Pulsed-Heating Calculations

The following calculations are based on the temperature-rise equation for zero skin-depth derived by Robert Siemann[1], which is

$$T(x,t) = \frac{1}{\sqrt{\pi}\rho c_\varepsilon} \int_{-\infty}^{t} \frac{dt'}{\sqrt{D(t-t')}} e^{-x^2/4D(t-t')} \frac{dP(t')}{dA} \qquad (1)$$

$$D \equiv k/\rho c_\varepsilon$$

where $\rho \equiv$ density, $c_\varepsilon \equiv$ specific heat, $k \equiv$ thermal conductivity, $dP/dA \equiv$ power per unit area on the surface. This equation assumes a semi-infinite plane at $x = 0$; vacuum for $x < 0$ and metal for $x > 0$. From Siemann's note, we know that that maximum temperature rise will occur on the surface of the metal at a time equal to the pulse length of the input power.

One can check the validity of eq. (1) by comparing it to the one used in Perry Wilson's note[2], which is

$$\Delta T = \frac{R_s}{k} \left(\frac{D \cdot T_p}{\pi}\right)^{1/2} \hat{H}^2 \qquad (2)$$

where $R_s \equiv$ surface resistance of the metal, $T_p \equiv$ pulse-length of the input power, $\hat{H} \equiv$ peak tangential magnetic field at the surface. Eq. (2) assumes a square-pulse, which means

$$\frac{dP}{dA} = \frac{R_s}{2} \hat{H}^2 \qquad 0 < t' < T_p \qquad (3)$$

Putting eq. (3) into eq. (1) and integrating from $t' = 0$ to $t' = T_p$ with $x = 0$ one arrives at eq. (2). Eq. (2) describes the heating on the surface of a cavity wall ignoring the fill-time of the cavity.

It is necessary to include the effects of the fill-time of the cavity and the coupling coefficient β between the cavity and the waveguide coupler in the analysis to receive a more realistic assessment of the pulsed-temperature rise of the cavity walls. It is the purpose of this analysis to determine the optimum β and the optimum pulse-length to use in a pulsed-heating experiment. It is seen from Siemann's note that the heat diffusion depth is an order of magnitude larger than the skin depth of copper at X-band, so one may take the skin-depth to be zero for this analysis. Thus, eq. (1) becomes applicable for this case.

---

[1] Siemann, "Heat Diffusion", ARDB-25

[2] Wilson, notes on pulsed-heating 7/31/96

Fig. 1 shows the heating on the end walls and the side walls of a TE_{011} copper cavity assuming a 1.0 μs pulse-length, an input peak power of 37 MW and a resonant frequency of 11.424 GHz. To facilitate a repeatable experiment, one would like to have the end walls dismountable. According to fig. 1, the optimal length of the cavity for the most heating on the end wall and less heating on the side wall is d = 1.90 cm. The corresponding radius for the resonant frequency of 11.424 GHz is a = 2.21 cm. These dimensions will be used to find an optimum β. The unloaded Q of the cavity is found to be Q = 21890. Other details for the cavity such as cooling and vacuum pumping are yet to be determined.

The power dissipated in the cavity taking into account the cavity fill-time τ and the coupling coefficient β is given by[3]

$$P(t) = \frac{4\beta}{(\beta+1)^2} P_i [1 - \exp(\frac{-t}{2\tau})]^2 \qquad (4)$$

$$\tau = \frac{Q_0}{\omega(1+\beta)}$$

where $P_i \equiv$ input peak power, $Q_0 \equiv$ unloaded Q, $\omega \equiv$ angular resonant frequency. Putting eq. (4) into eq. (1) with x = 0, since the largest temperature rise occurs at the surface, and normalizing the power per unit area to 1 W/m^2 gives

$$\frac{T(0,t)}{dP_i/dA} = \frac{4\beta}{(\beta+1)^2} \frac{1}{k} (\frac{D}{\pi})^{1/2} \int_{-\infty}^{t} \frac{dt'}{\sqrt{t-t'}} [1 - \exp(\frac{-t'}{2\tau})]^2$$

As shown in Siemann's note, the largest temperature-rise occurs at a time equal to the pulse-length T_p. Integrating this equation from t' = 0 to t' = T_p will give the desired maximum temperature rise per unit power per unit area on the surface of the cavity wall

$$\frac{T(0,T_p)}{dP_i/dA} = \frac{4\beta}{(\beta+1)^2} \frac{1}{k} (\frac{D}{\pi})^{1/2} \int_{0}^{T_p} \frac{dt'}{\sqrt{T_p-t'}} [1 - \exp(\frac{-t'}{2\tau})]^2 \qquad (5)$$

[3] Padamsee and Knobloch, "Issues in Superconducting RF Technology", USPAS 1/96

Eq. (5) can be numerically solved, for example, by using the routine 'quad8' in MATLAB, but the singularity at t' = T_p causes some inaccuracy. This equation, however, may be reduced to terms involving Dawson's integral F(x)

$$\frac{T(0,T_p)}{dP_i/dA} = \frac{4\beta}{(\beta+1)^2}\frac{1}{k}\left(\frac{D}{\pi}\right)^{1/2}\left[2\sqrt{T_p} - \sqrt{32\tau}\,F\left(\sqrt{T_p/2\tau}\right) + 2\sqrt{\tau}\,F\left(\sqrt{T_p/\tau}\right)\right] \tag{6}$$

$$F(x) = \exp(-x^2)\int_0^x \exp(t^2)\,dt$$

Dawson's integral can be computed using the IMSL routine DAWS or the function supplied in Numerical Recipes in C.

For copper, k = 391 W/m-K, ρ = 8.95x10^3 kg/m^3, c_ε = 385 J/kg-K, D = 1.135x10^{-4} m^2/s. The results for substituting these values into eq. (6) for various pulse-lengths is given in fig. 2. From the plot in fig. 2, one sees that the optimum pulse length is 1.5 µs and the optimum coupling coefficient is β = 1.2 which is close to critical coupling.

The temperature rise is found by multiplying eq. (3) with eq. (6). The tangential magnetic field H can be found knowing the input peak power and β. The corresponding temperature rise on the end and side walls of the cavity is plotted in fig. 3 for various peak powers.

Peak input power = 37 MW
Pulse Width = 1 μs

Solid line - End Wall
Dotted line - Side Wall

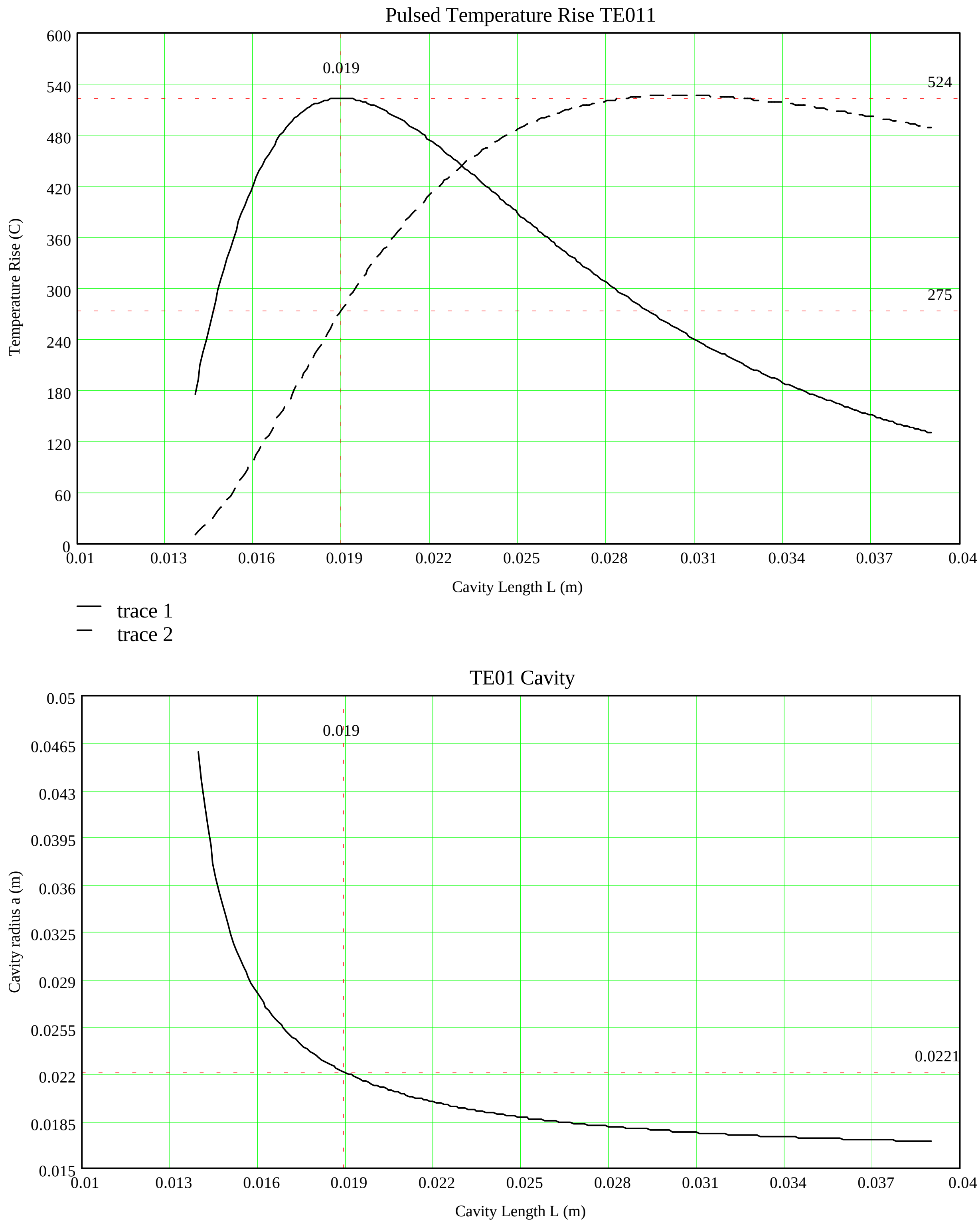

FIGURE 1

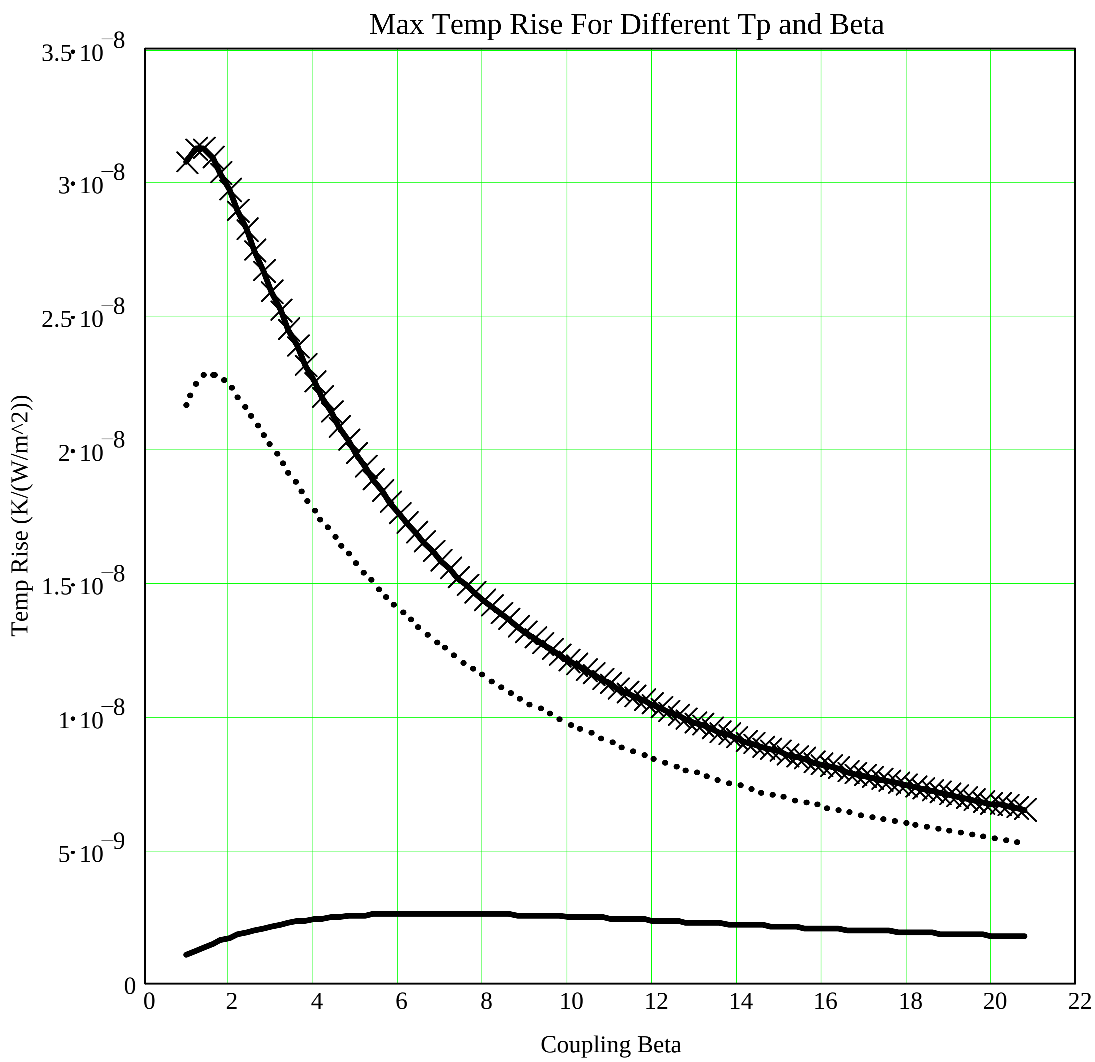

FIGURE 2

X's - 1.5 μs pulse length
Dotted line - 1.0 μs pulse length
Solid line - 150 ns pulse length

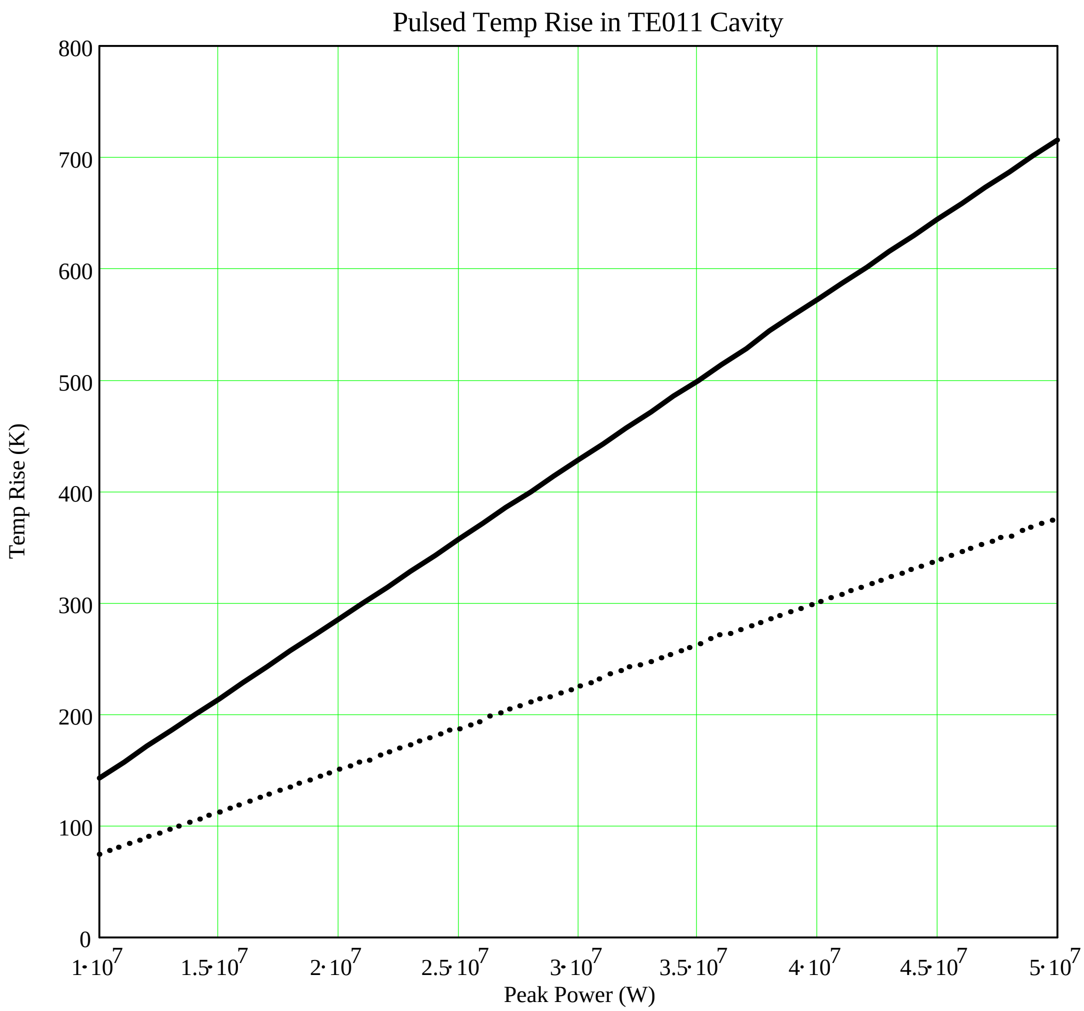

FIGURE 3

Solid Line - End Wall
Dotted Line - Side Wall