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Technical Note: Expected Waveforms, SFF BLM

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Abstract

In this note the time-domain video waveforms are computed for the 4-channel split, crystal-detected South Final Focus Bunch Length Monitor.

Table of Contents

This note will outline the time-domain video waveforms expected from the 4-channel split, crystal-detected SFF BLM output. This treatment will neglect mode-excitation by conversion in the bends along the waveguide network and mode-mixing due to wall-losses (hybrid modes).

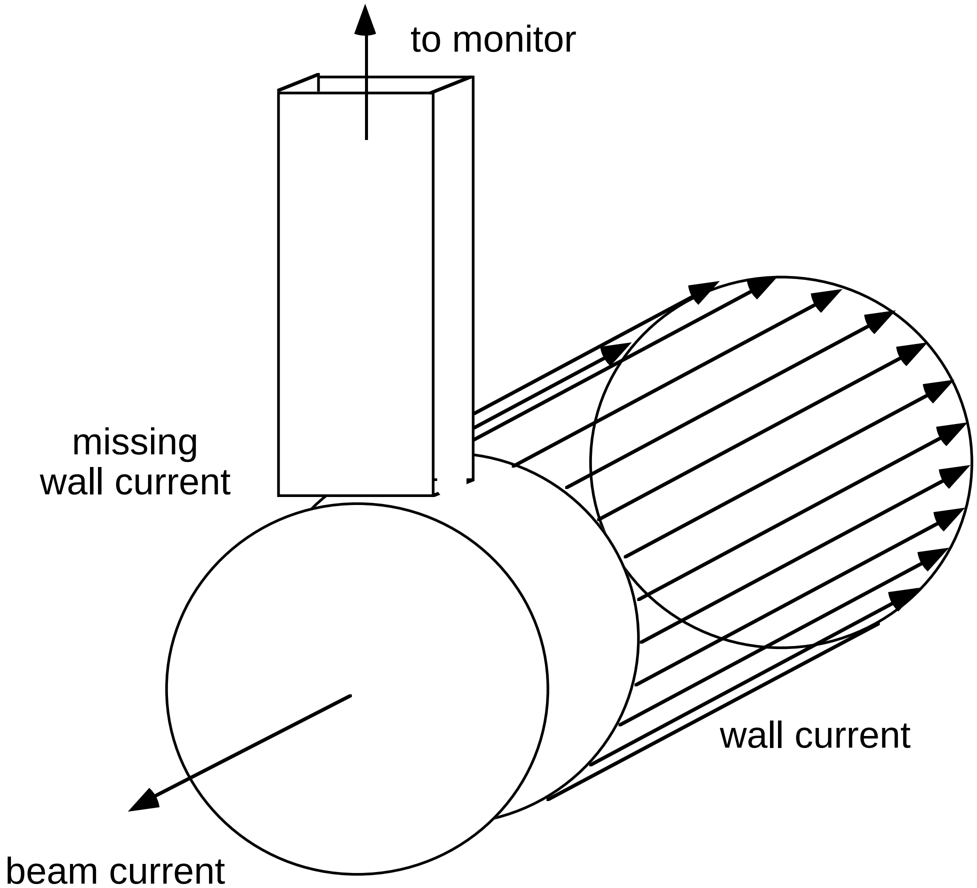

Fig.1 We calculate the voltage driven in each mode due to the virtual current source represented by the missing wall current. The current drive term is treated as uniform across the waveguide face.

Starting from the waveguide aperture situated over the dielectric gap on the beam pipe, we estimate the excitation of each mode according to the amplitude for that mode of the electric field component parallel to the virtual current source corresponding to the "missing" wall current induced by the beam. This current source is approximately uniform across the waveguide aperture, and thus the only modes excited by the beam are those with non-zero integral of parallel electric field across the aperture, when weighted with the transit time factor. The picture is that of Figs. 1 and 2.

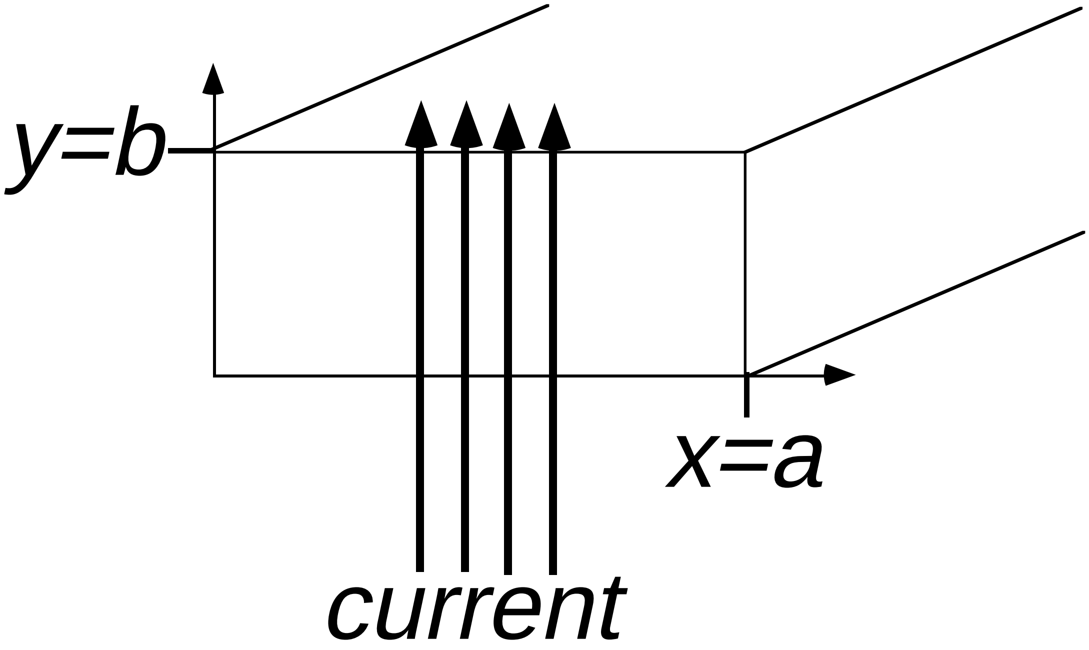

Fig. 2. We are considering WR90 waveguide with width *a*=90/100"=2.286cm, and height *b*=40/100"=1.016cm (inner dimensions, standard precision ±6mils quoted)

We then propagate these signals without mode conversion through a length of waveguide, with the appropriate attenuation and phase-shift for each mode, but neglecting hybridization of modes due to wall losses. The result is summed, filtered, and power-detected in four channels. The result consists of some useful plots, and a simple fortran program one can run to assess other effects, such as changes in waveguide length, other filtering schemes, a variety of bunch distributions, etc.

Our first interest in these calculations is to the relative amplitude of each guide mode, and the expected waveforms at the monitor location upstairs. (The absolute magnitude of the energy radiated has already been estimated, by a simpler method in a previous note, and found to be in good agreement with the MAFIA results.)

Review of Waveguide Modes

Solutions of Maxwell's Equations in smooth empty waveguide may be represented as superpositions of tranverse electric (TE) and transverse magnetic (TM) modes. A mode labelled "a" has cutoff wavenumber β_{ca}, such that

$$\beta_c^2 = \beta_0^2 - \beta^2,$$

where $\beta_0 = \omega/c$ with ω the angular frequency, and β is the axial wavenumber ($2\pi/\beta$ is the guide wavelength). Supposing that we have determined all the modes for our waveguide, and their various cut-off wavenumbers, β_c, let us form a list (an infinite list) of all these modes, and tally them with index "*a*". A general solution for a vacuum oscillation may then be represented as a sum over all these modes

$$\tilde{E}_t = \sum_a E_{\perp a}(\vec{r}_\perp) V_a(z,\omega) ,$$
$$\tilde{H}_t = \sum_a H_{\perp a}(\vec{r}_\perp) I_a(z,\omega) Z_{ca}(\omega).$$

where

$$Z_{ca} H_{\perp a} = \hat{z} \times E_{\perp a} .$$

The characteristic mode imepdance is

$$Z_{ca} = Z_0 \begin{cases} \dfrac{\beta_0}{\beta_a} & TE \text{ mode} \\ \dfrac{\beta_a}{\beta_0} & TM \text{ mode} \end{cases},$$

and Z_0 is the wave impedance of the medium, Z_0 ˜376.7Ω. We adopt the normalization

$$\int d^2 r_\perp E_{\perp a}(\vec{r}_\perp) \bullet E_{\perp a}(\vec{r}_\perp) = 1,$$ (Slater normalization)

where the integral is over the waveguide cross-section. This implies

$$\int \tilde{H}_{\perp a} \bullet \tilde{H}_{\perp b} d^2 r_\perp = \frac{\delta_{a,b}}{Z_{ca}^2}.$$

We choose the sign of Z_{ca} positive for positive β_a. We have chosen the transverse field components in this normalization to be real. The coefficients *V,I* take the forms

$$V_a(z,\omega) = V_a^+ e^{-j\beta_a z} + V_a^- e^{j\beta_a z},$$
$$Z_{ca} I_a(z,\omega) = V_a^+ e^{-j\beta_a z} - V_a^- e^{j\beta_a z}.$$

For rectangular waveguide, the TE mode transverse electric field is given by,

$$E_y = -E_0 k_x \sin(k_x x)\cos(k_y y),$$
$$E_x = E_0 k_y \cos(k_x x)\sin(k_y y),$$

with Slater norm,

$$E_0^{-2} = \begin{cases} \frac{1}{4}\left(k_x^2 + k_y^2\right)ab \ ; \ k_y \neq 0 \\ \quad \frac{1}{2}k_x^2 ab \qquad ; \ k_y = 0 \end{cases}.$$

Boundary conditions permit wavenumbers,

$$k_x = \frac{n\pi}{a},$$
$$k_y = \frac{m\pi}{b},$$

with n and m integers, and corresponding cutoff

$$\begin{aligned} \beta_{c-nm} &= \left[k_x^2 + k_y^2\right]^{1/2} \\ &= \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^{1/2}. \end{aligned}$$

The corresponding wavenumber is

$$\beta_{nm} = \left[\beta_0^2 - \beta_{c-nm}^2\right]^{1/2} = \frac{2\pi}{\lambda_{g-nm}},$$

where $\lambda_{g\text{-}nm}$ is the guide wavelength for the mode. In terms of these quantities, the norm for the TE_{nm} mode may be expressed as,

$$E_{nm}^{TE} = \left\{\frac{2\varepsilon_{om}}{\left(k_x^2 + k_y^2\right)ab}\right\}^{1/2} = \left\{\frac{2\varepsilon_{om}}{ab}\right\}^{1/2} \frac{1}{\beta_{c-nm}},$$

with ε_{0m}=1 for m=0, and ε_{0m}=2 for m>0.

The TM modes have transverse electric field given by,

$$E_y = E_0 k_y \sin(k_x x)\cos(k_y y),$$
$$E_x = E_0 k_x \cos(k_x x)\sin(k_y y),$$

with wavenumbers as in the TE case, and corresponding TMnm modes --- except that there are no n=0 or m=0 modes. The norm is conveniently given by an expression identical to that for the TE mode

$$E_{nm}^{TM} = \left\{\frac{2\varepsilon_{om}}{ab}\right\}^{1/2} \frac{1}{\beta_{c-mn}},$$

keeping in mind that the case m=0 does not arise for the TM modes.

The modes with lowest cutoff wavenumbers are listed in the following table,

Mode	β_c	$\frac{2\pi c}{\beta_c}$
TE_{10}	$\frac{\pi}{a}$	6.56 GHz
TE_{20}	$\frac{2\pi}{a}$	13.11 GHz
TE_{01}	$\frac{\pi}{b}$	14.75 GHz
TE_{11}, TM_{11}	$\sqrt{\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{a}\right)^2}$	16.14 GHz

A more detailed treatment of modes can be found in Ch.2, Microwave Electronics, of the course notes for *Microwave Linear Accelerators*, Applied Physics 453C.

Coupling to Waveguide Modes

For our purposes this decomposition into waveguide modes is quite convenient. We consider Maxwell's Equations in the guide, with a source term due to a current sheet at the waveguide face (z=0),

$$\vec{J}(y,z,t)=\frac{\delta(z)}{2\pi R}I_b\left(t-\tfrac{1}{c}y\right)\hat{y},$$

with I_b the beam current waveform, and R the pipe radius. (The inner pipe diameter is 1", and the dielectric liner thickness is about 5mm. We will take R~1.5cm). We neglect the curvature of the pipe over the waveguide face, any gap between the waveguide and the pipe, any beam offset, any reflections from the remainder of the geometry (that would amount to an Poynting flux into the guide), and take this source term alone to describe the mechanism of excitation of the waveguide modes. These approximations may be checked with MAFIA.

In the frequency domain we have,

$$\begin{aligned}\tilde{J}(y,z,\omega)&=\int_{-\infty}^{\infty}\frac{d\omega}{\sqrt{2\pi}}e^{-j\omega t}\frac{\delta(z)}{2\pi R}I_b\left(t-\tfrac{1}{c}y\right)\hat{y}\\&=\frac{\delta(z)}{2\pi R}\tilde{I}_b(\omega)e^{-j\beta_0 y}\hat{y}\end{aligned}.$$

The quantity $\tilde{I}_b$ is the Fourier transform of the beam current, *e.g.*, for a Gaussian bunch,

$$\tilde{I}_b(\omega)=Q_b\exp\left(-\tfrac{1}{2}\omega^2\sigma^2\right),$$

with Q_b the bunch charge, and σ the bunch length. We consider Maxwell's Equations in a small volume at the guide entrance, where

$$\tilde{E}_t = \sum_a E_{\perp a}(\vec{r}_\perp) V_a(0,\omega),$$
$$\tilde{H}_t = \sum_a H_{\perp a}(\vec{r}_\perp) Z_{ca}(\omega) I_a(0,\omega).$$

For the present problem, with waveguide excitation at z=0, and propagation to $z=L$ (the length of the waveguide network), we will assume a perfectly matched waveguide network (no reflection from bends --- or ample attenuation of these) so that no reflected signal is present). In this case,

$$V_a(0,\omega) = V_a^+ + V_a^- = V_a^+ = Z_{ca}(\omega) I_a(0,\omega),$$

(*i.e.*, $V_a^- = 0$) and we need solve merely for one phasor, V_a^+, for each waveguide mode. We do this by first dotting Ampere's Law,

$$\vec{\nabla} \times \tilde{H} = j\omega\varepsilon\tilde{E} + \tilde{J},$$

with a waveguide mode electric field,

$$\vec{E}_a^* \bullet \vec{\nabla} \times \tilde{H} = j\omega\varepsilon\vec{E}_a^* \bullet \tilde{E} + \vec{E}_{\perp a}^* \bullet \tilde{J},$$

and making use of a vector identity,

$$\vec{E}_a^* \bullet \vec{\nabla} \times \tilde{H} = \tilde{H} \bullet \vec{\nabla} \times \vec{E}_a^* - \vec{\nabla} \bullet \left(\vec{E}_a^* \times \tilde{H}\right),$$

and a mode relation,

$$\vec{\nabla} \times \vec{E}_a^* = j\omega\mu\vec{H}_a^*.$$

Combining these relations, the result is

$$\tilde{H}\bullet\left(j\omega\mu\vec{H}_a^*\right)-\vec{\nabla}\bullet\left(\vec{E}_a^*\times\tilde{H}\right)=j\omega\varepsilon\vec{E}_a^*\bullet\tilde{E}+\vec{E}_{\perp a}^*\bullet\tilde{J}.$$

This we integrate over the small volume, consisting of the waveguide cross-section, and a depth in *z* overlapping the current sheet at the waveguide face. We make use of the orthogonality properties of modes,

$$\eta_E\delta_{a,b}=\int\tilde{E}_a\bullet\tilde{E}_b^*d^2r=\delta_{a,b}\begin{cases}\frac{\beta_0^2}{\beta_a^2}\ ;TM\\ 1\ ;TE\end{cases},$$

$$\eta_H\delta_{a,b}=\int\tilde{H}_a\bullet\tilde{H}_b^*d^2r=\delta_{a,b}\begin{cases}1\ \ ;TM\\ \frac{\beta_0^2}{\beta_a^2}\ \ ;TE\end{cases},$$

$$\int d^2r_\perp\hat{z}\bullet\left(E_{\perp a}\times H_{\perp b}\right)=\delta_{ab}Z_{ca}^{-1},$$

and we neglect wall-losses in this small region in *z*. We find,

$$j\omega\mu\frac{I_a^+}{Z_{ca}}\eta_H-I_a^+=j\omega\varepsilon\eta_E V_a^++\int dV\,\vec{E}_{\perp a}^*\bullet\tilde{J},$$

or,

$$V_a^+=-Z_{ca}\int dV\,\vec{E}_{\perp a}^*\bullet\tilde{J}.$$

This we make more explicit, taking account of the form of our source term,

$$V_a^+(0,\omega)=-\frac{Z_{ca}\tilde{I}_b(\omega)}{2\pi R}\int_0^a dx\int_0^b dyE_{ya}(x,y)\bullet e^{-j\beta_0 y}.$$

We may abbreviate this as

$$\tilde{V}_a(\omega,0) = R_a \tilde{I}_b(\omega),$$

where R_a has units of Ohms, and gauges the coupling to mode *"a"*,

$$R_a = -\frac{Z_{ca}}{2\pi R}\int_0^a dx \int_0^b dy E_{ya}(x,y) \bullet e^{-j\beta_0 y}.$$

Note the relation to power flowing down the guide, across a plane near z=0,

$$\begin{aligned} P &= \frac{1}{2}\Re\sum_a V_a I_a^* \\ &= \frac{1}{2}\Re\sum_a \frac{|V_a|^2}{Z_{ca}} \\ &= \frac{1}{2}\left|\tilde{I}_b(\omega)\right|^2 \sum_{a'} \frac{|R_a|^2}{Z_{ca}} \end{aligned},$$

where the last sum is over propagating modes only. In the next section, we compute the integral for R_a, for each a.

Evaluation of Overlap Integrals

In this section, we compute the integrals,

$$R_a = -\frac{Z_{ca}}{2\pi R}\int_0^a dx \int_0^b dy E_{ya}(x,y) \bullet e^{-j\beta_0 y}.$$

Note that for any mode we have,

$$E_{ya} \propto \sin(k_x x)\cos(k_y y),$$

with $k_x = \frac{n\pi}{a}$. Thus the integral is non-zero only for TE_{nm} or TM_{nm}, with n odd. This derives from the symmetry of the geometry with the waveguide centered on the pipe, with z along the outward radial direction.

For TE modes, we have

$$\begin{aligned} R_a &= \int_0^a dx \int_0^b dy \left\{ \sin(k_x x)\cos(k_y y) \right\} \bullet e^{-j\beta_0 y} \\ &= \frac{Z_{ca}E_{0a}}{2\pi R} \int_0^b dy \cos(k_y y) e^{-j\beta_0 y} \qquad , \\ &= \frac{Z_{ca}E_{0a}}{2\pi R} \frac{2\beta_0}{k_y^2 - \beta_0^2} \frac{1}{2j} \left\{ (-1)^n e^{-j\beta_0 b} - 1 \right\} \end{aligned}$$

or,

$$R_a = \frac{Z_{ca}E_{0a}}{2\pi R} \frac{2\beta_0}{k_y^2 - \beta_0^2} e^{-j\left(\frac{\beta_0 b}{2}\right)} \times \begin{cases} j\cos\left(\frac{\beta_0 b}{2}\right); \; m \; odd \\ -\sin\left(\frac{\beta_0 b}{2}\right); \; m \; even \end{cases}.$$

We can make this still more explicit, using our expression for the TE mode impedance, and the normalization integral,

$$\frac{R_{nm}^{TE}}{Z_0} = \frac{(2\varepsilon_{om})^{1/2}}{\pi R (ab)^{1/2} \beta_{c-nm}\beta_{nm}} \frac{(\beta_0 b)^2}{(m\pi)^2 - (\beta_0 b)^2} e^{-j\left(\frac{\beta_0 b}{2}\right)} \begin{cases} j\cos\left(\frac{\beta_0 b}{2}\right); \; m \; odd \\ -\sin\left(\frac{\beta_0 b}{2}\right); \; m \; even \end{cases}.$$

For the TM modes the result is

$$R_{nm}^{TM} = \left(\frac{\beta_{nm}}{\beta_0}\right)^2 \left(\frac{m}{n}\right) R_{nm}^{TE}.$$

Signal Transmission Through the Network

The voltage driven in mode *"a"* at point z along the waveguide network takes the form,

$$\tilde{V}_a(\omega, z) = R_a \tilde{I}_b(\omega) \exp(-j\beta_a z - \alpha_a z),$$

where the quantity R_a represents the coupling to the mode at this frequency. The attenuation parameter for the TE_{mn} mode is,

$$\alpha_{nm}^{TE} = \frac{2}{b}\frac{R_s}{Z_0}\frac{\beta_0}{\beta_{nm}}\left\{\left(1+\frac{b}{a}\right)\frac{\beta_{c-nm}^2}{\beta_0^2} + \frac{b}{a}\left(\frac{\varepsilon_{0m}}{2} - \frac{\beta_{c-nm}^2}{\beta_0^2}\right)\frac{n^2ab+m^2a^2}{n^2b^2+m^2a^2}\right\},$$

where

$$R_s = \frac{1}{\sigma\delta} \approx 16.6 m\Omega \sqrt{f(GHz)},$$

for brass. For TM_{nm} modes, the attenuation parameter is,

$$\alpha_{nm}^{TM} = \frac{2}{b}\frac{R_s}{Z_0}\frac{\beta_0}{\beta_{nm}}\frac{n^2b^3 + m^2a^3}{n^2b^2a + m^2a^3}.$$

We represent the filtering of this signal according to

$$\tilde{V}_a(\omega, z) \to \tilde{V}_a^f(\omega, z) = R_a \tilde{I}_b(\omega) F_a(\omega) \exp(-j\beta_a z - \alpha_a z).$$

We will employ simple models for the filtering, neglecting the mode-specific features that are likely present (but have not been characterized), $F_a(\omega) = F(\omega)$.

Signal Pickup and Filtering

We consider a pickup at location z (~130') down the waveguide, and suppose, for illustration, that it couples to

$$\tilde{E}_t\left(x = \tfrac{a}{2}, y = b\right) = \sum_a \eta_a \tilde{V}_a^f(z,\omega),$$

where,

$$\eta_{TE_{nm}} = E_{y-TE_{nm}}\left(x = \tfrac{a}{2}, y = b\right) = (-1)^{m+1}\left(\frac{2\varepsilon_{om}}{ab}\right)^{1/2} \frac{\frac{n\pi}{a}}{\beta_{c-nm}},$$

$$\eta_{TM_{nm}} = E_{y-TM_{nm}}\left(x = \tfrac{a}{2}, y = b\right) = (-1)^{m}\left(\frac{2\varepsilon_{om}}{ab}\right)^{1/2} \frac{\frac{m\pi}{b}}{\beta_{c-nm}}.$$

The signal then takes the form

$$e(t,z) = \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} e^{j\omega t} \tilde{I}_b(\omega) \sum_a \eta_a R_a F_a(\omega) e^{-z\{j\beta_a(\omega) + \alpha_a(\omega)\}}.$$

The effect of the diode and video detection are taken into account simply by performing a low-pass filter on the modulus $|e|^2$

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right) S = \frac{1}{\tau}|e|^2,$$

where the integration time scale τ~1ns.

To calculate the integral for *e* we will employ a fast-Fourier transform. This entails a choice of time step Δ, and a number of points in time *N* (a power of 2). We employ the routine `four1` from Numerical Recipes, called as `four1(data,N,-1)`, where -1 indicates the inverse Fourier transform, and `data(1:2N)` is a real array, with assignments as follows: data(2k-1), data(2k), for $k=1,N$, correspond to the real and imaginary parts of $\tilde{V}(\omega_k, z)$, where $\omega_k=2\pi f_k$, and the frequencies are:

$$f_k = \begin{cases} \dfrac{k-1}{N\Delta} & ;k = 1,2,\ldots,\tfrac{1}{2}N+1 \\ \dfrac{k-1-N}{N\Delta} & ;k = \tfrac{1}{2}N+1 \end{cases}$$

We consider four flat filters: "U" corresponds to F=1 (unfiltered), "Ch1" corresponds to a LP 12 GHz filter, "Ch2" corresponds to a 12-19 GHz BP filter, "Ch3" corresponds to a 21.1 GHz HP filter, and "Ch4" corresponds to a 59.0 GHz HP filter. We may consider later the effect of slighlty more realistic filter functions appropriate to cut-off guide and the like.

[to be continued]