} \right] \\ - \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{\gamma} \right] e^{-(N-1)\gamma} & \Delta_1 - \frac{1}{2} \omega_1^2 \kappa_1 e^{-\gamma} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} 2j \frac{\omega_1 \Omega}{Q_{e1}} \tilde{V}_F,$$

where

$$\Xi = \left(\Delta_1 - \frac{1}{2} \omega_1^2 \kappa_1 e^{-\gamma} \right) \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{-\gamma} \right] e^{(N-1)\gamma} - \left(\Delta_1 - \frac{1}{2} \omega_1^2 \kappa_1 e^{\gamma} \right) \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{\gamma} \right] e^{-(N-1)\gamma} \quad (10)$$

The S-Matrix elements are then

$$S_{11}(\Omega) + 1 = \frac{A+B}{\tilde{V}_F(\Omega)} = 2j \frac{\omega_1 \Omega}{Q_{e1}} \frac{1}{\Xi} \left(\left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{-\gamma} \right] e^{(N-1)\gamma} - \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{\gamma} \right] e^{-(N-1)\gamma} \right), \quad (11)$$

$$S_{12}(\Omega) = \frac{Ae^{-(N-1)\gamma} + Be^{(N-1)\gamma}}{\tilde{V}_F(\Omega)} = 2j \frac{\omega_1 \Omega}{Q_{e1}} \frac{1}{\Xi} \left(\left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{-\gamma} \right] - \left[\Delta_N - \frac{1}{2} \omega_N^2 \kappa_N e^{\gamma} \right] \right). \quad (12)$$

These results are most easily visualized by numerical calculation for an example.

7-Cell W-Band Structure

Let us apply these results to a 7-cell W-Band Structure. For calculations I will use a Fortran code NMAT that I have written for the purpose of analyzing the secondary line of the matrix accelerator. It is described in Appendix A. Near the front of the code are calls to subroutines `strucinit` and `checkmatch`. With switches in the input namelist set appropriately, `strucinit` simply sets up the circuit parameters for a constant impedance structure, and `checkmatch` simulates the network analyzer by computing the S -matrix using both numerical tridiagonal matrix inversion, and the analytic form derived in the previous section. I put in an option to read data from the network analyzer to facilitate overlay and comparison.

The structure was intended to be a $0.09c$, 7-cell $2\pi/3$ mode, constant-impedance structure. I choose $Q_w=2200$ as a *guess* for a reasonable Q . This corresponds to the inputs (as described in more detail in the Appendix) listed in Table 1.

Table 1. "Ideal" parameter set : $0.09c$, 7-cell $2\pi/3$ mode, $Q_w=2200$

<code>iphase=2,</code>	constant phase-advance per interior cell
<code>bphase=120.</code>	phase advance is 120°
<code>qw0=2200.</code>	wall Q is 2200
<code>qem=0.</code>	let NMAT select external Q 's for cells
<code>idetune=0</code>	let NMAT detune coupler cells for match
<code>ncell=7,</code>	a 7-cell structure
<code>xcell=0.1093</code>	cell length (cm) for synchronism at 120°
<code>ibeta=2</code>	constant impedance structure
<code>beta0=0.09</code>	$0.09c$ group velocity
<code>freq0=91.392e9</code>	designed for 91.3892 GHz

There are other inputs but they are not relevant to this calculation (see Appendix for an example input namelist). Results are shown in Figs. 2-3. The result disagrees with measurement. However, before embarking on attempts to fit the results, let us note some of the related quantities alluded to in the previous section.

Among the outputs from NMAT are the following (ignoring many of the decimal places)

```
fill time (ns) = 0.2835665
expect attenuation parameter t = 3.7007552E-02
expect insertion loss (dB) at 91.39GHz = -0.3214436
transit angle wL/c (deg) = 119.9527
```

```
cell 1 freq (GHz)    90.22733    this is  $\omega_1/2\pi$ 
cell 2 freq (GHz)    89.09483    this is  $\omega_2/2\pi$ 
cell N-1 freq (GHz)  89.09483    this is  $\omega_6/2\pi$ 
cell N freq (GHz)   90.21564    this is  $\omega_7/2\pi$ 
```

```
external Q 1st cell    = 22.16693
external Q last cell   = 22.51004
```

```
Driving the input cell at 91.392GHz,
the steady-state reflection coefficient = 2.5868479E-03
VSWR looking into this port = 1.005187
|S21| = 0.9720609,          in dB = -0.2461303
```

```
identified # of modes = 7
```

mode#	f(GHz)	VSWR	IL(dB)	θ (deg)
1	84.784	1.514	1.974	-11.18
2	86.117	1.119	0.549	-40.18
3	88.017	1.050	0.283	-69.89
4	90.084	1.018	0.221	-99.60
5	91.384	1.006	0.246	-119.88
6	91.917	1.016	0.283	-129.23
7	93.184	1.149	0.813	-158.72

This initial attempt to compare with experiment didn't fair too well, and so I began tweaking parameters. One can see the results in Figs. 4-8. Parameters for each plot are those of Table 1, except for the changes marked directly on the plot. The parameters of Fig. 8 seem to provide a fair fit to measurement. They are given in Table 2 (and I list only the relevant namelist variables),

Table 2. "Best Eyeball Fit" parameter set : -0.05c, 7-cell $2\pi/3$ mode, $Q_W=500$

```
iphase=2,          constant phase-advance per interior cell
bphase=120.        phase advance is 120°
qw0=500.          wall Q is 2200
qem=0.            let NMAT select external Q's for cells
idetune=0         let NMAT detune coupler cells for match
ncell=7,          a 7-cell structure
xcell=0.1093      cell length (cm)
ibeta=2           constant impedance structure
beta0=-0.05c     backward wave
freq0=90.892e9   structure is detuned to operate at design - 0.5GHz
```

Conclusions---More Work To Do

The best fit to the results is seen in Fig. 8, and corresponds to a low $Q \sim 500$, a *backward* wave circuit ($-0.05c$), and a structure tuned to operate at the $91.39\text{GHz} - 0.5\text{GHz}$, off by 0.5GHz in tune. The S_{12} corresponding to this circuit is depicted in Fig. 9 and while qualitatively similar to the 4/11/97 data, it does not exhibit enough loss (-1.7dB insertion loss, not -3dB). This could likely be fixed by adjusting the wall Q to a still lower value. However---

The next logical step is to put aside the circuit model, and visit the MAFIA calculations that went into the structure design. I have made assumptions about wall Q , and cell symmetry that really shouldn't be assumptions.

Before leaving NMAT and the theory in this note; let us observe a few discrepancies I would like to clear up, I list them here.

(1) For the ideal case, insertion loss expected based on the attenuation parameter was -0.32 , while the value computed from the analytic result as well as the tridiagonal inversion was -0.25 .

(2) Theory as I have formulated it here states that a symmetric constant impedance structure, with losses, cannot be perfectly matched. This is either wrong or deep, and I need to convince myself of which. For the time-being I am not too distressed since the asymmetry should not be apparent to four digits in cell frequency, and 2 digits in external Q .

(3) I would like to devise a more precise matching algorithm, one that absolutely guarantees VSWR that is 1.00 at least in the circuit model. This would not translate directly, into an actual structure design that has this good match; but at least at the circuit level one would expect to be able to arrange this at one frequency at least (or is there something more here...).

One final matter concerns the upcoming beadpull measurements. For comparison with those results, I include figures of the mode profiles for both the "ideal" Table 1 parameters, and for the parameters of Table 2 (corresponding to Fig. 8, the case that fits pretty well). They can be found in Appendix B.

A good theoretical problem to pursue next is to run GDFIDL for the 7-cell structure dimensions, to check wall Q and circuit equivalent parameters.

A good experimental problem to pursue next is the beadpull measurement.

For the NLCTA experiment, we must get a handle on these features of W-Band structures, in particular the unacceptably high insertion loss.

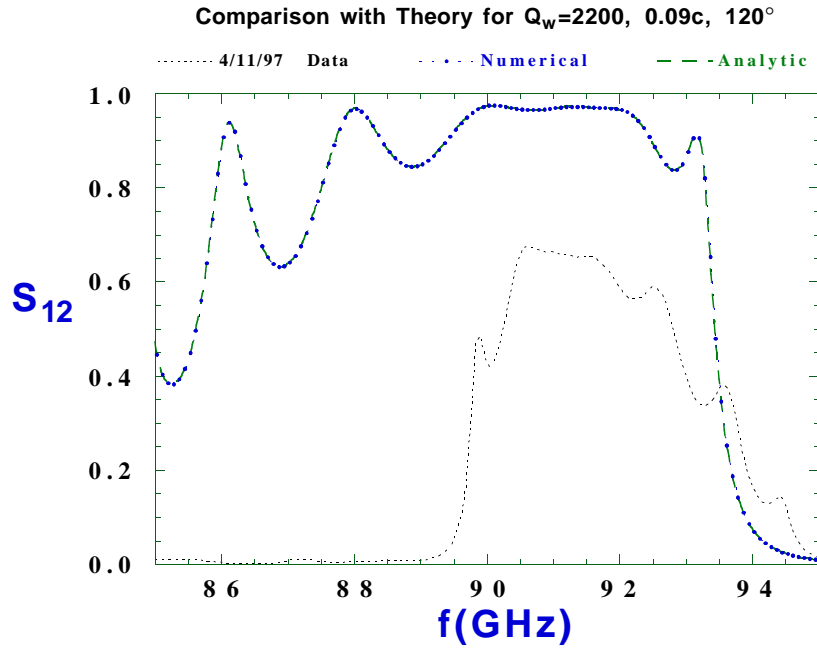
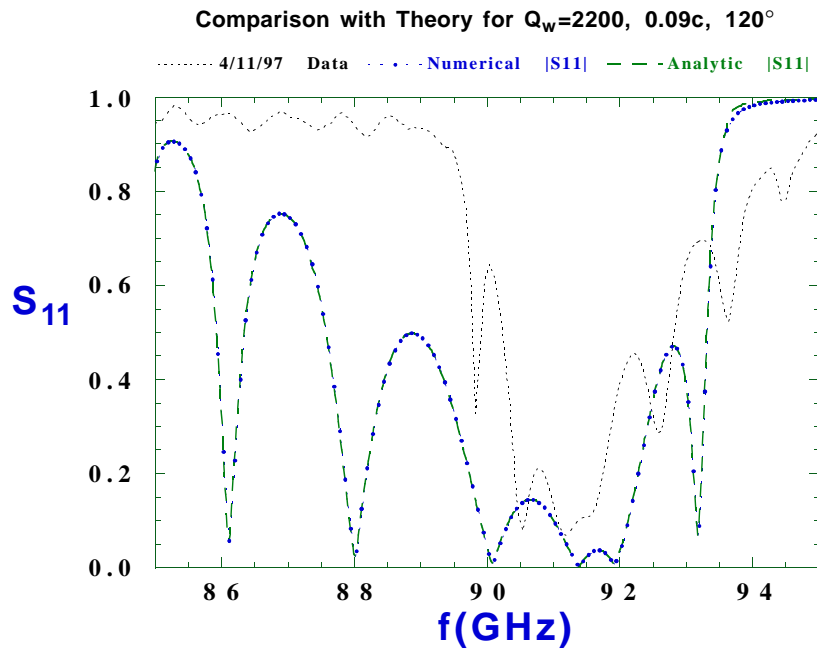


FIGURE 2. Overlay of the S_{12} component of the S-matrix for a 7-cell structure, looking into the input. Structure parameters as in Table 1, more or less the ideal case. The result of the tridiagonal matrix solver is overlaid with the analytic result. Also shown are results of measurements (see TN by P.J. Chou and R. Siemann).

FIGURE 3. S_{11} for the same parameters as in Fig. 2.



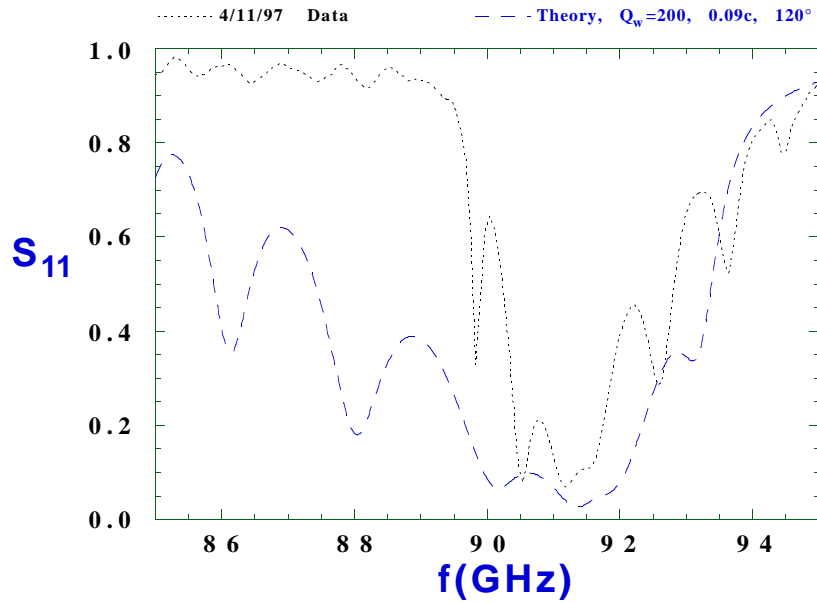
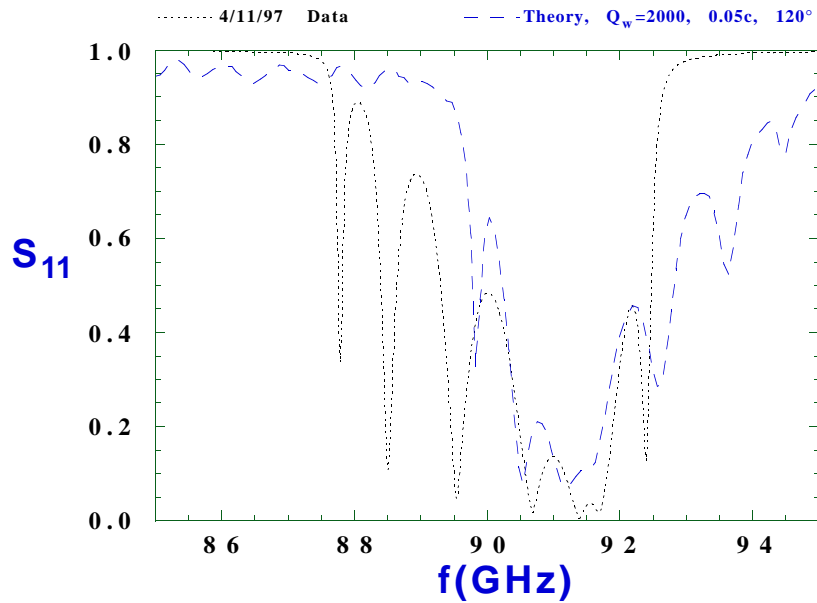


FIGURE 4. Parameters are as in Table 1, except that the wall Q has been lowered to 200. Note that this merely broadens the mode-width.

FIGURE 5. Parameters are as in Table 1, except that the wall Q is 2000 and the group velocity has been lowered to $0.05c$ to attempt to fit the observed bandwidth.



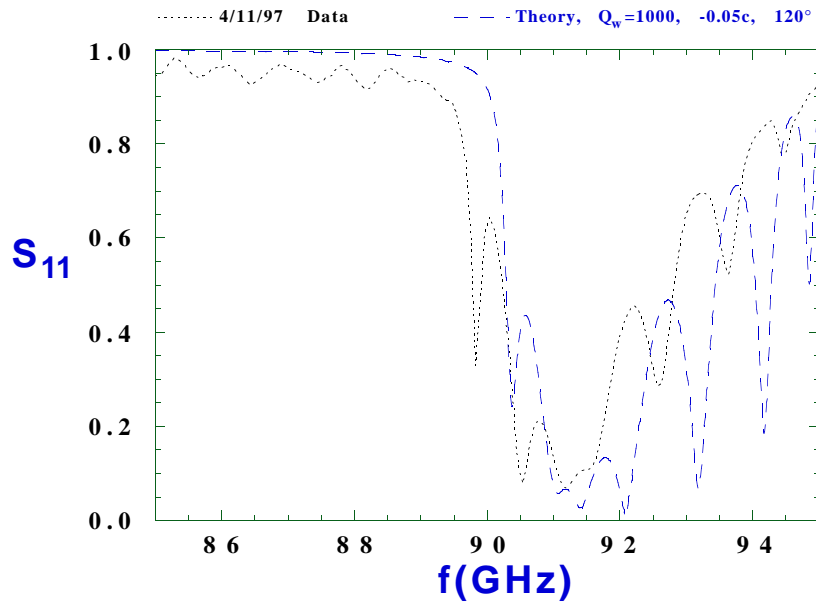
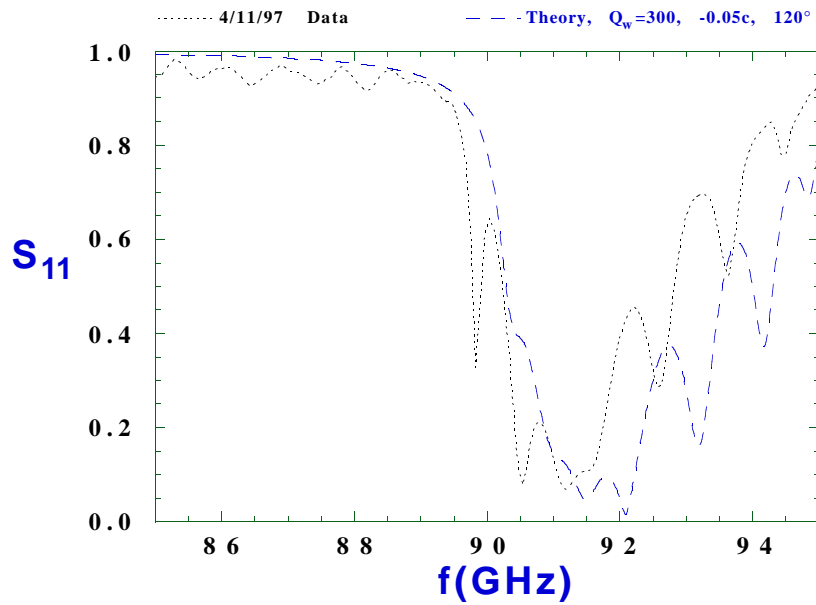


FIGURE 6. Parameters are as in Table 1, except that the wall Q has been lowered again, to 1000, attempting to fit the width of the modes, and the group velocity has been switched to $-0.05c$, corresponding to a *backward* wave, to attempt to match the observed symmetry.

FIGURE 7. Parameters are as in Table 1, except that the wall Q has been lowered again, to 300, attempting to fit the width of the modes; group velocity is $-0.05c$.



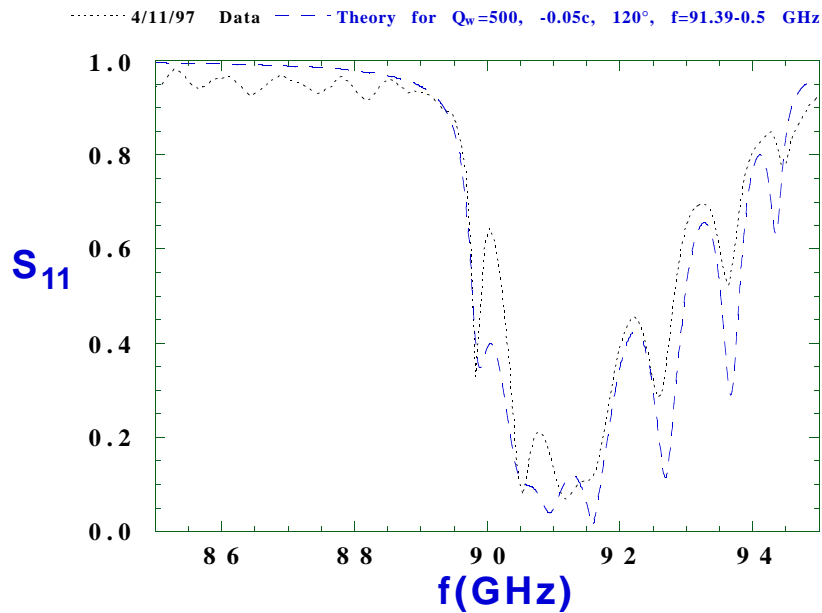
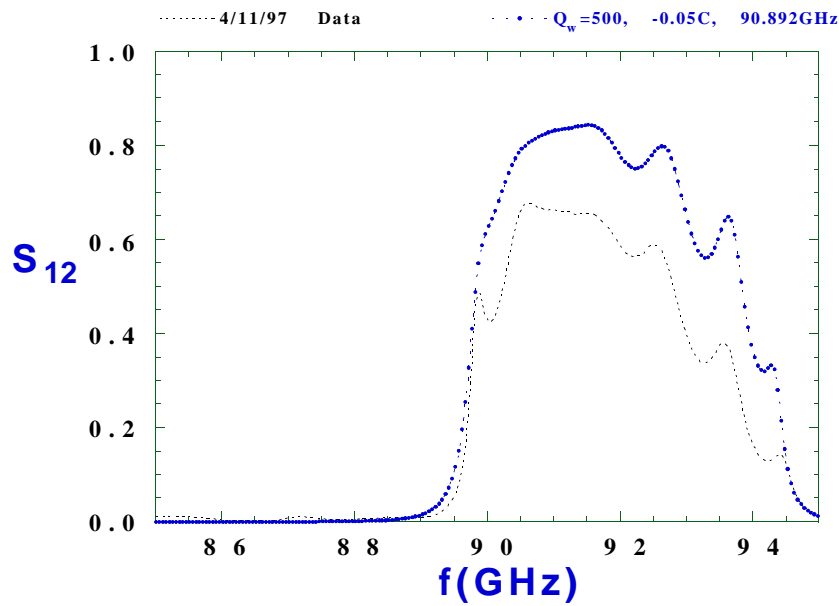


FIGURE 8. Parameters are as in Table 1, except that the wall Q has been set to 500 (with an eye on insertion loss, not shown). The group velocity is $-0.05c$. In this plot, the “design frequency” has been lowered by 0.5GHz, to adjust the curves to match. All adjustments illustrated in the foregoing figures were based on inspection of the plots, not a quantitative measure of error.

FIGURE 9. Comparison of 4/11/97 data with the S_{12} corresponding to Fig. 8 parameters.



Appendix A - NMAT.for

NMAT is portable, and provides output in the form of formatted ascii data written as a gnuplot command file, as well as formatted ascii data suitable for direct importation into Kaleidagraph. I hope to make it available soon as an α -test version in the group WWW space.

When NMAT runs, it first looks for an input namelist called NMAT.IN. Here is an example input file; variables are described more fully below (the comments in the file itself are just mnemonics).

```
$INPUTNML
!
  iphase=2,      !0=synch phase adv,1=zero dispersion,2=fixed
  bphase=120.,  !if iphase=2, use this phase-adv in degrees
!
  qw0=2200.,    !wall Q 2ndary cell
  iopt=1,       !0=decaying pulse, 1=square cw, 2=square pulse
  tfw=3.e-10,   !full-width square pulse (used if iopt=2)
  qep=100.,     !loaded Q for exponent. drive (iopt=0)
  qem=0.,       !if ne 0, use for external Q, last cell
  idetune=0,    !if ne 0, don't detune coupler cells
  ireadat=1,    !if ne 0, read data file for overlay
!
  ncell=7,      !number of cells secondary
  xcell=0.1093, !cm, cell length secondary
!
  ibeta=2,      !0=use beta1,1=compute CG beta1,2=CZ
  beta0=0.09,   !initial vg/c secondary
  beta1=0.09,   !final vg/c secondary (if ibeta=0)
!
  freq0=91.392e9,!design operating frequency
  fbase=91.392e9,!carrier frequency (a numerical parameter)
!
  tmin=0.,      !time starts here, sec
  tmax=10.e-9,  !time ends here, sec
  tswitch=1.e-10,!primary switched at this time >tmin
!
  ntin=2000,    !number of time steps
  imovie=0,     !0=no movie, 1,2-|V|
  iplot=1,      !0=no plots, 1=plots
  isolve=0,     !0=eikonal solve, 1 integrate exactly
  imodes=1,     !1=plot modes (if iplot ne 0)
$END
```

Here are expanded annotations explaining the notation. Some features of NMAT are not relevant here, and more detailed explanations will be found on notes on the matrix accelerator.

iphase=0 design a structure with synchronous phase-advance, based on the specified cell length, **xcell**, below
iphase=1 design a line with zero first-order dispersion
iphase=2 design a line with fixed phase-advance, in this case, use phase-advance per cell **bphase**.
qw0 wall Q of an accelerating cavity

NMAT also outputs transient waveforms

iopt=0 subject the structure to an exponentially decaying input pulse, with voltage time-constant specified by **qep**
iopt=1 use a cw input pulse with step turn on
iopt=2 use a square, “top-hat” pulse with length **tfw**
qem if $\neq 0$ then use this value for external Q for the last cell, rather than letting NMAT compute the matched value
idetune if $\neq 0$, don't NMAT will not detune the coupler cells to provide a match
ireadat if $\neq 0$, look for a two-column ascii data file called **example.dat**, and plot it as frequency, S11, compared to NMAT's computation
ncell number of cells in the structure
xcell period of the structure, assumed constant
ibeta=0 taper group velocity linearly from **beta0** to **beta1**
ibeta=1 design a constant gradient circuit
ibeta=2 group velocity is constant, equal to **beta0**, this is a constant impedance structure
beta0 initial V_g/c accelerator cavity
beta1 final V_g/c accelerator cavity (used only if **ibeta=0**)
freq0 design operating frequency
fbase a carrier frequency (a numerical parameter) used for eikonal integration of the transient problem

The remaining items are numerical and output control variables.

tmin a numerical parameter, start of time
tmax a numerical parameter, end of time
tswitch a numerical parameter, time at which drive turns on, used for transient simulation
ntin number of time steps
imovie if $\neq 0$, NMAT will make a movie using some gks routines
iplot if $\neq 0$, plots and verbose output are enabled
isolve=0 solve transient problem using eikonal integration
isolve=1 solve transient problem by integrating the full 2nd-order equations
imodes if $\neq 0$, plot all modes in this structure

An example of the output to screen, when NMAT is shown on the next page, comments I have added are in bold.

Simulating Constant Impedance Structure...
 will use EIKONAL integration (isolve=0)
 generating cw square wave input
 Primary Subr...
 writing plot file primout.dat
 expect fill time (ns) = 0.2835665
 expect atten parameter "tau" = 3.7007552E-02
 expect insertion loss (dB) = -0.3214436
 transit angle wL/c (deg) = 119.9527

ignore the following line

1st & last iris radius (cm) if cyl 4.8620652E-02

4.8620652E-02

cell 1 freq (GHz)	90.22733	this is $\omega_1/2\pi$
cell 2 freq (GHz)	89.09483	this is $\omega_2/2\pi$
cell N-1 freq (GHz)	89.09483	this is $\omega_6/2\pi$
cell N freq (GHz)	90.21564	this is $\omega_7/2\pi$

expected fill time (ns) = 0.2835665
 initial phase adv per cell (deg) 120.0000 **as asked**
 final phase adv per cell (deg) 120.0000
 external Q 1st cell, 2ndary = 22.16693 **NMAT computes**
 external Q last cell, 2ndary = 22.51004 **NMAT computes**
 plotting structure parms vs cell # ...
 writing plot file parm.dat **a gnuplot cmd file for structure parameters**

NMAT goes on to check match at design frequency

DRIVING INPUT CELL **computing S_{11} , by solution of a
 tridiagonal matrix equation**

INPUT CELL steady-state ref. coeff = 2.5868479E-03
 VSWR looking into this port = 1.005187
 |S21| = 0.9720609
 in dB = -0.2461303

**NMAT repeats this for a range of frequencies, simulating network analyzer
 S-Matrix - Input Coupler**

writing plot file nain.dat **a gnuplot cmd file for S_{11} , i.e.,
 simulated network analyzer output**
 writing tabular (KG) data to **S_{11} data formatted for importation to
 nainkg.dat **Kaleidagraph****

tabular (KG) data for analytic comp. to S_{11} **computed by analytic
 nainkg1.dat **formula also, if $i\beta=2$****

**NMAT surveys S_{11} result and tallies the modes, i.e., frequencies where
 there occur local dips in S_{11} provided the VSWR there is sufficiently low.**

At each such frequency it solves for and plots \tilde{V}_n vs. n.

identified # of modes = 7
writing plot file modesin.dat **plots each mode**

mode#	f(GHz)	VSWR	IL(dB)	Phase(deg)
1	84.784	1.514	1.974	-11.18
2	86.117	1.119	0.549	-40.18
3	88.017	1.050	0.283	-69.89
4	90.084	1.018	0.221	-99.60
5	91.384	1.006	0.246	-119.88
6	91.917	1.016	0.283	-129.23
7	93.184	1.149	0.813	-158.72

NMAT repeats the procedure, this time looking into the output cell.

OUTPUT CELL steady-state ref. coeff = 1.2116903E-02
DRIVING OUTPUT CELL

VSWR looking into this port = 1.024531
|S12| = 0.9573686
in dB = -0.3784168

S-Matrix - Output Coupler
writing plot file naout.dat
writing tabular (KG) data to
naoutkg.dat
tabular (KG) data for analytic comp. to
naoutkg1.dat
identified # of modes = 7
writing plot file modesout.dat

mode#	f(GHz)	VSWR	IL(dB)	Phase(deg)
1	84.784	1.522	2.106	11.20
2	86.117	1.126	0.681	40.20
3	88.017	1.056	0.415	69.91
4	90.084	1.025	0.354	99.63
5	91.400	1.025	0.379	120.12
6	91.917	1.009	0.416	129.25
7	93.184	1.142	0.945	158.73

NMAT next drives the circuit, from the input coupler, with the user selected waveform, and analyzes the peaking voltage in each cell (this is for matrix accelerator work).

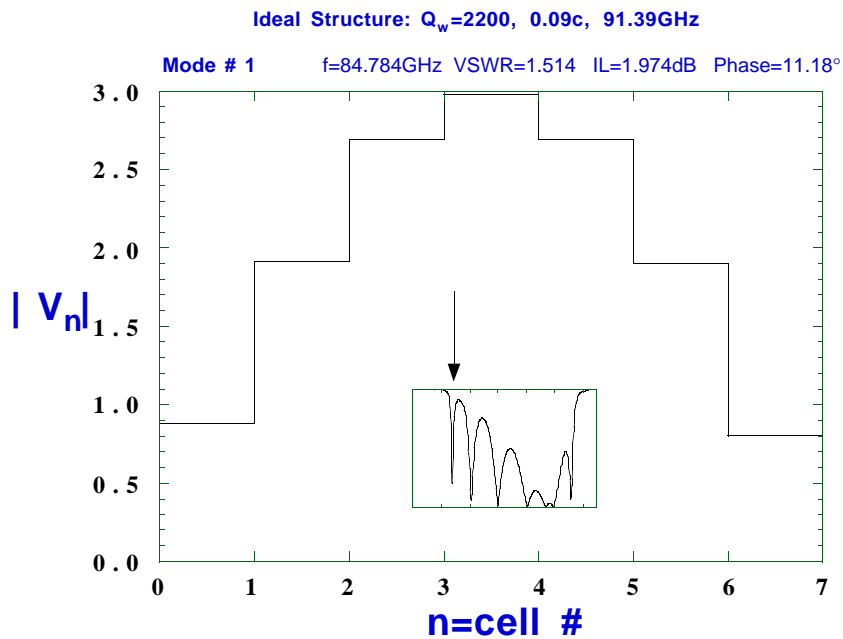
Analyzing Time-Domain Data w/ Specified Drive...
cell # Max V

1	1.077696
4	1.080962
5	1.254270
6	1.133917
7	1.045086

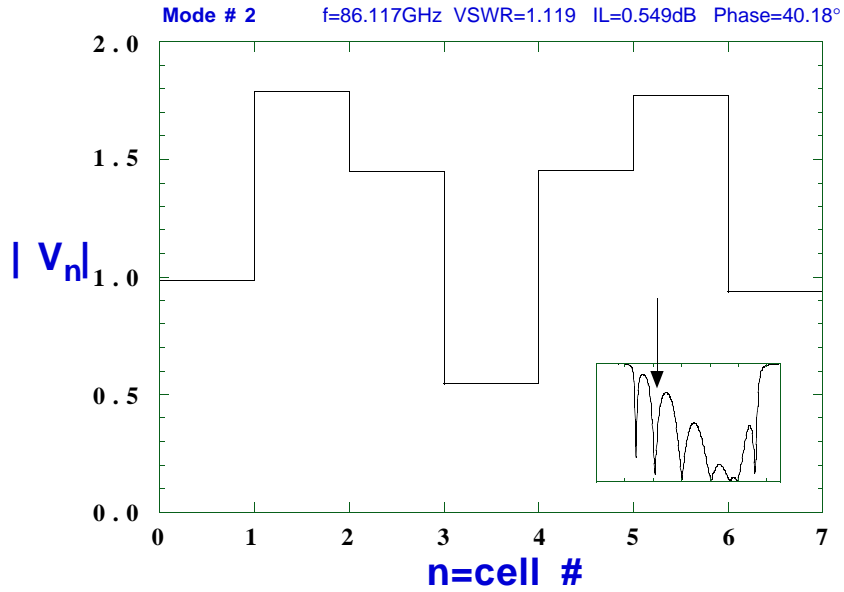
max V-Last/ max V-First 0.9697413
plotting waveforms for structure...
writing plot file str.dat

Appendix B - Mode Profiles

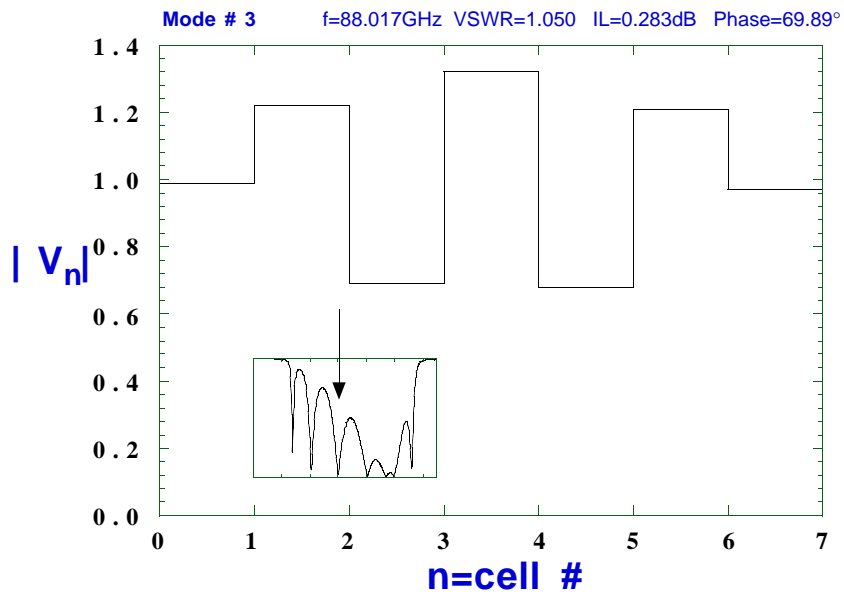
To make more concrete the modes as evidenced by the S_{11} data, it is helpful to make plots of cell amplitude. The following seven figures display the seven modes of the ideal case (Table 1 parameters) and the six figures following these display the six mode of the “fit” case (Table 2). There are six in this case simply because one mode was deformed enough that it failed NEMAT’s mode-search algorithm (requiring a local minimum in S_{11}). The plots labelled “ideal case” have parameters as in Table 1. The plots labelled “fit case” have parameters as in Table 2. With this in mind, and since the plots are clearly labelled, perhaps it is unnecessary to make a caption for each one.



Ideal Structure: $Q_w=2200$, $0.09c$, 91.39GHz

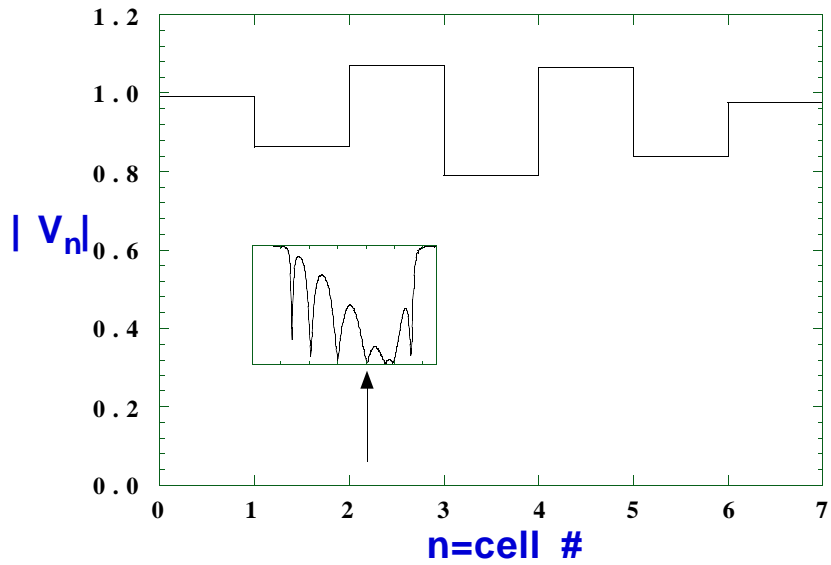


Ideal Structure: $Q_w=2200$, $0.09c$, 91.39GHz



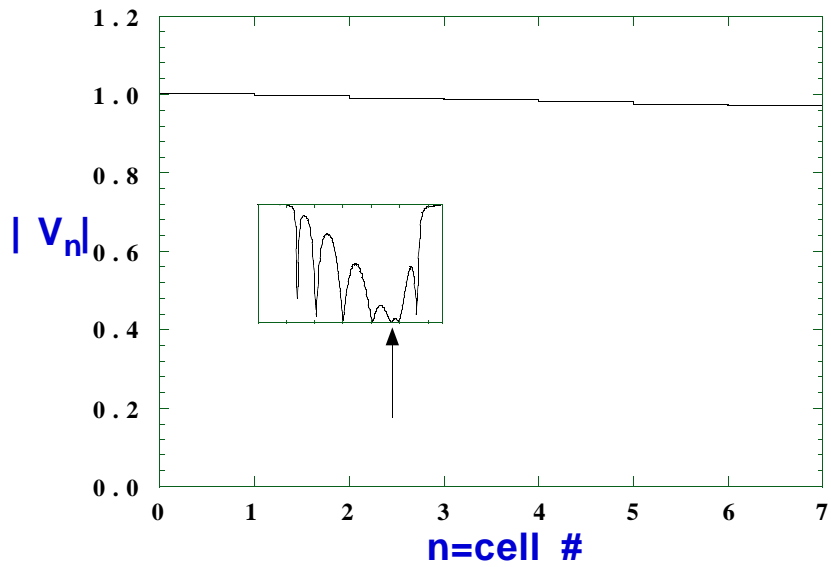
Ideal Structure: $Q_w=2200$, $0.09c$, 91.39GHz

Mode # 4 $f=90.084\text{GHz}$ $\text{VSWR}=1.018$ $\text{IL}=0.2213\text{dB}$ $\text{Phase}=99.60^\circ$



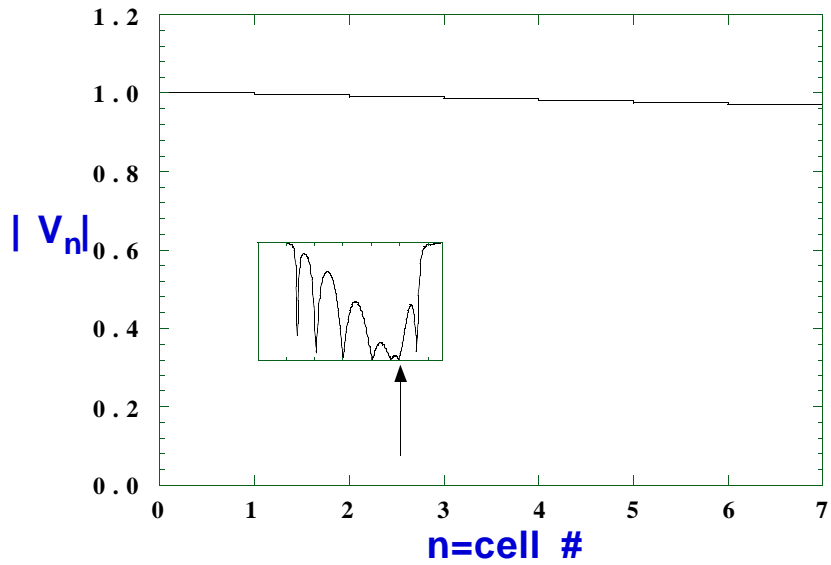
Ideal Structure: $Q_w=2200$, $0.09c$, 91.39GHz

Mode # 5 $f=91.384\text{GHz}$ $\text{VSWR}=1.006$ $\text{IL}=0.2463\text{dB}$ $\text{Phase}=119.88^\circ$



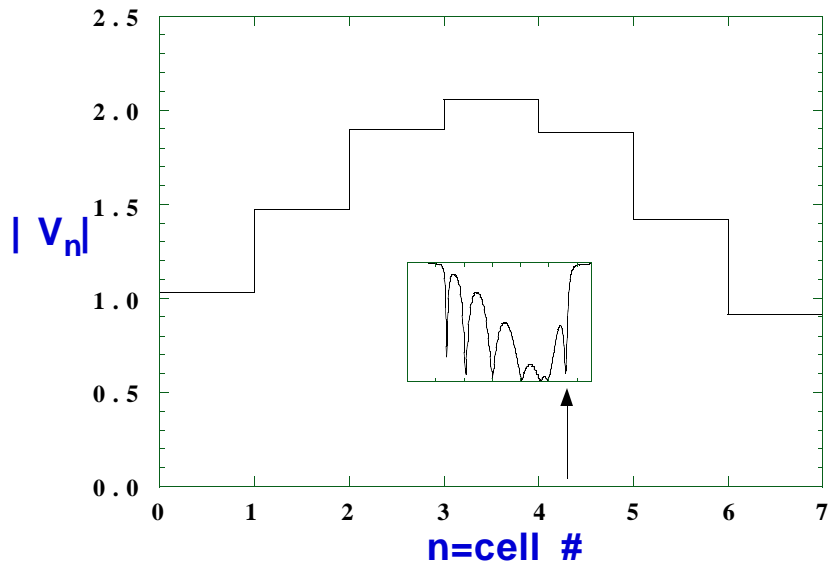
Ideal Structure: $Q_w=2200$, $0.09c$, 91.39GHz

Mode #6 $f=91.917\text{GHz}$ $\text{VSWR}=1.016$ $\text{IL}=0.283\text{dB}$ $\text{Phase}=129.23^\circ$

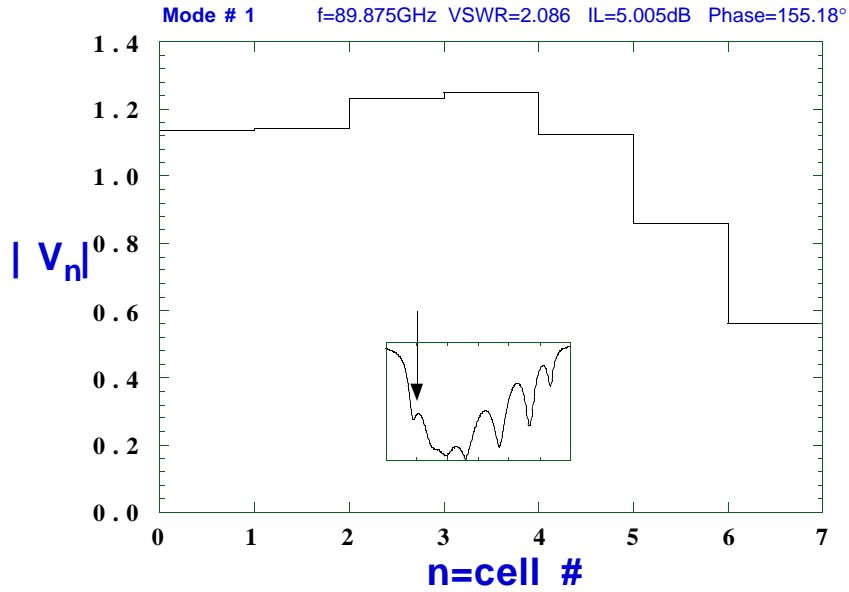


Ideal Structure: $Q_w=2200$, $0.09c$, 91.39GHz

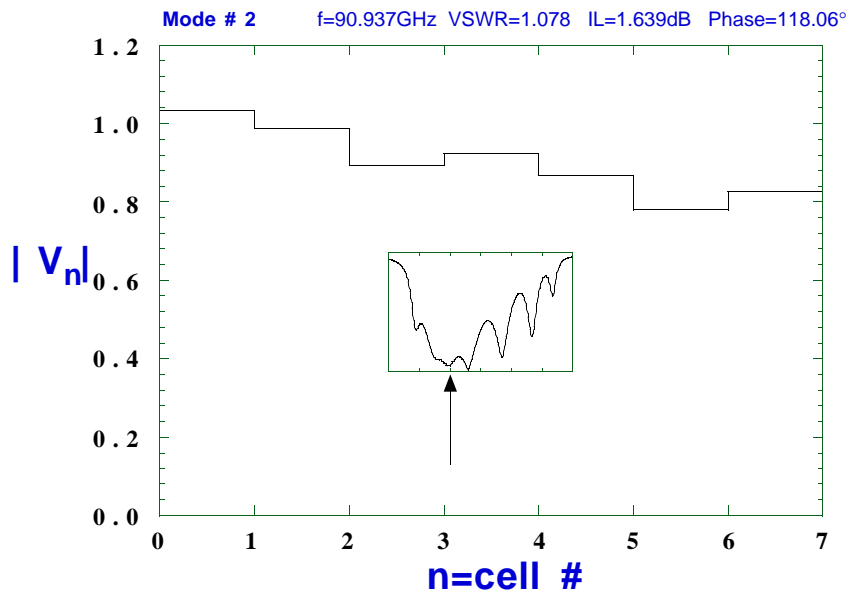
Mode #7 $f=93.184\text{GHz}$ $\text{VSWR}=1.149$ $\text{IL}=0.813\text{dB}$ $\text{Phase}=158.72^\circ$



"Fit" Structure: $Q_w=500$, $-0.05c$, 92.39 - 0.5 GHz

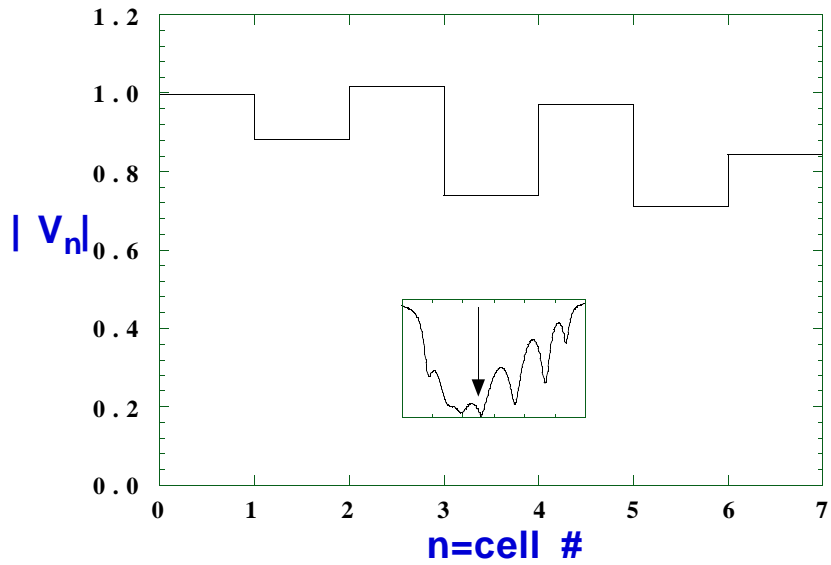


"Fit" Structure: $Q_w=500$, $-0.05c$, 92.39 - 0.5 GHz



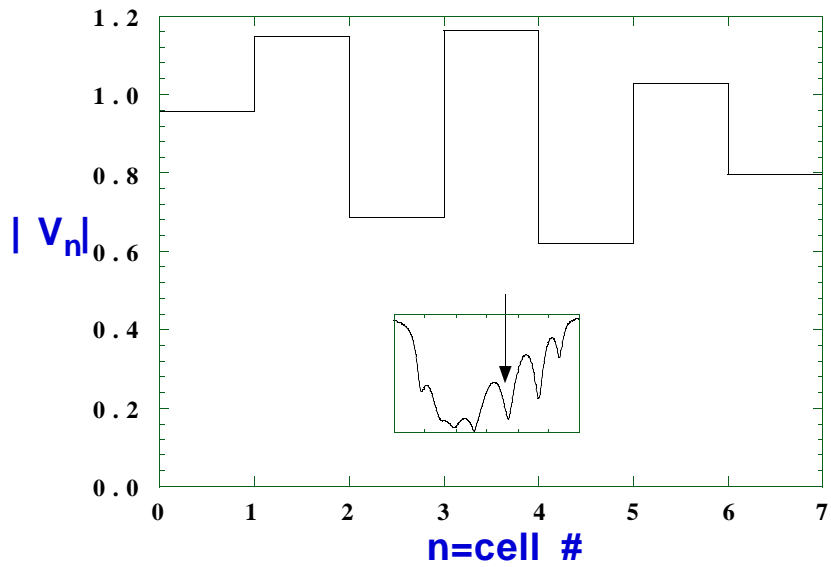
"Fit" Structure: $Q_w=500$, $-0.05c$, 92.39 - 0.5 GHz

Mode # 3 $f=91.558\text{GHz}$ VSWR=1.019 IL=1.493dB Phase= 100.07°

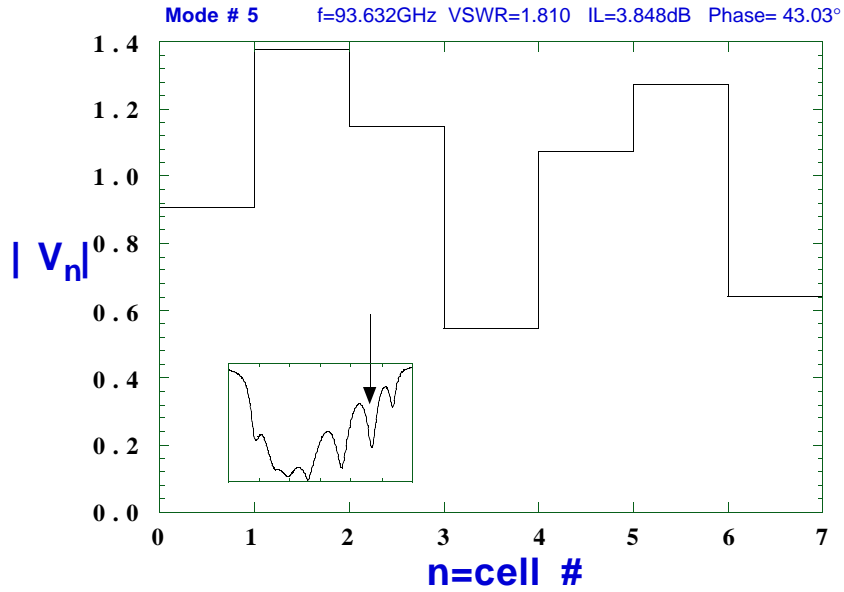


"Fit" Structure: $Q_w=500$, $-0.05c$, 92.39 - 0.5 GHz

Mode # 4 $f=92.650\text{GHz}$ VSWR=1.255 IL=1.982dB Phase= 71.18°



"Fit" Structure: $Q_w=500$, $-0.05c$, 92.39 - 0.5 GHz



"Fit" Structure: $Q_w=500$, $-0.05c$, 92.39 - 0.5 GHz

