

Physics of Free Electron Lasers - Homework 2 - ELF Experiment

The Electron Laser Facility experiment was a single-pass microwave FEL amplifier employing an induction linac and reversed coil pulsed wiggler.

(1) Referring to the attached article describing results from ELF, determine the following quantities

- microwave signal angular frequency as ω/c in cm^{-1}
- waveguide dimensions $a \times b$ in cm $a > b$, and the mode area $\Sigma = ab/2$.
- wavenumber k_z in cm^{-1} . Recall that for the TE_{01} mode,

$$k_z = \left\{ \left(\frac{\omega}{c} \right)^2 - \left(\frac{\pi}{b} \right)^2 \right\}^{1/2}$$

- wiggler wavenumber k_w in cm^{-1}
- beam energy, Lorentz factor γ , minimum beam pulse length τ , peak beam current at the FEL exit in A, charge transmitted through the FEL in C, and energy in the pulse
- using the waveguide-FEL resonance relation,

$$k_w + k_z - \frac{\omega}{c\beta} = 0$$

determine the wiggler parameter a_w (peak not rms) for resonance, and the corresponding wiggler field B_w in kG. Recall, the average axial speed is βc where,

$$\beta = 1 - \frac{1}{2\gamma^2} \left(1 + \frac{a_w^2}{2} \right)$$

•For the maximum rf power reported, compute the corresponding dimensionless signal amplitude a_s . Recall,

$$P_{rf} = \frac{P_0}{8\pi} \Sigma \left(\frac{\omega}{c} k_z \right) a_s^2$$

Repeat this for the input rf power. Also compute the efficiency of conversion of beam power to rf by taking the ratio of rf power to transmitted beam power.

(2) Next let's simulate the ELF experiment, using the parameters determined above as inputs. The FEL equations for a single wave-guide mode are

$$\frac{d\theta}{dz} = k_w + k_z - \frac{\omega}{c\beta},$$

$$\frac{d\gamma}{dz} = -\frac{1}{2} \frac{\omega}{c} \frac{a_w}{\gamma\beta} \text{Im}\{a \exp(i\theta)\},$$

$$\frac{da}{dz} = i \frac{2\pi a_w}{k_z \Sigma} \left(\frac{I}{I_0} \right) \left\langle \frac{\exp(-i\theta)}{\gamma\beta} \right\rangle.$$

(For future reference, we are neglecting the following: a “Bessel function factor” often denoted [JJ], space-charge, slippage, excitation of other modes, and a non-resonant term in the beam current density.) Recall that a is complex $a = a_r + ia_i$, and $a_s = (a_r^2 + a_i^2)^{1/2}$.

Use initial conditions (at $z=0$) for the beam corresponding to the loading routine you developed for homework 1, with $\pm 1\%$ energy spread. Assume that at $z=0$, $a = a_s$ (*i.e.* $a_i = 0$) with a_s as computed above for the input rf power. Compute and plot rf power as a function of z for $0 < z < 2\text{m}$, on resonance, and the error in power,

$$\Delta P_{tot} = \frac{mc^2}{e} I \langle \gamma(z) - \gamma(0) \rangle + P_{rf}(z) - P_{rf}(0)$$

This should start out 0 and remain a small fraction of the total rf power. Any integration technique or software tool that you prefer can be used for this integration.

2nd-Order Runge-Kutta

One reliable technique for integrating this is the 2nd-Order Runge-Kutta method. For the case of N macroparticles, we may express our equations in the form

$$\frac{dq_i}{dz} = p_i(\vec{q}),$$

where $q_{2i-1} = \theta_i$ and $q_{2i} = \gamma_i$, for $i=1,2,\dots,N$, and $q_{2N+1} = a_r$ and $q_{2N+2} = a_i$.

Given the value of \vec{q} , at z , $\vec{q}(z)$, the value at $z+dz$, $\vec{q}(z+dz)$, can be computed according to

$$d\vec{q}^{(1)} = \vec{p}(\vec{q})dz$$

$$d\vec{q}^{(2)} = \vec{p}\left(\vec{q} + \frac{1}{2}d\vec{q}^{(1)}\right)dz$$

$$\vec{q}(z+dz) = \vec{q}(z) + d\vec{q}^{(2)}$$

A routine for accomplishing this can be found at the class web site. For this I used $N=128$ particles, and $dz=2\text{cm}$.

Physics of Free Electron Lasers - Homework 2 - Solutions - ELF Experiment

(1) Input parameters for the simulation should be

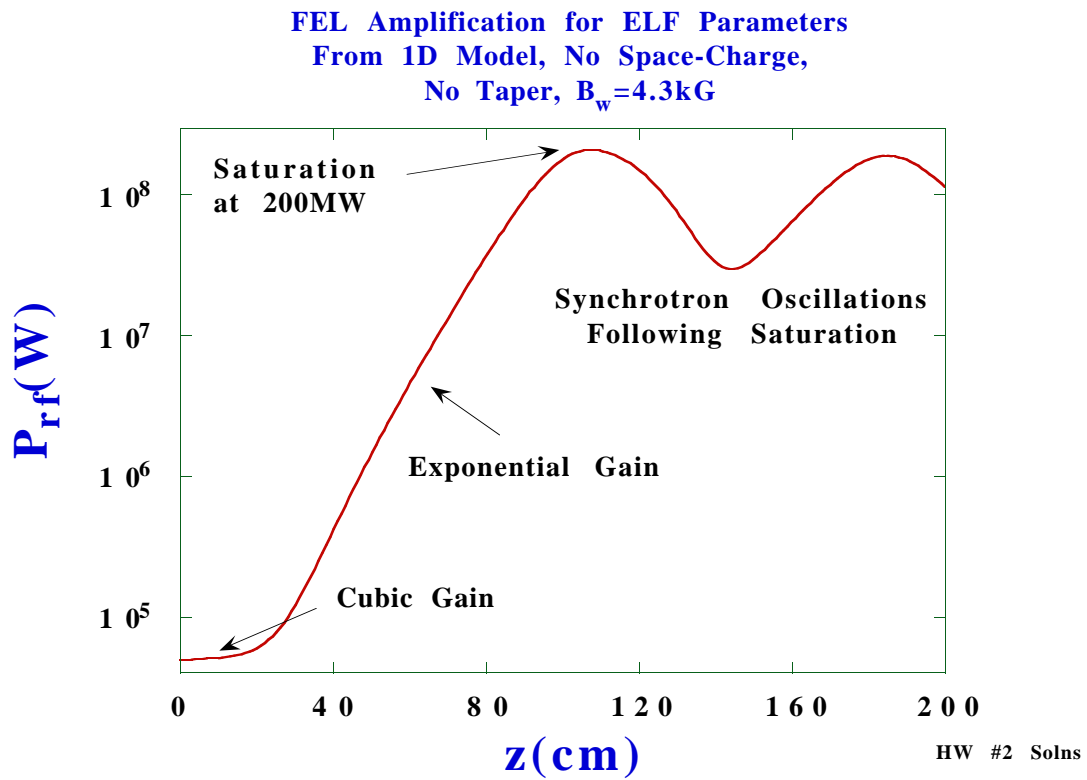
- microwave signal angular frequency: $\omega/c = 7.252\text{cm}^{-1}$
- waveguide dimensions are $a \times b = 10 \text{ cm} \times 3 \text{ cm}$, and the mode area $\Sigma = 15\text{cm}^2$
- wavenumber $k_z = 7.176\text{cm}^{-1}$
- wiggler wavenumber $k_w = 0.6411\text{cm}^{-1}$

Note that β_z , the average axial speed normalized by c is, for resonance,

$$\beta_z = \frac{\omega/c}{k_z + k_w} = \frac{7.252}{7.176 + 0.6411} = 0.93.$$

- The beam voltage for this experiment was **3.5MV**, Lorentz factor $\gamma = 7.849$, beam pulse length $\tau = 10\text{ns}$, current transmitted through the wiggler **I=850A**, beam charge **Q=8.5 μ C**, total energy in beam **E=QV=30J**. The rf energy input into the device is $50\text{kW} \times 10\text{ns} = 0.5\text{mJ}$ in a 10ns pulse, and $\sim 30\text{mJ}$ in a $0.55 \mu\text{s}$ magnetron pulse length. Thus the net rf energy input is minute compared to the energy in the beam. Note that for this γ , the beam speed in free space is $0.99c$, thus to achieve resonance, the wiggler must “slow” the beam down considerably.
- The wiggler parameter a_w (peak not rms) for resonance, and the corresponding wiggler field B_w in kG are **$a_w = 3.98$, $B_w = 4.3\text{kG}$** .
- For the maximum rf power reported, 1 GW, the corresponding dimensionless signal amplitude **$a_s = 6.1 \times 10^{-2}$** at the output, **$a_s = 4.3 \times 10^{-4}$** at the input. Beam power is **$0.85\text{kA} \times 3.5\text{MV} = 2.98\text{GW}$** , and 1GW rf output corresponds to **34%** efficiency. (This method of computing efficiency neglects beam loss en route from the injector, and omits efficiency from the wallplug to the beam).

(2) The result of a 2nd-order Runge Kutta integration with $N=128$ particles, and $dz=2\text{cm}$ is depicted below.



The error in power, starts out 0 and remains small compared to the rf power.

Numerical Error in Total Power
(Linear Scale)

