



ARDB Tech Note No. 63

Scalings for the Ring-Linac

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In this note we set down the basic design scalings for the ring linac, absent tuning errors. Breakdown threshold is also discussed. A best estimate is set down for the timeline for the preparation for this experiment.

Scalings for a Tuned Structure and a VSWR=1 Ring	2
Breakdown	6
Additional Work	7
Timeline	8

Scalings for a Tuned Structure and a VSWR=1 Ring

The picture is that of Fig. 1, consisting of a constant impedance structure of length L to be determined, with the output coupled, through a phase-shift, ϕ , and an attenuation, A , back to the structure input.

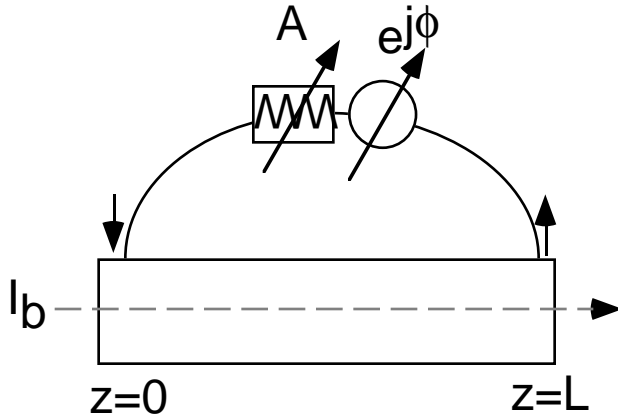


Fig.1 Idealized picture of the constant impedance structure embedded in a resonant ring.

The gradient induced by the beam in steady-state is

$$G_b(z) = rl_b(1 - e^{-\alpha z}),$$

with r the shunt impedance per unit length, and α the attenuation per unit length. The gradient produced by an input signal may be represented as

$$G_{in}(z) = G_{in}(0)e^{-\alpha z}.$$

The gradient experienced by the beam is the sum of these two,

$$G(z) = G_b(z) + G_{in}(z) = G_{in}(0)e^{-\alpha z} + rl_b(1 - e^{-\alpha z}).$$

The microwave power flowing through the structure at a point z is

$$P(z) = \frac{|G(z)|^2}{2\alpha r},$$

and thus the gradient at $z=0$, may be expressed in terms of the gradient at the structure exit according to

$$G(0) = \tilde{A}G(L),$$

where $\tilde{A} = Ae^{j\phi}$. This permits one to solve for

$$\frac{G(0)}{rl_b} = \tilde{A} \left(\frac{1 - e^{-\tau}}{1 - \tilde{A}e^{-\tau}} \right),$$

where $\tau = \alpha L$ is the attenuation parameter, with L the structure length. At any point z in the structure, one then has,

$$\frac{G(z)}{rl_b} = 1 - \left(\frac{1 - \tilde{A}}{1 - \tilde{A}e^{-\tau}} \right) e^{-\alpha z}.$$

Note that

$$\frac{G(L)}{rl_b} = \frac{1 - e^{-\tau}}{1 - \tilde{A}e^{-\tau}}.$$

Evidently the gradient increases toward the end of the structure, except in the case $|\tilde{A}| = 1$. Integrating this gradient over the structure, we obtain the net voltage,

$$\frac{V}{Rl_b} = \frac{1}{Rl_b} \int_0^L dz G(z) = 1 - \left(\frac{1 - e^{-\tau}}{\tau} \right) \left(\frac{1 - \tilde{A}}{1 - \tilde{A}e^{-\tau}} \right).$$

Here $R = rL$ is the shunt impedance of the structure. We may quantify the range of voltage variation achievable with the phase-knob, ϕ , by considering $\tilde{A} = +A$ and $\tilde{A} = -A$. After a bit of algebra one finds,

$$\frac{V_+}{Rl_b} = 1 - \left(\frac{1 - e^{-\tau}}{\tau} \right) \left(\frac{1 - A}{1 - Ae^{-\tau}} \right),$$

$$\frac{V_-}{Rl_b} = 1 - \left(\frac{1 - e^{-\tau}}{\tau} \right) \left(\frac{1 + A}{1 + Ae^{-\tau}} \right),$$

and the range of phase-knob adjustable voltage variation is

$$\frac{\Delta V}{Rl_b} = \frac{V_+ - V_-}{Rl_b} = \frac{A}{1 - A^2 e^{-2\tau}} \frac{2}{\tau} (1 - e^{-\tau})^2.$$

For purposes of practical, worst-case estimates, we may take the shunt impedance of an idealized W-Band cell ($0.6\text{M}\Omega$) and derate this by a factor of 3, to obtain, for a 60-cell (6cm) structure $R\sim 12\text{M}\Omega$, and $r\sim 200\text{M}\Omega/\text{m}$. In terms of group velocity v_g and wall Q , Q_w , the attenuation constant is

$$\alpha = \frac{\omega}{2Q_w v_g}.$$

For a group velocity $v_g=0.09c$, and a wall Q of $Q_w\sim 2000$, the attenuation constant is $\alpha\sim 5.3\text{m}^{-1}$, or $\tau=0.32$. For $I_b\sim 0.5\text{A}$, $R/I_b\sim 6\text{MV}$, and this is the tuning range ΔV for $A=1$. For 1dB of loss in the recirculation network, one has $A\sim 0.89$, and $\Delta V\sim 4.3\text{MV}$ (72%). For 3dB loss, $A\sim 0.71$ and $\Delta V\sim 2.7\text{MV}$ (45%). The peak gradient achievable is just $r/I_b\sim 100\text{MV}/\text{m}$, corresponding to no attenuation, and perfect phasing. This corresponds to 4.7MW flowing to the structure output. For 1dB of loss, and perfect phasing, these figures are: 77MV/m and 2.8MW. For 3dB of loss: 56MV/m and 1.5MW. Without recirculation: 27MV/m and 0.3MW.

These scalings are summarized in a spreadsheet (see WWW), with an example of the output depicted below.

Scallings for a Tuned Structure and a VSWR=1 Ring w/Simple Loss		4/17/07	
Inputs			
betag	0.00	group velocity /c	
lambda	0.0033	m, rf wavelength	3.1+150265
ncell	60	# of cells	200700000 m/s
qwall	2000	wall Q of cell	
theta	120	transit angle in degrees	
ringatten	3	dB	
rshuntl	2.00E+08	Ohm/m, shunt impedance per unit length	For 1 dB Attenuation
beamcurr	0.5	Ampere, beam current	Optimum Difference Voltage at 136 cells Difference = 0.805 x Vmax = 12MV -6.90 dB in structure
Derived			For 3 dB Attenuation
omega	5.708E+11	rad/sec	Optimum Difference Voltage at 178 cells Difference = 0.607 x Vmax = 12MV
alpha	5.28887652	m ⁻¹	-0.58 dB in structure
length	0.066	m, structure length	For 5 dB Attenuation
tau	0.34906585	attenuation parameter	Optimum Difference Voltage at 195 cells Difference = 0.472 x Vmax = 10.1MV
aparm	0.70794578	voltage reduction factor	-0.87 dB in structure
rshunt	1.32E+07	Ohm	
vmax	6.60E+06	Volts, max voltage, ideal case	
vmax_actual	3.35E+06	V, at optimal phase	
vmin_actual	2.54E+05	V, at -180 deg	
vdiff	3.10E+06	V, voltage variation versus 180 deg phase knob	
gmin	4.17E+07	V/m at z=0, gradient (opt phase)	
gmax	5.89E+07	V/m at z=L, gradient (opt phase)	
gavg	5.08E+07	V/m, average (opt phase)	
powerin	8.21E+05	W, power at z=0	
powerout	1.64E+06	W, power at z=L	
ekilpatrick	2.38E+08	V/m, Kilpatrick	
ephenom	4.29E+08	V/m, Phenomenological, 0.1 microsec pulse	

Breakdown

Given that the pulse length for this experiment will be much longer than the natural fill-time for a W-Band travelling wave structure, it is good to revisit the phenomenology we have been adopting for breakdown. This is simply the empirical fit

$$E_{br} \approx E_K \left(1 + \frac{4.5}{T_p (\mu s)^{1/4}} \right) \approx 1.8 E_K,$$

with T_p the pulse length, and E_K the breakdown field according to Kilpatrick's criterion for a continuous wave (CW) pulse,

$$E_K \approx 25 \frac{MV}{m} \sqrt{f(GHz)} \approx 240 MV / m.$$

It should be recognized too that for power tubes, breakdown thresholds are known to be lower. According to this criterion, we might expect a hard limit in the range of 430MV/m. For a reasonable ratio of surface field to gradient, and for favorable ring attenuation, it would not be surprising to experience breakdown in our circuit, and to find ourselves pursuing a conditioning cycle for the ring. This suggests that instrumentation of the kind employed on the resonant ring at Test-Stand #4 should be considered (scintillator & PMT, RGA, gated CCD). Note that if one considers, as we have, running at 1A of beam current, it might be possible to reach a hard-limit in the conditioning cycle.

Additional Work

We are pursuing the following additional items along these lines:

- (1)** analysis of the amplification of the backward wave in the presence of multiple reflections in the recirculation arm.
- (2)** effect of structure tuning errors
- (3)** effect of transverse gradients
- (4)** ring layout and error budget based on manufacturer spec for VSWR and loss, or ARDB-measured where available
- (5)** design of a phase-shifter --- this is being pursued by Mikhail Gershteyn and Michael Shapiro, Insight Product Co. These are the folks who do the design work for the MIT mode converters.

Timeline

