Microwave Electronics

An Introduction to the Notion of Equivalent Circuit Starting from Maxwell's Equations

in six lectures

- ☞ **Maxwell's Equations & Modes in a Guide**
- ❒ **Equivalent Circuit for Waveguide Modes**
- ❒ **Modes of a Cavity**
- ❒ **Cavity with a Port & External Q**
- ❒ **Microwave Networks**
- ❒ **Slater's Perturbation Theorem**

Microwave Electronics I

Maxwell's Equations

and

Waveguide Modes

① Lorentz Force Law

② Maxwell’s Equations

③ Skin Depth

④ Orthogonal Modes

⑤ Phase & Group Velocity

⇨ Quiz

①Lorentz Force Law

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\vec{E}$ Newtons per Coulomb=V/m
"electric field strength"

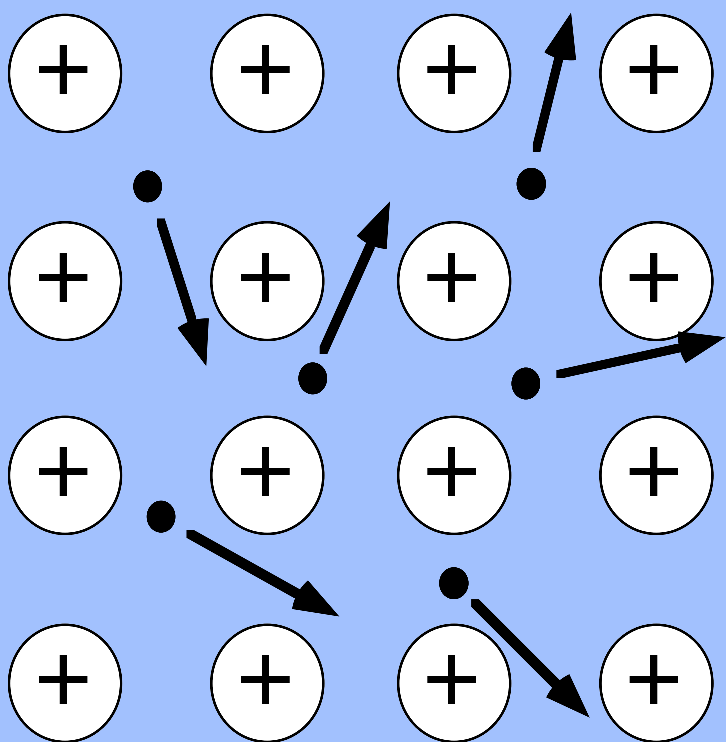
$\vec{B}$ Newtons per Ampere-meter=T=Wb/m^2
"magnetic flux density"
or "magnetic induction"

- defines the fields & abstracts them from the sources
- describes "test particle" motion
- describes response of media

Hendrik Antoon **Lorentz**
b. July 18, 1853, Arnhem, Netherlands
d. Feb. 4, 1928, Haarlem

Example: Conductivity

$$m\frac{d\vec{v}}{dt} = q\vec{E} - m\frac{\vec{v}}{\tau} \qquad \vec{v} = \frac{q\tau}{m}\vec{E}$$



$$J = nq\vec{v}$$

$$= \frac{nq^2\tau}{m}\vec{E}$$

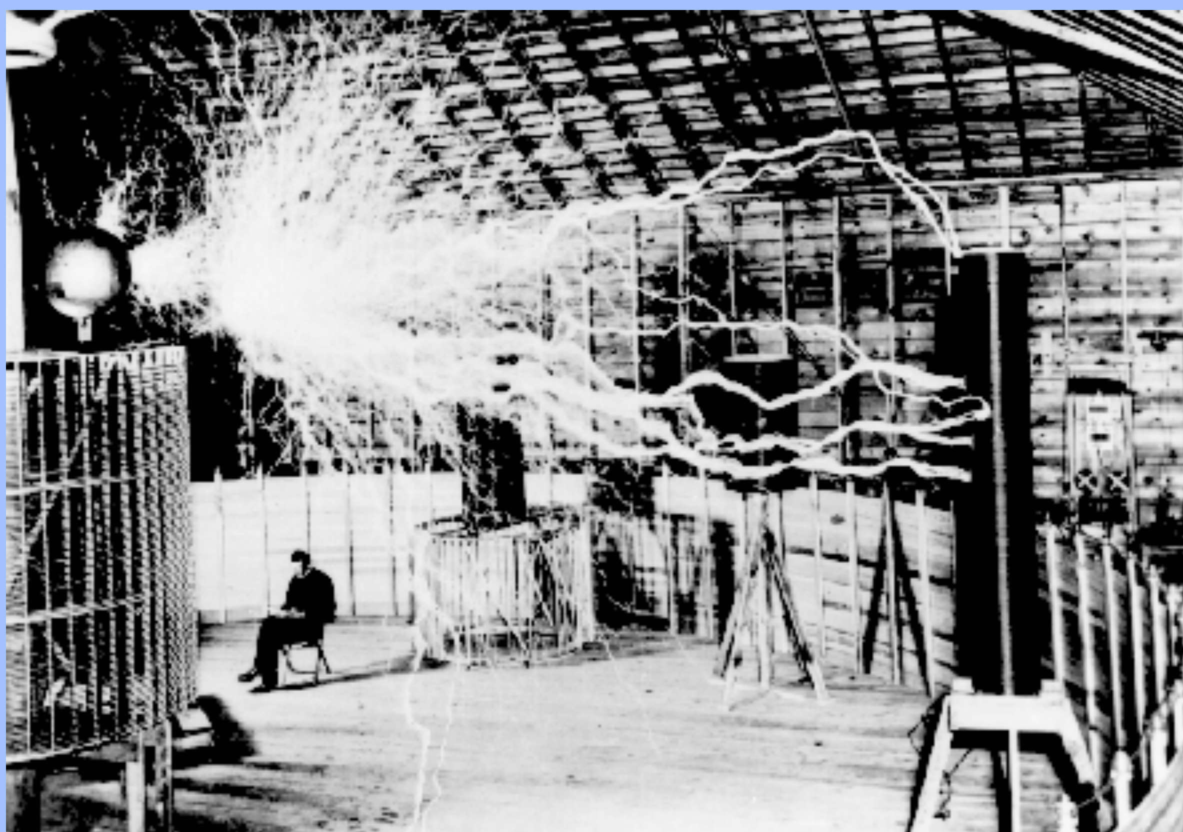
$$= \sigma\vec{E}$$

$\sigma = "\sigma_{DC}" \approx 5.8 \times 10^7\, mho/m$ for Cu

Georg Simon **Ohm**
(b. March 16, 1789, Erlangen, Bavaria --d. July 6, 1854, Munich)

P. **Drude**, 1900

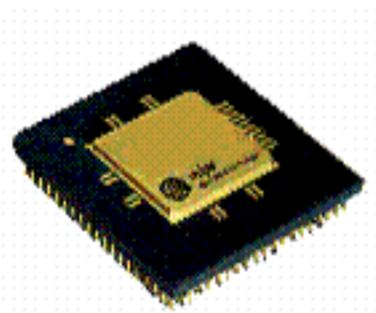
NB This is a simplified picture of a normal conductor...occasionally this picture breaks down...



and of course this model cannot be applied to all materials...

METALS Metals are chemical elements which form solids that are opaque, lustrous, good conductors of electricity and heat, and when polished, good reflectors of light. Most metals are strong, ductile, and malleable, and, in general, of high density. Metals are the primary structural materials of technology, and include the great range of iron-based alloys (e.g. cast iron, plain carbon steels, alloy steels).

Ceramics This class may be defined as any inorganic, nonmetallic solids processed or used at high temperatures. We immediately think of such things as pottery, sanitary whiteware, tiles, table china, etc. We often overlook the the more " high tech " applications of oxides, carbides and nitrides. Many of these are of great industrial interest. Ceramics also include materials such as glass, graphite, and cement (concrete).



Semiconductors This is a special category of non-metallic inorganic material. They are insulators in which the energy gap between the state of the valence electrons and the electron states required for electrical conduction are much smaller than in conventional insulators and can be bridged by thermal excitation or by introducing small levels of impurities with electron states in the gap. These material are the building blocks of transistors, solid electronics and computers.

POLYMERS These cover a group of materials with the common feature that the binding is covalent. At one end of the scale are the linear polymers in which the simple molecules, often carbon-hydrogen groups, are joined into very long chains by strong covalent bonds and the binding between the chains is due to weak van der Waals forces. They are never fully crystalline. They are the basis of wood and thermoplastics. At the other end of the scale are the close-network polymers in which three-dimensional covalent molecules are formed by polymerization of monomer units. Besides plastics, polymers are the basis of the paint, rubber, and synthetic fiber industries.

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②Maxwell’s Equations

Electricity & Magnetism before Maxwell...

- Charges repel or attract
- Current carrying wires repel or attract
- Time-varying currents can induce currents in surrounding media

After Maxwell...

- Light is an electromagnetic phenomenon
- Nature is not Galilean
- Thermodynamics applied to electromagnetic fields gives divergent results
- Matter appears not to be stable
- Questions arise concerning gravitation...

“Ordinary” Electronics

•voltages vary slowly on the scale of the
transit time = circuit size / speed of light
•circuit size small compared to wavelength
•voltage between two points independent of path
•may treat elements as “lumped”
•unique notion of impedance of an element
•bring a multimeter

Microwave Electronics

•circuit size appreciable compared to a wavelength
•voltage between two points depends on path
•elements are “distributed”, spatial phase-shifts occur between them
•if the word “impedance” is used, you may always ask how it was defined...
•if any result of test & measurement is quoted, you may always ask how the equipment was calibrated
•bring crystal detectors, filters, mixers, a signal generator, a spectrum analyzer, and, if you have them a network analyzer, calibration kit, vector voltmeter

History

Henry **Cavendish**

(b. Oct. 10, 1731, Nice, France--d. Feb. 24, 1810, London, Eng.)

Charles-Augustin de **Coulomb**

(b. June 14, 1736, Angoulême, Fr.--d. Aug. 23, 1806, Paris)

André-Marie **Ampere**

(b. Jan. 22, 1775, Lyon, France--d. June 10, 1836, Marseille)

Karl Friedrich **Gauss**

(b. April 30, 1777, Brunswick--d. 1855)

Hans Christian **Ørsted**

(b. Aug. 14, 1777, Rudkøbing, Den.--d. March 9, 1851, Copenhagen)

Siméon-Denis **Poisson**

(b. June 21, 1781, Pithiviers, Fr.--d. April 25, 1840, Sceaux)

Michael **Faraday**

(b. Sept. 22, 1791, Newington, Surrey --d. August 25, 1867, Hampton Court)

James Clerk **Maxwell**

(b. June 13 or Nov. 13, 1831, Edinburgh--d. Nov. 5, 1879, Glenlair)

Heinrich (Rudolf) **Hertz**

(b. Feb. 22, 1857, Hamburg--d. Jan. 1, 1894, Bonn)

Guglielmo **Marconi**

(b. April 25, 1874, Bologna, Italy--d. July 20, 1937, Rome)

Gauss's Law

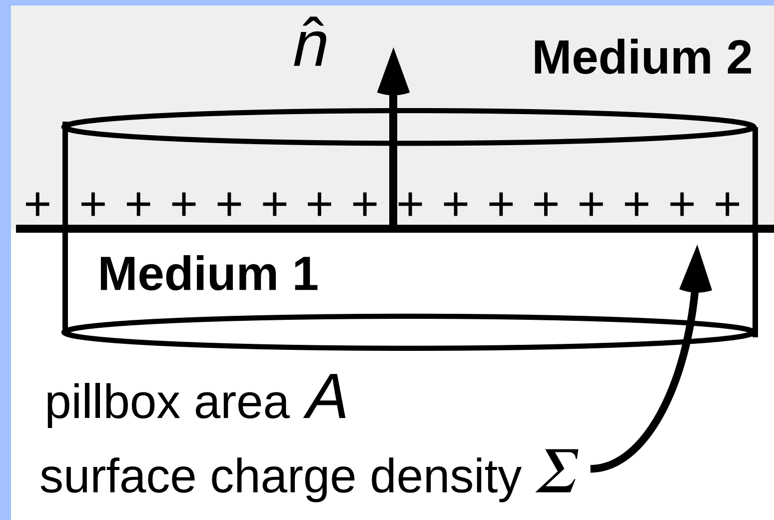
$$\oint_{\partial V} \vec{D} \bullet d\vec{S} = \int_V \rho dV$$

$$\left(\vec{D}_2 - \vec{D}_1\right) \bullet \hat{n} A = \Sigma A$$

$$D \Leftrightarrow \Sigma \Leftrightarrow C / m^2$$

electric displacement

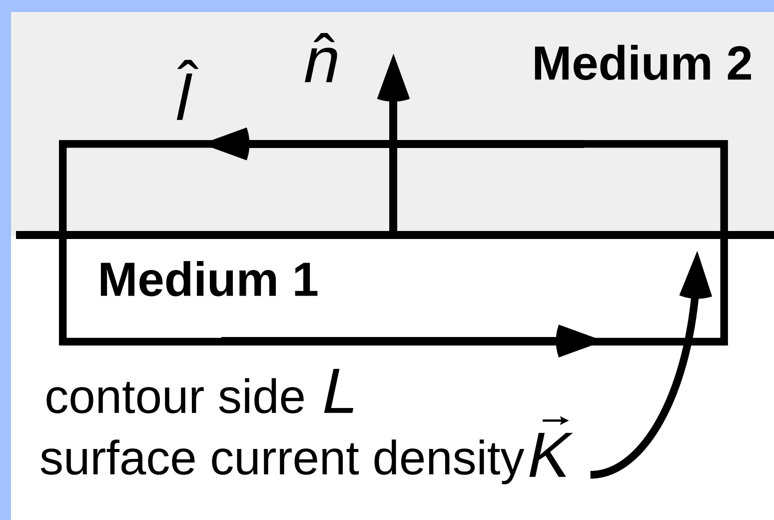
or electric flux density



Ampere's Law

before Maxwell

$$\oint_{\partial S} \vec{H} \bullet d\vec{l} = \int_S \vec{J} \bullet d\vec{S}$$



$$\left(\vec{H}_2 - \vec{H}_1\right) \bullet \hat{l} = \vec{K} \times \hat{n} \bullet \hat{l}$$

$$\hat{n} \times \left(\vec{H}_2 - \vec{H}_1\right) = \vec{K}$$

$$H \Leftrightarrow K \Leftrightarrow A / m$$

magnetic field strength

or magnetic flux

In Vacuum...

$$\vec{D} = \varepsilon_0 \vec{E} \qquad \vec{H} = \vec{B} / \mu_0$$

$$\varepsilon_0 \approx 8.85 \times 10^{-12} \; farad \; per \; meter$$

$$\mu_0 = 4\pi \times 10^{-7} \; henry \; per \; meter$$

or

$$\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c = 2.9979 \times 10^8 \; m/s$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

In Media...

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

electric dipole moment density $\vec{P} = \chi_e \varepsilon_0 \vec{E}$

$$\vec{H} = \vec{B} / \mu_0 - \vec{M} = \vec{B} / \mu$$

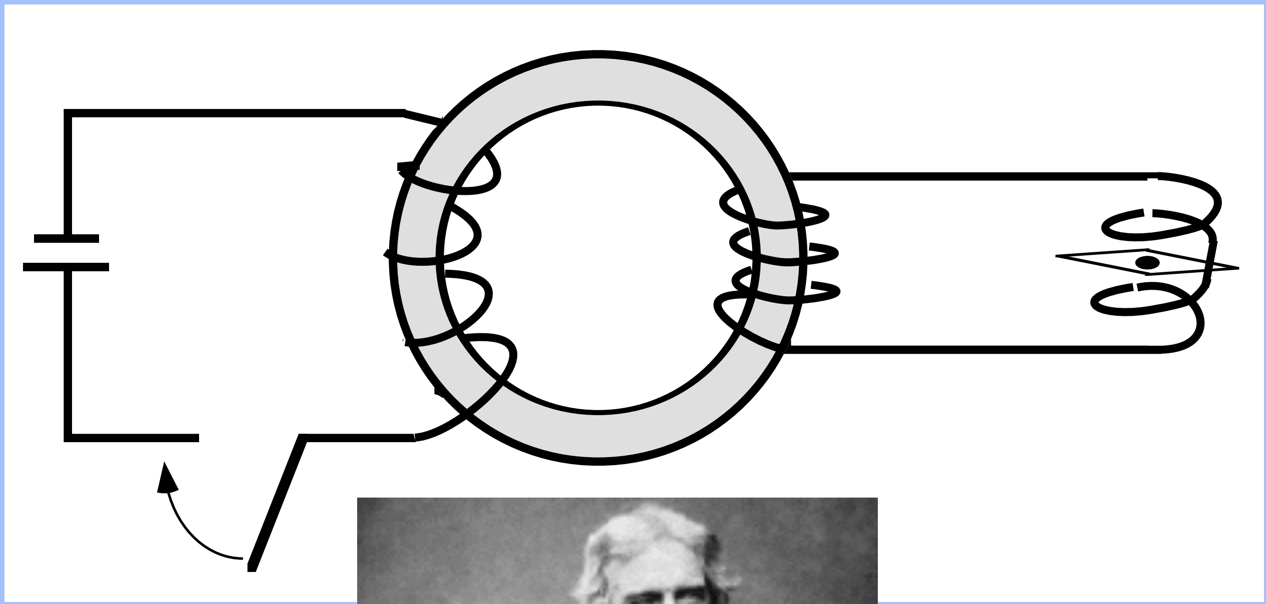
magnetic dipole moment density $\vec{M} = \chi_m \vec{H}$

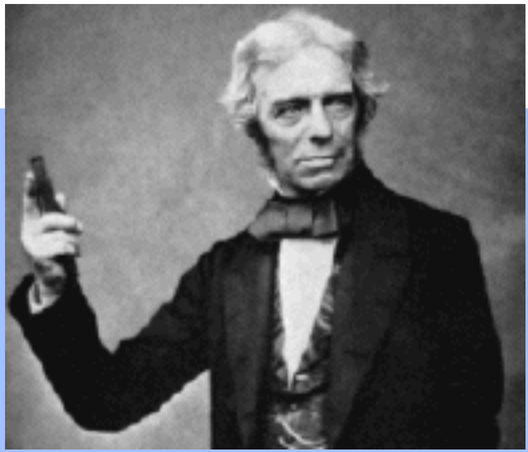
Finer Points:

- These are really frequency domain expressions
- In general ε,μ are tensors
- μ may be non-linear & biased by a DC field
- H,D depend on your point of view

Faraday's Law

$$\oint_{\partial S} \vec{E} \bullet d\vec{l} = -\int_{S} \frac{\partial \vec{B}}{\partial t} \bullet d\vec{S}$$





●*Before Maxwell*, Ampere's Law was inconsistent with conservation of charge

●*After Maxwell*, the fields didn't need charge to support them, they could propagate on their own

●*Of course no one believed Maxwell, but the fields didn't mind*

Charge Conservation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \vec{J} = 0$$

or

$$\frac{\partial}{\partial t}\int_V \rho dV = -\oint_{\partial V} \vec{J} \bullet d\vec{S}$$

Ampere's Law (before Maxwell's addition of Displacement Current) implied

$$\frac{\partial \rho}{\partial t} = -\nabla \bullet \vec{J} = -\nabla \bullet \nabla \times \vec{H} = 0$$

...actually not a bad approximation in conductors, or a dense plasma.. excellent for electrostatics, magnetostatics

Maxwell's Equations

Gauss's Law

$$\vec{\nabla} \bullet \vec{D} = \rho$$

Ampere's Law
with
Maxwell's Displacement Current

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Faraday's Law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

No Magnetic Charge

$$\nabla \bullet \vec{B} = 0$$

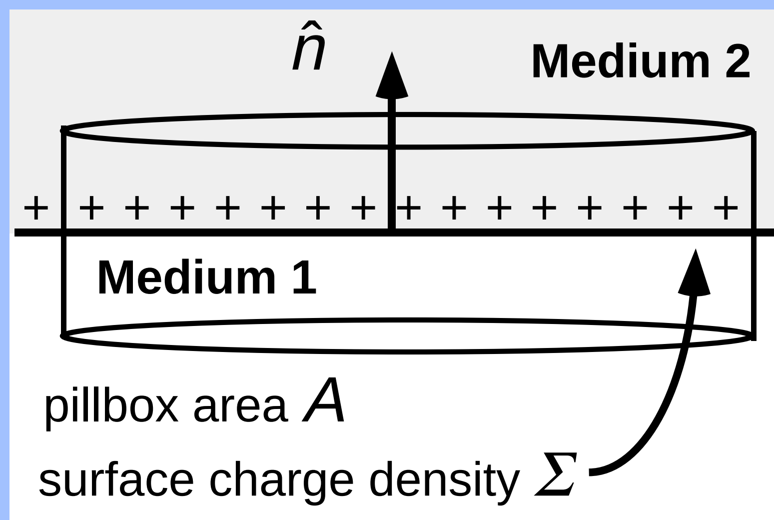
James Clerk Maxwell
b. June 13 or Nov. 13, 1831, Edinburgh
d. Nov. 5, 1879, Glenlair

Boundary Conditions

apply Maxwell's equations in integral form...

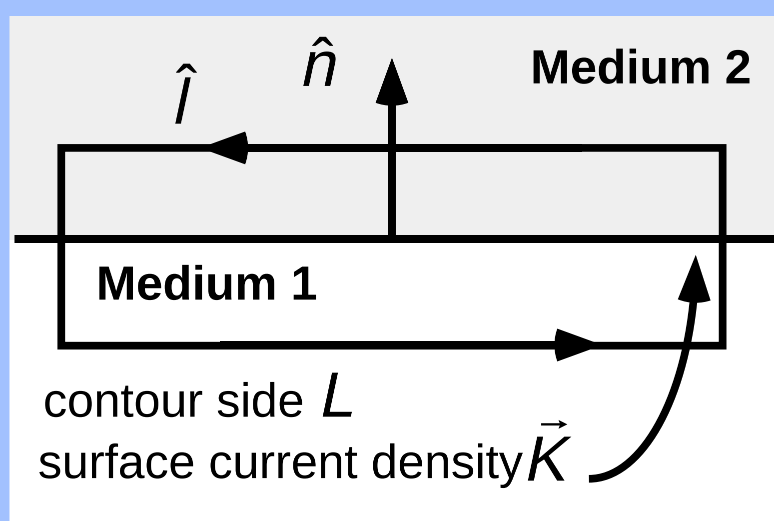
$$\left(\vec{D}_2 - \vec{D}_1\right) \bullet \hat{n} = \Sigma$$

$$\left(\vec{B}_2 - \vec{B}_1\right) \bullet \hat{n} = 0$$



$$\hat{n} \times \left(\vec{H}_2 - \vec{H}_1\right) = \vec{K}$$

$$\hat{n} \times \left(\vec{E}_2 - \vec{E}_1\right) = 0$$



Example to Illustrate ε: Unmagnetized Plasma

$$m\frac{d\vec{v}}{dt} = q\vec{E} - m\frac{\vec{v}}{\tau}$$

$$\vec{E} = \Re\left(\tilde{E}e^{j\omega t}\right) = \frac{1}{2}\left(\tilde{E}e^{j\omega t} + \tilde{E}^{*}e^{-j\omega t}\right)$$

$$\tilde{v} = \frac{1}{(1+j\omega\tau)}\frac{q\tau}{m}\tilde{E} \Rightarrow \tilde{J} = nq\tilde{v} = \frac{\sigma_{DC}}{(1+j\omega\tau)}\tilde{E}$$

Apply charge conservation & Gauss's Law...

$$\frac{\partial\rho}{\partial t} + \nabla\bullet\vec{J} = 0 \Rightarrow j\omega\tilde{\rho} + \nabla\bullet\tilde{J} = 0$$

$$\nabla\bullet\varepsilon_0\tilde{E} = \tilde{\rho} = -\frac{1}{j\omega}\nabla\bullet\tilde{J} = -\frac{1}{j\omega}\nabla\bullet\frac{\sigma_{DC}}{(1+j\omega\tau)}\tilde{E}$$

$$\nabla\bullet\left\{\varepsilon_0 + \frac{1}{j\omega}\frac{\sigma_{DC}}{(1+j\omega\tau)}\right\}\tilde{E} = 0 \Rightarrow \varepsilon = \varepsilon_0 + \frac{1}{j\omega}\frac{\sigma_{DC}}{(1+j\omega\tau)}$$

N.B. $\lim_{\tau\to\infty}\frac{\varepsilon}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\omega^2}$ *with* $\omega_p^2 = \frac{nq^2}{m\varepsilon_0}$

Evidently...

•electric displacement depends on what you consider to be the “external” circuit, electric field does not

View #1

$$\nabla \bullet \varepsilon_0 \tilde{E} = \tilde{\rho}_{plasma} + \tilde{\rho}_{other}$$

$$\tilde{D} = \varepsilon_0 \tilde{E}$$

View #2

$$\nabla \bullet \varepsilon\, \tilde{E} = \tilde{\rho}_{other}$$

$$\tilde{D} = \varepsilon\, \tilde{E}$$

•electric permittivity is a frequency-domain concept...

$$\varepsilon = \varepsilon_0 + \frac{1}{j\omega} \frac{\sigma_{DC}}{(1 + j\omega\tau)} = \varepsilon(\omega)$$

Polarization in the time domain...

$$\vec{P}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} e^{j\omega t} \tilde{P}(\omega)$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} e^{j\omega t} \varepsilon_0 \chi_e(\omega) \tilde{E}(\omega)$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} e^{j\omega t} \varepsilon_0 \chi_e(\omega) \int_{-\infty}^{+\infty} \frac{dt'}{\sqrt{2\pi}} e^{-j\omega t} \vec{E}(t')$$

$$= \varepsilon_0 \int_{-\infty}^{+\infty} dt' G(t-t') \vec{E}(t')$$

where the Green's function is

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \, e^{j\omega t} \chi_e(\omega)$$

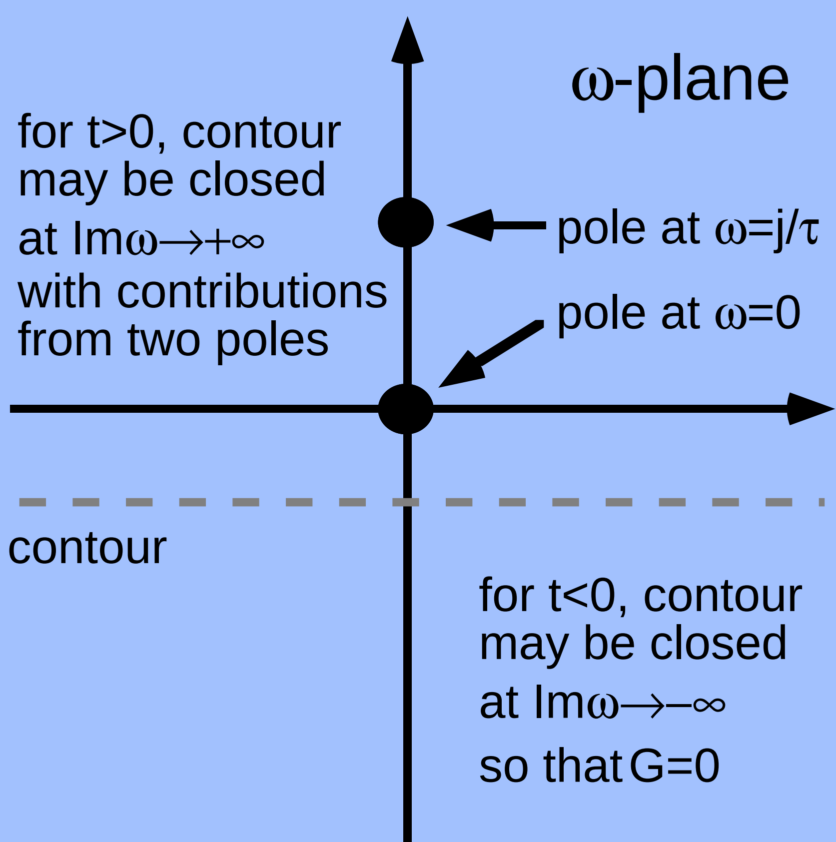
$$\chi_e(\omega) = \int_{-\infty}^{+\infty} dt \, e^{-j\omega t} G(t)$$

Example...unmagnetized plasma...

$$G(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{j\omega t}\chi_e(\omega)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{j\omega t}\frac{\omega_p^2\tau}{j\omega(1+j\omega\tau)}$$

$$= \omega_p^2\tau\left(1-e^{-t/\tau}\right)H(t)$$



Susceptibility in the High Frequency Limit

when the Green's function is analytic near t>0,

$$\chi_e(\omega) = \int_{-\infty}^{+\infty} dt\, e^{-j\omega t} G(t)$$

$$= \int_{-\infty}^{+\infty} dt\, e^{-j\omega t} H(t) \sum_{n=0}^{\infty} \frac{G^{(n)}(0)}{n!} t^n$$

$$= \sum_{n=0}^{\infty} \frac{G^{(n)}(0)}{n!} \left(j \frac{\partial}{\partial \omega} \right)^n \int_0^{+\infty} dt\, e^{-j\omega t}$$

$$= \sum_{n=0}^{\infty} \frac{G^{(n)}(0)}{n!} \left(j \frac{\partial}{\partial \omega} \right)^n \frac{1}{j\omega}$$

$$= -j \frac{G(0)}{\omega} - \frac{G^{(1)}(0)}{\omega^2} + j \frac{G^{(2)}(0)}{\omega^3} \ldots$$

Example: unmagnetized plasma

$$\chi_e(\omega) = -\frac{\omega_p^2}{\omega^2} - j \frac{\omega_p^2}{\omega^3 \tau} + \ldots$$

Kramers-Kronig Relations

Since G is causal χ_e must be analytic in the Imω<0 half-plane, so that

$$\chi_e(\omega) = \frac{1}{2\pi j}\oint \frac{\chi_e(\omega')}{\omega' - \omega}$$

for points ω,ω' and contour in the lower half-plane. Let the contour lie just below the real axis and use

$$\lim_{\varepsilon\to 0} \frac{1}{\omega' - \omega - j\varepsilon} = P\left(\frac{1}{\omega' - \omega}\right) + \pi j \delta(\omega' - \omega)$$

then

$$\chi_e(\omega) = \frac{1}{\pi j} P \int_{-\infty}^{+\infty} \frac{\chi_e(\omega')}{\omega' - \omega}$$

$$\Re\chi_e(\omega) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\Im\chi_e(\omega')}{\omega' - \omega}$$

$$\Im\chi_e(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\Re\chi_e(\omega')}{\omega' - \omega}$$

relates dispersion & absorption

Scalar & Vector Potentials

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi$$

Maxwell's Equations...

$$\vec{\nabla} \bullet \vec{B} = \vec{\nabla} \bullet \vec{\nabla} \times \vec{A} = 0$$ ✔

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left\{ -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi \right\} = -\frac{\partial}{\partial t}\vec{\nabla} \times \vec{A} = -\frac{\partial \vec{B}}{\partial t}$$ ✔

$$\vec{\nabla} \times \mu\vec{H} = \vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \boxed{\nabla\left(\nabla \bullet \vec{A}\right) - \nabla^2 \vec{A}}$$

$$= \mu\vec{\nabla} \times \vec{H} = \mu\frac{\partial \varepsilon\vec{E}}{\partial t} + \mu\vec{J}$$

$$= \boxed{\mu\varepsilon\frac{\partial}{\partial t}\left(-\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi\right) + \mu\vec{J}}$$

$$\vec{\nabla} \bullet \vec{D} = \vec{\nabla} \bullet \varepsilon\vec{E} = \boxed{\varepsilon\vec{\nabla} \bullet \left(-\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi\right) = \rho}$$

Gauge Invariance $\vec{A} \to \vec{A} - \vec{\nabla}\psi$

$$\varphi \to \varphi + \frac{\partial \psi}{\partial t}$$

leaves E,B unchanged

Lorentz Gauge $\nabla \bullet \vec{A} + \mu\varepsilon \frac{\partial \varphi}{\partial t} = 0$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} \quad \nabla^2 \varphi - \mu\varepsilon \frac{\partial^2 \varphi}{\partial t^2} = -\rho / \varepsilon$$

evidently the characteristic speed of propagation in the medium is $v = (\mu\varepsilon)^{-1/2}$

Coulomb Gauge $\vec{\nabla} \bullet \vec{A} = 0$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} + \mu\varepsilon \vec{\nabla} \frac{\partial \varphi}{\partial t}$$

$$\nabla^2 \varphi = -\rho / \varepsilon$$

related to the Lorentz Gauge potentials via

$$\nabla^2 \psi = -\mu\varepsilon \left(\frac{\partial \varphi}{\partial t} \right)_{Lorentz\ Gauge}$$

Hertzian Potentials

in a homogeneous, isotropic source-free region...

$$\vec{\nabla} \bullet \tilde{E} = \vec{\nabla} \bullet \tilde{H} = 0$$

Magnetic Hertzian potential $\tilde{E} = -j\omega\mu\vec{\nabla} \times \Pi_m$

$$\nabla \times \tilde{H} = j\omega\varepsilon\tilde{E} = k_0^2\vec{\nabla} \times \Pi_m \text{ with } k_0^2 = \mu\varepsilon\omega^2$$

$$\Rightarrow \tilde{H} = k_0^2\Pi_m + \vec{\nabla}\psi$$

$$\nabla \times \tilde{E} = -j\omega\mu\left(\vec{\nabla}\vec{\nabla} \bullet \Pi_m - \nabla^2\Pi_m\right)$$

$$= -j\omega\mu\tilde{H} = -j\omega\mu\left(k_0^2\Pi_m + \vec{\nabla}\psi\right)$$

choice of gauge $\psi = \vec{\nabla} \bullet \Pi_m \Rightarrow \nabla^2\psi + k_0^2\psi = 0$

$$\nabla^2\Pi_m + k_0^2\Pi_m = 0 \qquad \tilde{H} = \vec{\nabla}\vec{\nabla} \bullet \Pi_m + k_0^2\Pi_m$$

Electric Hertzian potential $\tilde{H} = j\omega\varepsilon\vec{\nabla} \times \Pi_e$

$$\nabla^2\Pi_e + k_0^2\Pi_e = 0 \qquad \tilde{E} = \vec{\nabla}\vec{\nabla} \bullet \Pi_e + k_0^2\Pi_e$$

Energy Conservation

$$-\vec{J}\bullet\vec{E} = \textit{rate of work done on fields}$$

$$= \left\{\frac{\partial\vec{D}}{\partial t} - \vec{\nabla}\times\vec{H}\right\}\bullet\vec{E}$$

$$= \frac{\partial\vec{D}}{\partial t}\bullet\vec{E}\underbrace{-\vec{\nabla}\times\vec{H}\bullet\vec{E} + \vec{\nabla}\times\vec{E}\bullet\vec{H}}_{\vec{\nabla}\bullet\left(\vec{E}\times\vec{H}\right)} - \vec{\nabla}\times\vec{E}\bullet\vec{H}$$

$$= \frac{\partial\vec{D}}{\partial t}\bullet\vec{E} + \vec{\nabla}\bullet\left(\vec{E}\times\vec{H}\right) + \frac{\partial\vec{B}}{\partial t}\bullet\vec{H}$$

$$\vec{S} = \vec{E}\times\vec{H} = \textit{Poynting Flux}$$

In a linear medium, with ε and μ ***independent of frequency*****:**

$$-\vec{J}\bullet\vec{E} = \vec{\nabla}\bullet\vec{S} + \frac{\partial u}{\partial t}$$

where

$$u = \vec{E}\bullet\vec{D} + \vec{B}\bullet\vec{H} = \textit{field energy density}$$

When ε and μ are *not independent of frequency* we should work in the frequency domain...

$$\vec{E} = \Re\left(\tilde{E}e^{j\omega t}\right) = \frac{1}{2}\left(\tilde{E}e^{j\omega t} + \tilde{E}^{*}e^{-j\omega t}\right)$$

and similarly for J, H, etc...let us compute averages over the rapid rf oscillation...can show that

$$\overline{S} = \frac{1}{2}\Re\left(\tilde{E} \times \tilde{H}^{*}\right)$$

somewhat more challenging is the calculation of the rate of change of field energy density

$$r = \frac{\partial \vec{D}}{\partial t} \bullet \vec{E} + \frac{\partial \vec{B}}{\partial t} \bullet \vec{H}$$

Questions arise...is this integrable?

$$\overline{r} = \frac{\partial \overline{u}}{\partial t} = ?$$

and if so, what is the
average stored energy density $\overline{u}$?

To address this problem we first compute $\dfrac{\partial \vec{D}}{\partial t} \bullet \vec{E}$

take $\vec{E}(t) = \Re\left\{\tilde{E}(\omega,t)e^{j\omega t}\right\} = \Re\left\{\left(\tilde{E}_0(\omega) + \tilde{E}_1(\omega)t\right)e^{j\omega t}\right\}$

$\vec{D}(t) = \varepsilon_0 \vec{E}(t) + \Re\left\{\tilde{P}(\omega,t)e^{j\omega t}\right\}$

and compute P...

$$\tilde{P}(\omega,t)e^{j\omega t} = \varepsilon_0 \int_{-\infty}^{+\infty} dt' G(t-t')\left(\tilde{E}_0(\omega) + \tilde{E}_1(\omega)t'\right)e^{j\omega t'}$$

$$= \varepsilon_0 \tilde{E}_0(\omega) \int_{-\infty}^{+\infty} dt' G(t-t') e^{j\omega t'} + \varepsilon_0 \tilde{E}_1(\omega) \frac{\partial}{j\partial\omega} \int_{-\infty}^{+\infty} dt' G(t-t') e^{j\omega t'}$$

$$= \varepsilon_0 \tilde{E}_0(\omega)\chi_e(\omega)e^{j\omega t} + \varepsilon_0 \tilde{E}_1(\omega)\frac{\partial}{j\partial\omega}\chi_e(\omega)e^{j\omega t}$$

or

$$\tilde{P}(\omega,t) = \varepsilon_0 \tilde{E}_0(\omega)\chi_e(\omega) + \varepsilon_0 \tilde{E}_1(\omega)\left(\chi_e t - j\frac{\partial \chi_e}{\partial\omega}\right)$$

$$= \varepsilon_0 \chi_e(\omega)\tilde{E}(\omega,t) - \varepsilon_0 j \frac{\partial \chi_e}{\partial \omega}\tilde{E}_1(\omega)$$

then

$$\frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \varepsilon_0 \Re\left\{ j\omega \tilde{P}(\omega,t)e^{j\omega t} + e^{j\omega t}\frac{\partial \tilde{P}}{\partial t}\right\}$$

$$= \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \varepsilon_0 \Re\left\{ j\omega\left(\chi_e(\omega)\tilde{E}(\omega,t) - j\frac{\partial \chi_e}{\partial\omega}\tilde{E}_1(\omega)\right)e^{j\omega t} + e^{j\omega t}\chi_e(\omega)\frac{\partial \tilde{E}(\omega,t)}{\partial t}\right\}$$

$$= \Re\left\{ e^{j\omega t}\left(\varepsilon \frac{\partial \tilde{E}(\omega,t)}{\partial t} + j\varepsilon\omega \tilde{E}(\omega,t) + \varepsilon_0 \tilde{E}_1(\omega)\omega \frac{\partial \chi_e}{\partial\omega}\right)\right\}$$

$$= \Re\left\{ e^{j\omega t}\left(\frac{\partial \tilde{E}(\omega,t)}{\partial t}\frac{\partial(\omega\varepsilon)}{\partial\omega} + j\varepsilon\omega\tilde{E}(\omega,t)\right)\right\}$$

finally

$$\overline{\frac{\partial \vec{D}}{\partial t} \bullet \vec{E}} = \frac{1}{2}\Re\left\{ \frac{\partial(\omega\varepsilon)}{\partial\omega}\frac{\partial \tilde{E}(\omega,t)}{\partial t} \bullet \tilde{E}^*(\omega,t) + j\varepsilon\omega\left|\tilde{E}(\omega,t)\right|^2\right\}$$

Finally

$$\overline{\frac{\partial \vec{D}}{\partial t} \bullet \vec{E}} = \frac{1}{4}\left\{ \frac{\partial(\omega\varepsilon_r)}{\partial\omega} \frac{\partial}{\partial t} \left|\tilde{E}(\omega,t)\right|^2 - \varepsilon_i \omega \left|\tilde{E}(\omega,t)\right|^2 - 2\frac{\partial(\omega\varepsilon_i)}{\partial\omega} \Im\left(\frac{\partial \tilde{E}(\omega,t)}{\partial t} \bullet \tilde{E}^*(\omega,t) \right) \right\}$$

so that, in the absence of losses,

$$\bar{r} = \overline{\frac{\partial \vec{D}}{\partial t} \bullet \vec{E} + \frac{\partial \vec{B}}{\partial t} \bullet \vec{H}} = \frac{\partial \bar{u}}{\partial t}$$

where the field energy density is

$$\boxed{\bar{u} = \frac{1}{4}\left\{ \frac{\partial(\omega\varepsilon)}{\partial\omega} \left|\tilde{E}(\omega,t)\right|^2 + \frac{\partial(\omega\mu)}{\partial\omega} \left|\tilde{H}(\omega,t)\right|^2 \right\}}$$

Energy conservation takes the form

$$-\overline{\vec{J} \bullet \vec{E}} = \nabla \bullet \bar{S} + \frac{\partial \bar{u}}{\partial t}$$

with time-averaged Poynting flux

$$\bar{S} = \frac{1}{2}\Re\left(\tilde{E} \times \tilde{H}^*\right)$$

③Skin Depth

Start from Maxwells Equations

$$\vec{\nabla} \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J} \approx \tilde{J} = \sigma \tilde{E}$$

$$\vec{\nabla} \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t} = -j\omega \tilde{B}$$

Fields ∝ exp(jωt)

$$\vec{\nabla} \bullet \tilde{E} = \vec{\nabla} \bullet \tilde{B} = 0$$

Reduce to an equation for H

$$\vec{\nabla} \times \left(\vec{\nabla} \times \tilde{H} \right) = \vec{\nabla} \left(\vec{\nabla} \bullet \tilde{H} \right) - \nabla^2 \tilde{H} = -\nabla^2 \tilde{H}$$

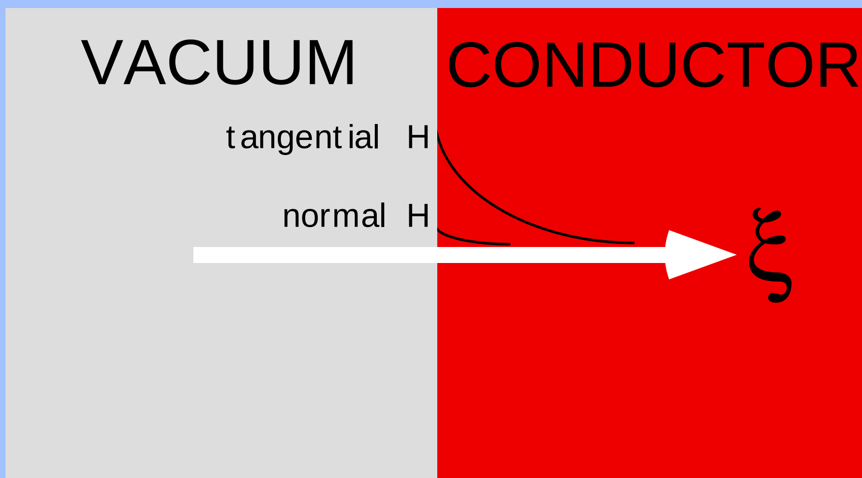
$$= \vec{\nabla} \times \left(\sigma \tilde{E} \right) = -j\omega\sigma \tilde{B} = -j\mu\omega\sigma \tilde{H}$$

$$\nabla^2 \tilde{H} - j\,\mathrm{sgn}\,\omega \frac{2}{\delta^2} \tilde{H} = 0$$

$$\delta = \sqrt{\frac{2}{\mu|\omega|\sigma}} = \text{"}skin - depth\text{"}$$

$$\approx 2\mu m \; at \; 1GHz \; in \; Cu$$

Let's solve for fields in conductor...



select coordinate along outward normal

$$\frac{d^2}{d\xi^2}\tilde{H}_t - j\,\mathrm{sgn}\,\omega\,\frac{2}{\delta^2}\tilde{H}_t = 0$$

$$\tilde{H}_t(\xi) = \tilde{H}_t(0)\exp\left\{-\frac{\xi}{\delta}(1+j\,\mathrm{sgn}\,\omega)\right\}$$

$$\tilde{E} = \frac{1}{\sigma}\vec{\nabla}\times\tilde{H} \approx \frac{1}{\sigma}\hat{n}\times\frac{\partial\tilde{H}}{\partial\xi} = -Z_s\hat{n}\times\tilde{H}_t$$

"impedance boundary condition"

$$Z_s = \frac{1+j\,\mathrm{sgn}\,\omega}{\sigma\delta} = R_s(1+j\,\mathrm{sgn}\,\omega)$$

$$R_s = \frac{1}{\sigma\delta} = \text{surface resistance}$$

$$\approx 8.3\mathrm{m}\Omega \textit{ at } 1\,GHz \textit{ in } Cu$$

Power per m^2 into the conductor...

$$\bar{S} \bullet \hat{n} = \frac{1}{2}\Re\left(\tilde{E} \times \tilde{H}^* \bullet \hat{n}\right)$$

$$= -\frac{1}{2}\Re\left(Z_s\left(\hat{n} \times \tilde{H}_t\right) \times \tilde{H}^* \bullet \hat{n}\right)$$

$$= \frac{1}{2}\Re\left(Z_s\left|\hat{n} \times \tilde{H}_t\right|^2\right)$$

$$= \frac{1}{2}R_s\left|n \times \tilde{H}\right|^2 = \frac{1}{2}R_s\left|\tilde{K}\right|^2$$

where in the last line we have made use of the result for surface current density,

$$\tilde{K} = \int_0^\infty \tilde{J} d\xi = -\hat{n} \times \tilde{H}(0)$$

④Orthogonal Modes

(Uniform Waveguide)

From Maxwell's Equations, with fields $\propto e^{j\omega t - jk_z z}$

$$\vec{\nabla} \bullet \tilde{E} = \vec{\nabla} \bullet \tilde{B} = 0$$

$$\vec{\nabla} \times \tilde{E} = -j\omega \tilde{B}$$

$$\vec{\nabla} \times \tilde{H} = j\omega \tilde{D}$$

we can see that E & H satisfy the wave equation,

$$\vec{\nabla} \times \left(\vec{\nabla} \times \tilde{H} \right) = -\nabla^2 \tilde{H}$$

$$= j\omega \vec{\nabla} \times \tilde{D} = j\omega\varepsilon \left(-j\omega \tilde{B} \right)$$

$$= \omega^2 \varepsilon\mu \tilde{H}$$

and similarly for E, so that

$$\left(\nabla_\perp^2 + k_c^2 \right) \tilde{H} = 0$$

$$\left(\nabla_\perp^2 + k_c^2 \right) \tilde{E} = 0$$

where $k_0^2 = \omega^2 \varepsilon\mu$ $\quad k_c^2 = k_0^2 - k_z^2$

The divergence conditions take the form

$$\tilde{E}_z = \frac{1}{jk_z}\nabla_\perp \bullet \tilde{E}_\perp \qquad \tilde{H}_z = \frac{1}{jk_z}\nabla_\perp \bullet \tilde{H}_\perp$$

so that the longitudinal field components may be determined from the transverse components. In addition,we may write the curl equations

$$\tilde{E} = \frac{1}{jk_0}\vec{\nabla}\times Z_0\tilde{H} \qquad Z_0\tilde{H} = -\frac{1}{jk_0}\vec{\nabla}\times\tilde{E} \qquad Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

to express the transverse components in terms of the longitudinal

$$-jk_0 Z_0\tilde{H}_\perp = -\hat{z}\times\vec{\nabla}_\perp\tilde{E}_z - jk_z\hat{z}\times\tilde{E}_\perp$$

$$jk_0\tilde{E}_\perp = -\hat{z}\times\vec{\nabla}_\perp Z_0\tilde{H}_z - jk_z\hat{z}\times Z_0\tilde{H}_\perp$$

or

$$\tilde{E}_\perp = -\frac{k_0}{jk_c^2}\left(\hat{z}\times\vec{\nabla}_\perp Z_0\tilde{H}_z - \frac{k_z}{k_0}\vec{\nabla}_\perp\tilde{E}_z\right)$$

$$Z_0\tilde{H}_\perp = -\frac{k_0}{jk_c^2}\left(-\hat{z}\times\vec{\nabla}_\perp\tilde{E}_z - \frac{k_z}{k_0}\vec{\nabla}_\perp Z_0\tilde{H}_z\right)$$

TE Modes E_z=0

transverse fields may be determined from H_z

$$Z_0\tilde{H}_\perp = \frac{k_z}{jk_c^2}\vec{\nabla}_\perp Z_0\tilde{H}_z$$

$$\tilde{E}_\perp = -\frac{k_0}{jk_c^2}\hat{z}\times\vec{\nabla}_\perp Z_0\tilde{H}_z = -\frac{k_0}{k_z}\hat{z}\times Z_0\tilde{H}_\perp$$

longitudinal field satisfies

$$\left(\nabla_\perp^2 + k_c^2\right)\tilde{H}_z = 0$$ with boundary condition

$$0 = \hat{n}\bullet Z_0\tilde{H}_\perp = \frac{k_z}{jk_c^2}\hat{n}\bullet\vec{\nabla}_\perp Z_0\tilde{H}_z \quad i.e.\ \frac{\partial\tilde{H}_z}{\partial n} = 0$$

TM Modes H_z=0

transverse fields may be determined from E_z

$$\tilde{E}_\perp = \frac{k_z}{jk_c^2}\vec{\nabla}_\perp\tilde{E}_z$$

$$Z_0\tilde{H}_\perp = \frac{k_0}{jk_c^2}\hat{z}\times\vec{\nabla}_\perp\tilde{E}_z = \frac{k_0}{k_z}\hat{z}\times\tilde{E}_\perp$$

longitudinal field satisfies

$$\left(\nabla_\perp^2 + k_c^2\right)\tilde{E}_z = 0$$ with boundary condition $\tilde{E}_z = 0$

Cut-Off & Characteristic Impedance Z_c

Boundary conditions restrict the permissible values of cut-off wavenumber k_c to a discrete set. Each mode has a corresponding minimum wavelength λ_c beyond which it is “cut-off” in the waveguide.

$$\lambda_c = 2\pi / k_c$$

The guide wavelength is

$$\lambda_g = 2\pi / k_z = \lambda_0 / \sqrt{1 - \lambda_0^2 / \lambda_c^2}$$

$$\lambda_0 = 2\pi / k_0$$

In general for a given mode we have

$$Z_c \tilde{H}_\perp = \hat{z} \times \tilde{E}_\perp$$

where

$$Z_c = \begin{cases} Z_0 \dfrac{k_0}{k_z} = Z_0 \dfrac{\lambda_g}{\lambda_0} & TE \text{ mode} \\[2ex] Z_0 \dfrac{k_z}{k_0} = Z_0 \dfrac{\lambda_0}{\lambda_g} & TM \text{ mode} \end{cases}$$

Modal Decomposition

A general solution for a given geometry may be represented as a sum over modes

$$\tilde{E}_t = \sum_a E_{\perp a}(\vec{r}_\perp) V_a(z,\omega)$$

$$\tilde{H}_t = \sum_a H_{\perp a}(\vec{r}_\perp) I_a(z,\omega) Z_{ca}(\omega)$$

where $Z_{ca} H_{\perp a} = \hat{z} \times E_{\perp a}$

and we adopt the normalization

$$\int d^2 r_\perp E_{\perp a}(\vec{r}_\perp) \bullet E_{\perp a}(\vec{r}_\perp) = 1$$

where the integral is over the waveguide cross-section.
We choose the sign of Z for positive k_z.
The coefficients V,I take the forms

$$V_a(z,\omega) = V_a^+ e^{-jk_{za}z} + V_a^- e^{jk_{za}z}$$

$$Z_{ca} I_a(z,\omega) = V_a^+ e^{-jk_{za}z} - V_a^- e^{jk_{za}z}$$

Relation between Power, V & I

One can also show, for non-degenerate modes, that

$$\int d^2 r_\perp E_{\perp a}(\vec{r}_\perp) \bullet E_{\perp b}(\vec{r}_\perp) = \delta_{ab}$$

$$Z_{ca} Z_{cb} \int d^2 r_\perp H_{\perp a}(\vec{r}_\perp) \bullet H_{\perp b}(\vec{r}_\perp) = \delta_{ab}$$

$$\int d^2 r_\perp \hat{z} \bullet \left(E_{\perp a} \times H_{\perp b} \right) = \delta_{ab} Z_{ca}^{-1}$$

This requires Green's Theorem,

$$\int \left(\psi_1 \nabla^2 \psi_2 + \vec{\nabla} \psi_1 \bullet \vec{\nabla} \psi_2 \right) d^2 r = \oint \psi_1 \frac{\partial \psi_2}{\partial n} dl$$

and the eigenvalue equations for H_z & E_z

As a result, one may express the power flow in the waveguide, in terms of V & I according to

$$P = \int d^2 r \frac{1}{2} \Re\left(\tilde{E}_t \times \tilde{H}_t^* \right)$$

$$= \sum_a \frac{1}{2} \Re\left(V_a I_a^* \right)$$

Meaning of V,I

Given the orthogonality relations, one can determine *V,I* from the transverse fields at a point *z*

$$V_a(z,\omega) = \int d^2 r_\perp \tilde{E}_t(\vec{r}_\perp, z) \bullet E_{\perp a}(\vec{r}_\perp)$$

$$I_a(z,\omega) = Z_{ca} \int d^2 r_\perp \tilde{H}_t(\vec{r}_\perp, z) \bullet H_{\perp a}(\vec{r}_\perp)$$

and this is enough to determine the solution everywhere in the uniform guide, since this fixes the right & left-going amplitudes.

Given the uniqueness of V,I, their relation to power, and the units (volts, amperes) it is natural to refer to them as voltage & current.

It is important to keep in mind however that they appear as complex mode amplitudes,not work done on a charge or time rate of change of charge.

at the same time, for particular geometries and applications, V & I can often be related to these more conventional concepts

⑤Phase & Group Velocity

consider a narrow-band drive at z=0

$$V(t,0) = f(t)e^{j\omega_0 t}$$

$$\tilde{V}(\omega,0) = \int_{-\infty}^{\infty} \frac{dt}{\sqrt{2\pi}} f(t)e^{j(\omega_0-\omega)t}$$

$$f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{2\pi}} \tilde{V}(\omega,0)e^{j(\omega-\omega_0)t}$$

compute the voltage down-range

$$V(t,z) = \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{2\pi}} \tilde{V}(\omega,0)e^{j\omega t - jk_z z}$$

$$\approx \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{2\pi}} \tilde{V}(\omega,0)e^{j(\omega-\omega_0)t - jk_z(\omega_0)z - j\frac{dk_z}{d\omega}(\omega_0)(\omega-\omega_0)z}$$

$$= e^{j\omega_0 t - jk_z(\omega_0)z} \int_{-\infty}^{\infty} \frac{d\omega}{\sqrt{2\pi}} \tilde{V}(\omega,0)e^{j(\omega-\omega_0)\left(t - \frac{dk_z}{d\omega}z\right)}$$

$$= e^{j\omega_0 t - jk_z(\omega_0)z} f\left(t - \frac{dk_z}{d\omega}z\right)$$

can see that constant phase-fronts travel at

$$v_\varphi = \frac{\omega}{k_z} = phase - velocity$$

while the modulation *f* travels at

$$v_g = \frac{d\omega}{dk_z} = group - velocity$$

For a mode in uniform guide,

$$v_\varphi = \frac{\omega}{\sqrt{k_0^2 - k_c^2}} = \frac{c}{\sqrt{\mu\varepsilon}} \frac{1}{\sqrt{1 - k_c^2 / k_0^2}} > \frac{c}{\sqrt{\mu\varepsilon}}$$

$$v_g = \frac{c}{\sqrt{\mu\varepsilon}} \frac{k_z}{k_0} = \frac{c}{\sqrt{\mu\varepsilon}} \sqrt{1 - k_c^2 / k_0^2} < \frac{c}{\sqrt{\mu\varepsilon}}$$

Summary

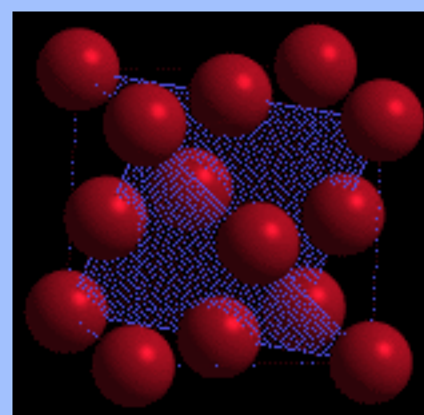
- ✓ Lorentz Force Law
- ✓ Maxwell's Equations
- ✓ Skin Depth δ
- ✓ Modes in a Waveguide
- ✓ Phase Velocity v_ϕ & Group Velocity v_g

Acknowledgements

Encyclopedia Brittanica
http://www.eb.com

100 Years of Radio
http://www.alpcom.it/hamradio/

Dept. Materials Science, MIT
http://tantalum.mit.edu/



Varian Associates
http://www.varian.com/

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http://www.neuronet.pitt.edu/~bogdan/tesla/

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- Dept. of Physics and Astronomy, Michigan State University
- University of Guelph

Vocabulary

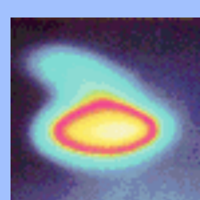
- Electric Field E
- Magnetic Field H
- Energy U, Power P
- Frequency f, or Angular Frequency ω
- Conductivity, σ or Resistivity ρ
- Phase Velocity v_ϕ, Group Velocity v_g
- Distributed vs Lumped Elements
- E & H Fields, Charged Particles
- Behavior of Fields in Media

For More Information...

http://beam.slac.stanford.edu/

W3 Virtual Library of Beam Physics

links to all accelerator labs on the planet
...conferences...schools...news...jobs...
companies...vendors...databases...
researchers...preprints...



Recommended Reading

- RF Engineering for Accelerators, **Turner**, (CERN 92-03)
  An introduction to RF as applied to accelerators.
- Microwave Electronics, **Slater**
  The classic introduction to microwave electronics.

Related Texts

- Classical Electrodynamics, **Jackson**
  A graduate level electrodynamics text.
- Field Theory of Guided Waves, **Collin**
  A modern introduction to microwave electronics.
- Foundations for Microwave Engineering, **Collin**
  An overview of the elements of microwave electronics.
- Microwave Measurements, **Ginzton**
  An introduction to practical microwave work.
- Principles of Microwave Circuits, **Montgomery, Dicke, Purcell**
  An introduction to common network elements.
- Waveguide Handbook, **Marcuvitz**
  Analysis of the circuit parameters for network elements.

SI Units

Elemental units			
Length	metre	—	m
Mass	kilogram	—	kg
Time	second	—	s
Electric current	ampere	—	A
Temperature	kelvin	—	K
Luminous intensity	candela	—	cd
Plane angle	radian	—	rad
Solid angle	steradian	—	sr
Derived units			
Acceleration	metre/second squared	m/s^2	
Area	square metre	m^2	
Capacitance	farad	$A \cdot s/V$	F
Charge	coulomb	$A \cdot s$	C
Density	kilogram/cubic metre	kg/m^3	
Electric field strength	volt/metre	V/m	
Energy	joule	$N \cdot m$	J
Force	newton	$kg \cdot m/s^2$	N
Frequency	hertz	s^{-1}	Hz
Illumination	lux	lm/m^2	lx
Inductance	henry	$V \cdot s/A$	H
Kinematic viscosity	square metre/second	m^2/s	
Luminance	candela/square metre	cd/m^2	
Luminous flux	lumen	$cd \cdot sr$	lm
Magnetic field strength	ampere/metre	A/m	
Magnetic flux	weber	$V \cdot s$	Wb
Magnetic flux density	tesla	Wb/m^2	T
Power	watt	J/s	W
Pressure	pascal (newton/square metre)	N/m^2	Pa
Resistance	ohm	V/A	Ω
Stress	pascal (newton/square metre)	N/m^2	Pa
Velocity	metre/second	m/s	
Viscosity	newton-second/square metre	$N \cdot s/m^2$	
Voltage	volt	W/A	V
Volume	cubic metre	m^3	

Handy Numbers

$$10\log_{10}(1/2) \approx -3dB$$

$$10\log_{10}(1/3) \approx -5dB$$

$$20\log_{10}(0.99) \approx -0.1dB$$

$$1mW \equiv 0dBm$$

$$\left.\begin{array}{l} \kappa \approx 401\, W/{}^\circ K\ m \\ C \approx 385 J/{}^\circ K\ kG \\ \alpha \approx 1.7\times 10^{-5}/{}^\circ K \\ \rho \approx 1.56\times 10^{-8}\Omega - m \end{array}\right\}$$

Copper

Joint Accelerator School

RF Engineering for Particle Accelerators

To

- ⇨Understand
- ⇨Invent
- ⇨Design
- ⇨Build
- ⇨Operate

RF Systems

Outline for Morning Lectures

1 Microwave Electronics 1 -**Maxwell's Equations & Modes in a Guide**
2 M.E. 2 - **Equivalent Circuit Representation for Modes in a Guide**
3 M.E. 3 - **Modes of a Cavity**
4 Cavity Design
5 M.E. 4 - **Cavity with a Port & External Q**
6 M.E. 5 - **Microwave Networks**
7 M.E. 6 - **Slater's Perturbation Theorem**
8 **Superconducting Cavities**
9 **Beam-Cavity Interaction, Beam-Loading**
10 **Klystron 1 - Space-Charge Limited Flow, Guns**
11 Structure 1-Standing-Wave
12 SLED Pulse Compression
13 **Wakefields 1 - Fundamentals**
14 **Klystron 2 - Bunching, Space-Charge**
15 Structure 2-Travelling Wave
16 Ferrite Loaded Cavity 1
17 **Wakefields 2 - in SW & TW Structures**
18 **Klystron 3 - Simulation**
19 **Structure 3-Fabrication and Conditioning**
20 **Structure 4 -Surface fields, Breakdown, Multipactor, Dark Current**
20 **Wakefields 3 - Other Sources of Impedance**
21 Other RF Sources
22 **High Gradients in Superconducting Cavities**
23 Modulators
24 Windows & High-Power Transmission
25 Ferrite Loaded Cavity 2
26 **Design for System Stability - Heavy Beam Loading**

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An Introduction to the
Equivalent Circuit
Starting from Maxwell's Equations

in six lectures

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- ❒ **Slater's Perturbation Theorem**

Why are you here?

What do you want?

Understanding of fundamentals...

- Distributed vs. Lumped Elements
- The Meaning of Current & Voltage
- Transit Time & Retardation

Familiarity with the language...

- V, I, Z, δ, R_s, Q_w, Q_e, R/Q...
- π mode, Travelling Wave,...
- Tee, Load, Circulator, 3dB Coupler...

Ability to Solve Problems...

- How to design, build & tune my cavity?
- What is the right power source to use?
- My system isn't working, what to do?