

Notes on "Proposed Structure for a Crossed-Laser Beam, GeV per Meter Gradient, Vacuum Electron Linear Accelerator"

Summary of Article

This article by Y. C. Huang, D. Zheng, W. M. Tulloch and R. L. Byer is published in Applied Physics Letters **68** 753 (1996). The accelerator geometry is shown on the right. If Beam 1 is the beam propagating from bottom to top, the transverse coordinates in the frame of the laser beam (x_1 and z_1) in terms of the coordinates in the frame of the electron beam are

$$x_1 = x \cos \theta - z \sin \theta$$

$$z_1 = x \sin \theta + z \cos \theta$$

and the x and z components of the electric field in the electron beam frame, in terms of the components in the laser beam frame are

$$E_z = -E_{x1} \sin \theta + E_{z1} \cos \theta$$

$$E_x = E_{x1} \cos \theta + E_{z1} \sin \theta$$

Davis (see below) gives a relation between the x - and z -components of the fields at a laser focus. Using it and the small angle approximation gives $E_x = E_{x1}$ and

$$\begin{aligned} \frac{E_z}{E_{x1}} &= -\theta + \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \frac{x_1}{z_r} = -\theta + \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \left(\frac{x}{z_r} - \frac{z\theta}{z_r} \right) \\ &= \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \frac{x}{z_r} - \theta \left(1 + \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \frac{z}{z_r} \right) \\ &= \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \frac{x}{z_r} - \frac{i\theta e^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} = \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \left(\frac{x}{z_r} - \theta \right) \end{aligned}$$

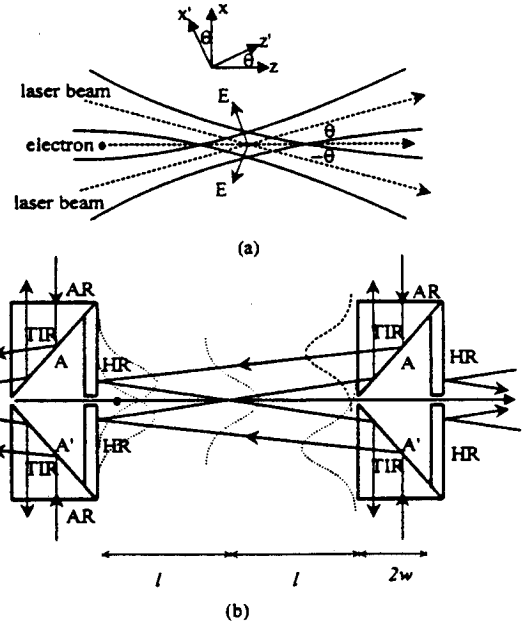
The coordinate and field transformations for Beam 2, the one propagating from top to bottom, are

$$x_2 = x \cos \theta + z \sin \theta \quad \text{and} \quad E_x = E_{x2} \cos \theta - E_{z2} \sin \theta$$

$$z_2 = -x \sin \theta + z \cos \theta \quad \text{and} \quad E_z = E_{x2} \sin \theta + E_{z2} \cos \theta$$

The fields in the small angle approximation are $E_x = E_{x2}$ and

$$\frac{E_z}{E_{x2}} = \frac{ie^{i\phi_g}}{\sqrt{1+(z/z_r)^2}} \left(\frac{x}{z_r} + \theta \right)$$



The laser beam fields are calculated using the results of Davis. The results for an electron of energy γ are

$$E_{x1} = E_1 e^{i\phi_p} \frac{1}{\sqrt{1+(z/z_r)^2}} \exp(i(\phi_g + \phi_{r1})) \exp\left(-\frac{(x-z\theta)^2 + y^2}{w^2}\right)$$

$$E_{x2} = E_2 e^{i\phi_p} \frac{1}{\sqrt{1+(z/z_r)^2}} \exp(i(\phi_g + \phi_{r2})) \exp\left(-\frac{(x+z\theta)^2 + y^2}{w^2}\right)$$

where

$$\phi_g = \tan^{-1}(z/z_r); \phi_p = \frac{kz}{2} \left(\theta^2 - \frac{1}{\gamma^2} \right); \quad w^2 = w_0^2 \left(1 + z^2/z_r^2 \right);$$

$$\phi_{r1} = -\frac{z}{z_r} \frac{(x-z\theta)^2 + y^2}{w^2}; \quad \phi_{r2} = -\frac{z}{z_r} \frac{(x+z\theta)^2 + y^2}{w^2};$$

Putting $E_2 = -E_1$ cancels the x-component of the net field and gives a net accelerating component. Setting $x = y = 0$ gives eq. (2) in Huang *et al.*

The parameters and dimensions of the structure they consider in detail are:

$\theta = 40$ mrad	$\lambda = 1$ μm	$l = 100$ μm (see figure for definition)	$w = 17$ μm
stage length = $2l + 2w = 334$ μm	$z_r = 908$ μm	$w_0 = 17$ μm	waist size at reflectors = 17.1 μm
beam hole size ~ a few μm	gradient (G) = 0.7 GeV/m	ΔE per stage = 0.24 MeV	

Comments

CHARGE LIMIT & BUNCH LENGTH: The loss factor for a short Gaussian bunch passing from a pipe of radius b through an iris of radius a is*

$$k = \frac{Z_0 c}{\pi^{3/2} \sigma_z} \ln(b/a)$$

Using the approximate value $\sigma_z \sim 0.01\lambda$ ($= 10^{-8}$ m) and estimating $b/a = 10$ gives $k \sim 5 \times 10^6$ V/pC. The energy loss per stage equals the energy gain for a charge of 0.05 pC = 3×10^5 electrons. Any bunch would have to have much smaller charge.

A subsequent paper "A Proposed High Gradient Laser-driven Electron Accelerator using Crossed Cylindrical Laser Focusing" by Y. C. Huang and R. L. Byer switches from spherical to cylindrical optics. The beam geometry becomes planar, and the charge limit should be substantially higher.

VARIATION OF GRADIENT ACROSS THE BUNCH: The acceleration in Gaussian in the transverse positions with the RMS of the Gaussian roughly equal to $w_0/\sqrt{2}$ when the Rayleigh length is greater than the stage length. Small variation of gradient across the bunch requires that the beam size be $< w_0/10$.

* P. B. Wilson, SLAC-PUB-4547 (Jan., 1989)

DEFLECTING FIELDS: The acceleration comes the longitudinal field in the focus of a laser beam and a projection of the transverse fields onto the beam axis. Each laser beam has a deflecting component that is $\sim 1/\theta$ times the accelerating component, and the beams have to be out of phase to give no net deflection on axis. There can be deflections of particles offset from the axis and from errors - a phase error with the beams not 180° out of phase or unequal amplitudes.

Just as in any accelerator there is a phase shift, $\exp(i(\phi_g + \pi/2))$, between the longitudinal and transverse fields. This phase shift is 90° for distances much shorter than the Rayleigh length, so if the bunch is near the crest of the accelerating wave, it is near the zero crossing in the deflecting force.

For the case of no laser errors but particles off axis in the x-direction the deflecting field is

$$\frac{dp_x}{dt} = qx \left(\frac{\partial E_{x1}}{\partial x} + \frac{\partial E_{x2}}{\partial x} \right) \Big|_{x=0} = \frac{2qxz\theta}{w^2} (E_{x1} - E_{x2}) \approx \frac{2qxz}{iw^2} E_z$$

where it was assumed that the structure is much less than the Rayleigh length in the last step. There is no net deflection for ballistic trajectories; the deflection on one side of the focus are canceled by the deflection on the other side.

Making the approximation that the acceleration is uniform, i.e. $E_z \approx G$, and that the 90° phase shift can be accounted for with a factor σ_z/λ , the synchrotron radiation power due to the transverse fields is

$$P_\gamma \approx \frac{8r_e c}{3mc^2} \gamma^2 (qG)^2 \left(\frac{xz}{w^2} \right)^2 \left(\frac{\sigma_z}{\lambda} \right)^2$$

The ratio of synchrotron radiation loss to energy gain traversing one cell is

$$\frac{\Delta E_{SR}}{\Delta E_{Acc}} = \frac{\Delta E_{SR}}{2lqG} = \frac{8r_e}{9mc^2} \gamma^2 qG \left(\frac{x}{w} \right)^2 \left(\frac{1}{w} \right)^2 \left(\frac{\sigma_z}{\lambda} \right)^2$$

For the sample parameters in the table above and $\sigma_z/\lambda = 0.01$

$$\frac{\Delta E_{SR}}{\Delta E_{Acc}} = 1.2 \times 10^{-14} \gamma^2 \left(\frac{x}{w} \right)^2$$

This could introduce energy spread and/or energy limits, but these seem negligible. For example, a $\gamma = 10^6$ particle at $x/w = 0.1$ would lose 10^{-4} of its energy gain to synchrotron radiation.

The next steps are to do jitter and emittance growth calculations to determine tolerances on laser intensity and phase (June 17, 1996)

USE OF FREE SPACE EXPRESSIONS FOR THE FIELDS: The calculation is done using free space fields and ignoring the change in the field pattern due to the irises. Yen-Cheih Huang has recently performed a calculation for a structure with apertures. He finds a 15 - 20% reduction in energy gain for apertures with $a/\lambda = 2 - 3$.

Summary of Reference 9 - L. W. Davis, Phys Rev A, 19, 1177 (1979).

Davis solves the equation for the focus of a laser beam by writing the vector potential as

$$A(\vec{r}) = e^{i(\omega t - kz)}\Psi(\vec{r})$$

Ψ is expanded in even powers of kw_0 where w_0 is the laser waist size. The lowest order solution is

$$\Psi_0 = \frac{1}{\sqrt{1+(z/z_r)^2}} \exp(i \tan^{-1}(z/z_r)) \exp\left(-\frac{x^2+y^2}{w_0^2 \sqrt{1+(z/z_r)^2}} \exp(i \tan^{-1}(z/z_r))\right)$$

where $z_r = \pi w_0^2/\lambda$ is the optical Rayleigh length. The first term is the Guoy phase advance

$$\phi_g = \tan^{-1}(z/z_r)$$

that arises from the wavefront being described by incoming spherical waves before the focus and by outgoing spherical waves after the focus. Rewriting the second term

$$\begin{aligned} \Psi_0 &= \frac{1}{\sqrt{1+(z/z_r)^2}} \exp(i\phi_g) \exp\left(-\frac{x^2+y^2}{w_0^2 \sqrt{1+(z/z_r)^2}} (\cos \phi_g + i \sin \phi_g)\right) \\ &= \frac{1}{\sqrt{1+(z/z_r)^2}} \exp(i\phi_g) \exp\left(-\frac{x^2+y^2}{w_0^2 \sqrt{1+(z/z_r)^2}} \left(\frac{z_r}{\sqrt{z_r^2+z^2}} + i \frac{z}{\sqrt{z_r^2+z^2}}\right)\right) \\ &= \frac{1}{\sqrt{1+(z/z_r)^2}} \exp(i(\phi_g + \phi_r)) \exp\left(-\frac{x^2+y^2}{w^2}\right) \end{aligned}$$

where

$$w^2 = w_0^2 \left(1 + (z/z_r)^2\right)$$

is the waist size a distance z from the focus and

$$\phi_r = -\frac{z}{z_r} \frac{(x^2+y^2)}{w^2}$$

is the radial phase due to the curvature of the wavefronts.

In terms of Ψ_0 , the x- and z-components of the electric field are

$$\begin{aligned} E_x &= -ik\Psi_0 \exp(i(\omega t - kz)) \\ E_z &= \frac{i}{\sqrt{1+(z/z_r)^2}} e^{i\phi_g} \frac{x}{z_r} E_x \end{aligned}$$